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A STUDY OF COMBINED BENDING AND AXIAL LOAD IN REINFORCED CONCRETE MEMBERS

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ABSTRACT

This bulletin presents the methods and results of an experimental and analytical investigation undertaken in an attempt to throw new light on the behavior of reinforced concrete members subject to combined bending and axial load. It is the purpose of this report to describe observations regarding the basic behavior of such members and to develop mathematical expressions for ultimate loads.

A total of 120 column specimens were tested, 90 of which were 10-in. square tied columns with 1.46 to 4.8 percent reinforcement, and 30 were 12-in. cylindrical spiral columns with 4.25 percent longitudinal reinforcement. The concrete quality was varied from 1500 to 5500 p.s.i., and the eccentricity of loading varied from 0 to 1¼ times the lateral dimension of the columns.

An inelastic flexural theory was developed, by means of which the behavior of the test columns may be explained and the measured ultimate loads may be predicted with a satisfactory accuracy.

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The standard theory, on the other hand, became so widely used that the approximative character of this theory was forgotten, and applications beyond its range of validity resulted. When beams were designed with an allowable concrete compressive stress, f_c, equal to

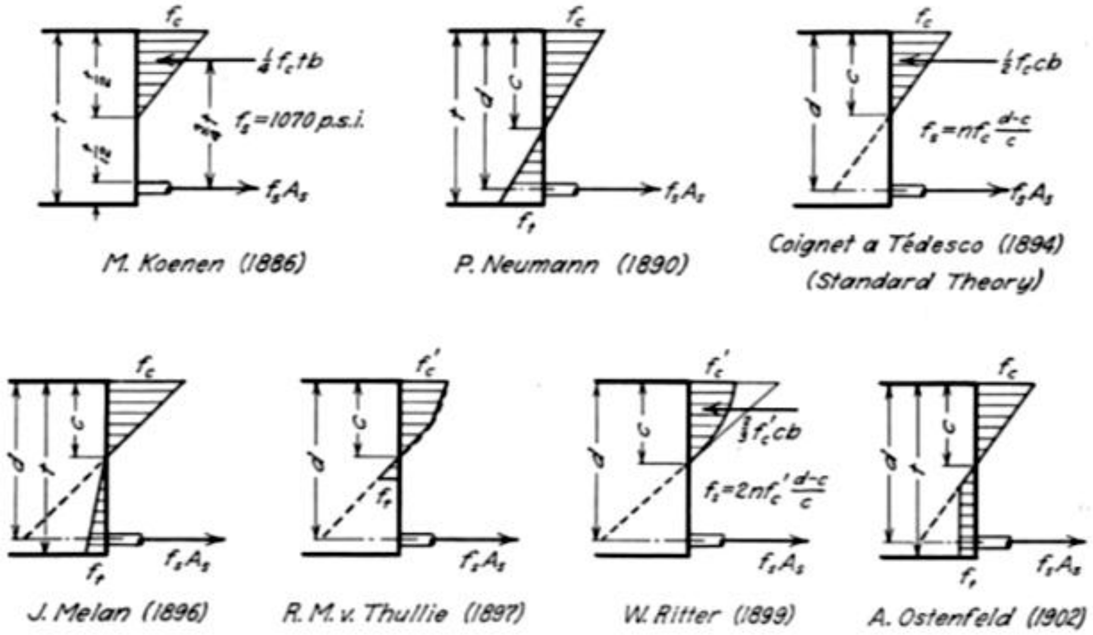

Fig. 1. Early Assumptions in Flexural Analysis

0.325 times the strength of 6- by 12-in. cylinders, f_c', it was at times erroneously concluded that the safety factor against a compression failure was near three. In the period from 1920 to 1930, Slater, Zipprodt and Lyse made valuable contributions in pointing out that the safety factor in the case mentioned above generally is considerably larger than that indicated by the ratio f_c'/f_c, thus re-emphasizing the actual inelastic behavior of concrete.(16, 24)

Another important development took place in the decade from 1920 to 1930. Before that time, bending stresses were generally neglected in the design of concrete building columns, and such stresses were assumed to be provided for, together with other effects, in an all-inclusive factor of safety. This neglect was mainly due to the lack of suitable methods for structural analysis of monolithic structures. Two major methods were developed to meet this lack of information. The Slope-Deflection Method appeared in 1918,(15a) and the Moment Distribution Method followed.(23a, 35a) The moments of inertia used in these methods have generally been based on the uncracked section. Recent extensive tests of structures(65, 69a) have shown that this application of P. Neumann's 60-year-old theory (Fig. 1) is satisfactory for the structural analysis of indeterminate reinforced concrete structures at working loads if proper values of the modulus ratio are chosen.

About 1900, centrically loaded reinforced concrete columns were generally designed after the following formula for the allowable load:

$$P = f_c A_c + f_s A_{st} \tag{1}$$

where f_c and f_s were allowable stresses, and A_c and A_{st} were areas of concrete and steel, respectively. Later the standard theory was used, and the transformed area formula resulted

$$P = A_g f_c [1 + (n - 1) p_{st}] \tag{2}$$

in which p_{st} is the ratio of effective longitudinal reinforcement area to the gross area of concrete, and n is the modular ratio.

In 1921, McMillan made a study of column test data[17] which showed that building columns under load may develop steel stresses, due to plastic action, considerably higher than those predicted by the standard theory, Eq. (2). This study led to the ACI Column Investigation in the 1930's which was carried out by Lyse, Slater, and Richart. Through their work, rational equations for the strength of centrically loaded reinforced concrete columns were developed.[23, 25, 26, 27, 28, 30, 31, 32, 33, 36, 40, 43] The ultimate load of tied columns and the yield point of spiral columns were expressed as

$$P = 0.85 f_c' A_c + f_{yp} A_{st}. \tag{3}$$

For the ultimate load of spiral columns, the following equation was developed

$$P = 0.85 f_c' A_{core} + f_{yp} A_{st} + 2.0 p_{sp} A_{core} f_{sp} \tag{4}$$

in which

f_{yp} = yield point stress of longitudinal reinforcement

f_{sp} = useful limit stress of spiral reinforcement, generally assumed as the stress at a strain of 0.005

A_{core} = area of the column core (out-to-out of spiral)

p_{sp} = ratio of volume of spiral reinforcement to the volume of concrete core.

Following the publication of the results of this investigation, an inelastic design formula of the same form as Eq. (1) became much used in this country as well as in several countries abroad. The basic formula of the 1947 ACI Building Code[93] may be written

$$P = 0.225 f_c' A_c + 0.4 f_{yp} A_{st}. \tag{5}$$

This design equation is, however, characterized by a larger "factor of safety" for the concrete than for the steel. Furthermore, the inelastic design is used only for columns loaded centrically or with small eccentricities. Beams and columns with large eccentricities are still being designed after the standard theory. In beams, however, the inelastic properties of concrete are recognized to some extent by allowing the effectiveness of compression reinforcement in resisting bending to be taken as twice the value obtained from the standard theory.

Another milestone of progress in the theory of reinforced concrete was passed in 1931 when Emperger wrote a critical study of the modular ratio and the allowable stresses.[29] This paper initiated intense studies of the ultimate strength of reinforced concrete beams in bending, which soon spread over the world. A large number of ultimate theories, some of which are described in detail in Section 10, were developed.

In recent years it has been claimed repeatedly that our knowledge of the entire field of reinforced concrete design has advanced so far that a transition to inelasticity rather than elasticity and to ultimate loads rather than working stresses is necessary in order to continue progress. It has also been argued that equal factors of safety should be used for concrete and steel, and that different safety factors should be used for live and dead loads. A transition to ultimate design has been made in some countries such as USSR and Brazil, and several European authors have claimed that the ultimate theories are "ripe for the specification form now."

There are two major phases involved in an ultimate design of reinforced concrete structures: (1) the structural analysis of indeterminate structures, and (2) the process of dimensioning sections. In the past, the major interest has been focused on the dimensioning problem. Hence, the ultimate design methods adopted in USSR and Brazil, while specifying dimensioning procedures based on ultimate loads, have still maintained the theory of elasticity for most purposes of structural analysis. In recent years, some studies have been devoted to the inelastic analysis of reinforced concrete structures. Examples of such studies are the Fracture Line Theory for slabs,[99a] the Stringer Theory for cylindrical shells,[110b] and the Method of Partial Restraints for continuous beams.[39] The present Danish specifications for reinforced concrete structures[110a] permit an inelastic analysis of indeterminate structures, while the dimensioning methods are based on the standard theory.

In this country it is generally felt, however, that the ultimate dimensioning theory is worthy of primary interest. For concentrically loaded columns, and beams failing in bending, this theory has been rather well established as the result of a large number of tests. Our knowledge of some other types of members, among them eccentrically loaded columns, is incomplete because tests made on such members of large dimensions are too few to be conclusive.

Thus, the investigation reported herein was undertaken in order

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bent around the longitudinal bars as shown to the left in Fig. 5a. In the cases of large eccentricities, however, a heavy reinforcement welded to the longitudinal bars was necessary to prevent tension, diagonal tension, or bond failures in the capitals. Such a reinforcement unit is shown to the right in Fig. 5a. Except for the top capital of one column which failed in bond, all capitals proved to be satisfactory, as they were perfectly sound after the prismatic part of the columns had failed.

The columns were cast in a vertical position. Some previous investigators[44, 51, 94] cast their specimens horizontally in order to avoid a differential in concrete quality along the column length. Such a differential

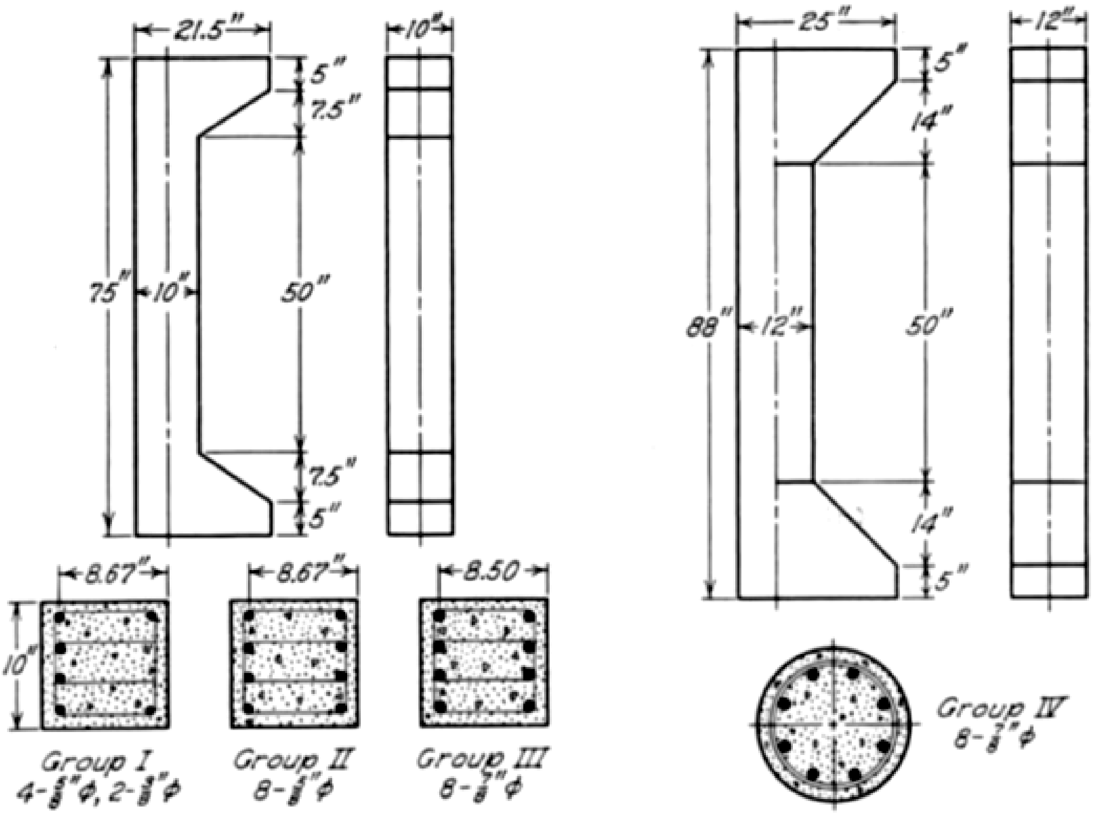
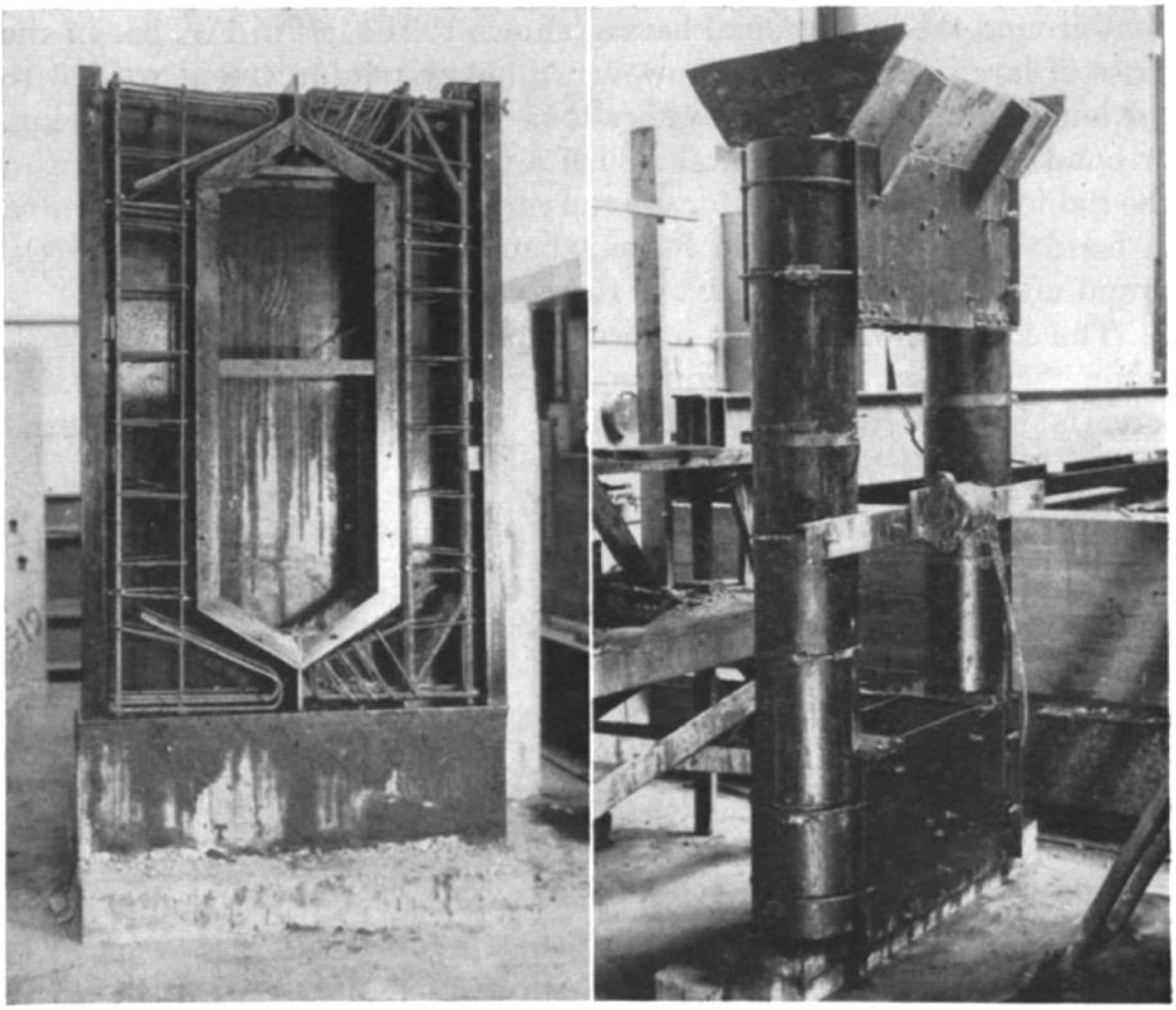


Fig. 4. Types of Specimens

will exist in a vertically cast column because the concrete in the bottom part generally will be compacted better than in the top part. Also, even a moderate water gain or bleeding will decrease the c/w ratio and hence the strength of the concrete in the upper part. Horizontal casting, on the other hand, will cause a strength differential across the cross section of the columns.

Furthermore, the orientation of the members during casting will determine whether the longitudinal stresses during testing will be parallel or perpendicular to the direction of casting. Since all columns tested at



(a) (b)

Fig. 5. Types of Forms

this laboratory in earlier investigations[22, 43, 95] were cast vertically, since most columns in practice are cast vertical or inclined, and since it is very difficult to cast cylindrical columns horizontally, all columns described herein were cast and tested in a vertical position.

A view of the two types of steel forms is given in Fig. 5a and 5b. It can be seen that the columns were cast in pairs in forms built from plates, shapes, and split pipes bolted together. The lateral dimensions of the prismatic parts of the columns were found to be true to ± 0.1 in. at the time of testing. Spacers were provided to keep the reinforcement units centered in the forms with a 1-in. cover over the longitudinal bars of the tied columns, and a 1-in. cover over the spirals of the cylindrical columns. Since a heavy external vibrator was attached to the forms to compact the concrete, this spacing was found to vary about $\pm 3/8$ in. The forms were carefully leveled and plumbed.

In all columns a reasonably fluid consistency of concrete was used to facilitate its placing around the heavy capital reinforcements and to

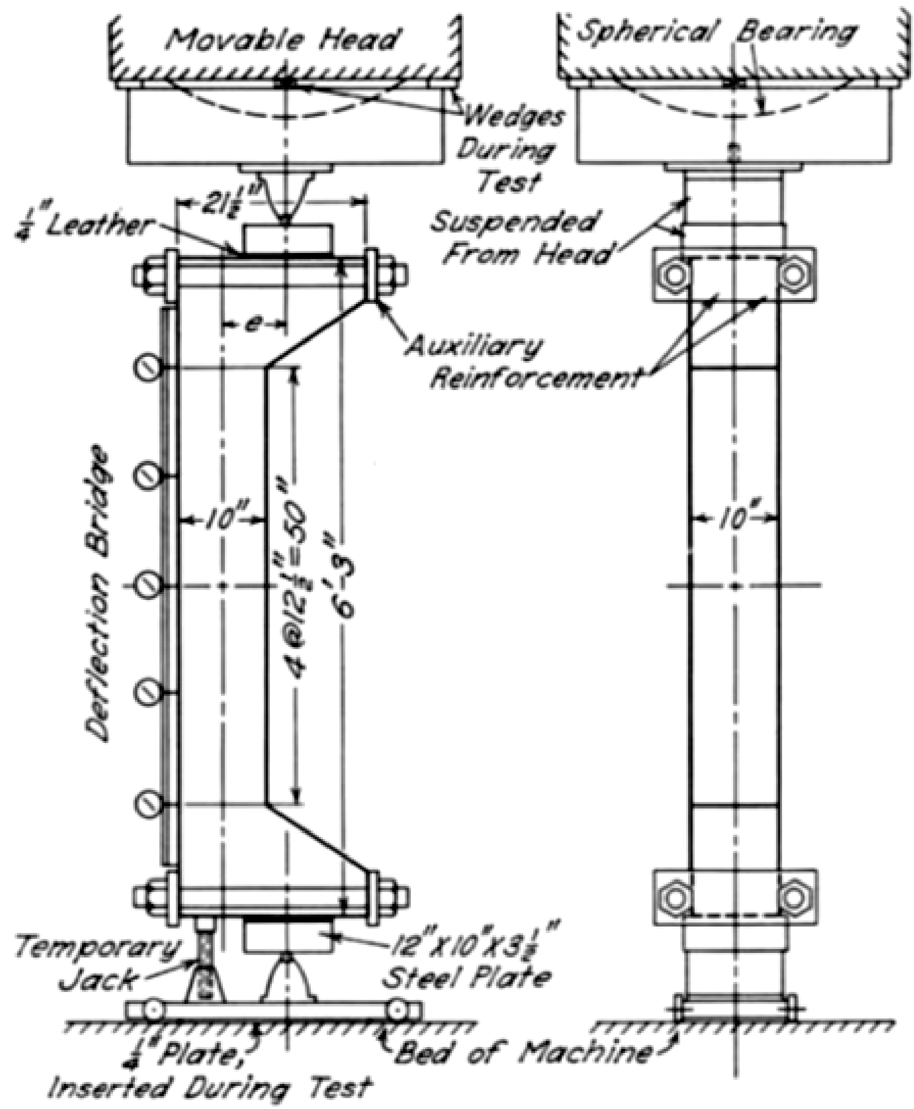


Fig. 7. Testing Arrangement

shows a square column. The bottom knife edge rested on a wheeled carriage, thus permitting easy placing, centering, and removal of the columns in the testing machine. The upper knife edge and base plate were suspended from the movable head of the machine. Two auxiliary tension rods were attached to each capital of the square columns, because it was found impracticable to reinforce these capitals sufficiently with only embedded bar reinforcement. A deflection bridge, carrying five dial indicators, furnished deflection readings in the plane of eccentricity.

The arrangement for testing the spirally reinforced columns was very similar; however no auxiliary rods were used. The concentrically loaded spiral columns were tested with flat ends.

All columns were carefully centered and plumbed in the testing machine. However, since the knife edges were ½ in. thick and since no optical instruments were used to center the columns, the actual eccentricities of loads during testing probably deviated up to ±¼ in. from

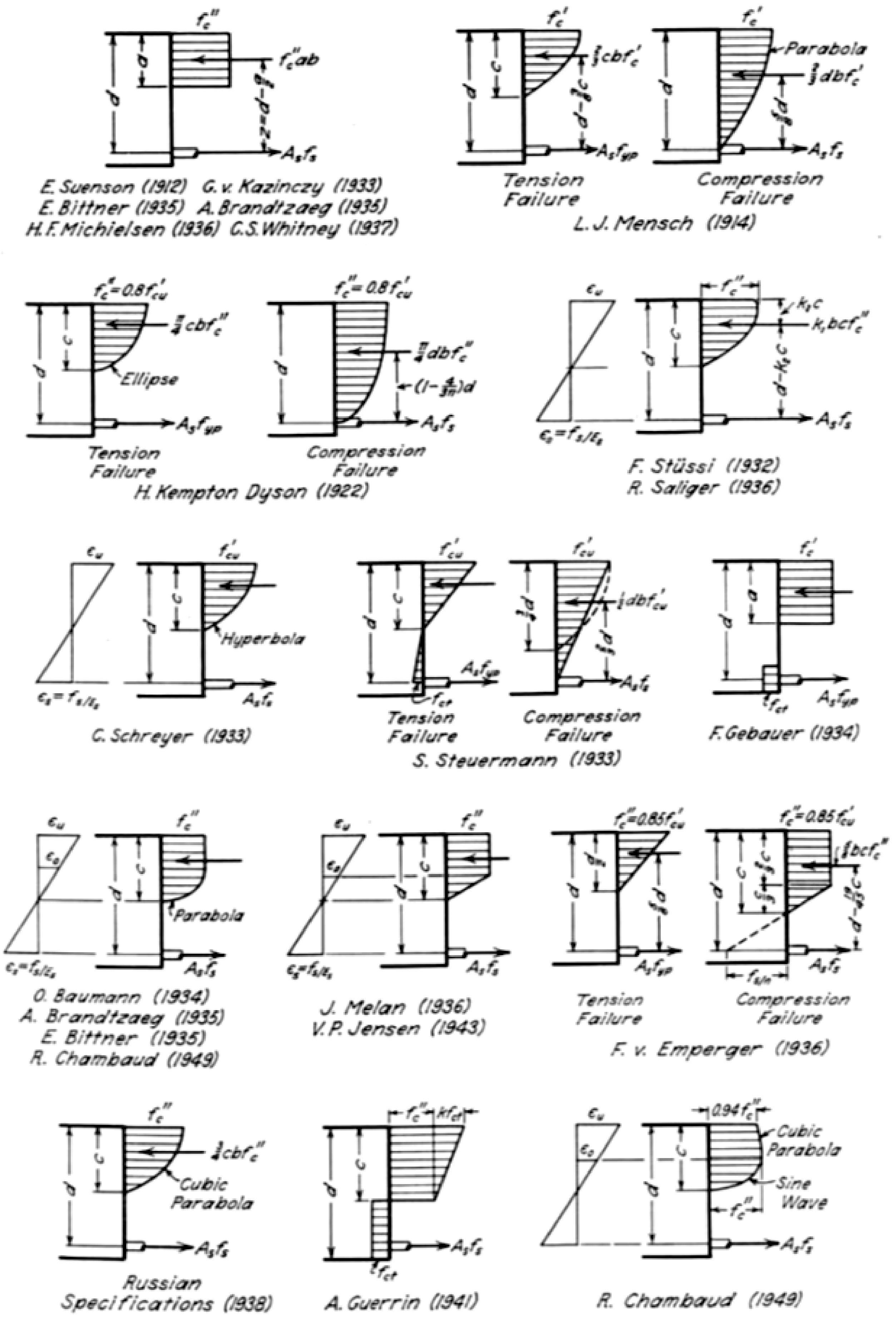


Fig. 8. Assumptions in Flexural Analysis

nized that the quality of concrete in beams and columns cast from the same concrete mixture may differ, since beams generally are cast in a horizontal position while columns are cast vertically.

In later years it has been attempted to measure concrete strength in structural elements directly by means of indentation tests(101) or wave-velocity methods.(103) Since such methods are still in the experimental stage, however, the strength of 6- by 12-in. cylinders, f_c', has been adopted as a measure of concrete strength in the present tests. The maximum stress in flexure, f_c'', corresponding to the column specimens was chosen equal $0.85 f_c'$. This value was found as an average in numerous tests of vertically-cast concentrically loaded columns tested with flat ends.(43) Effects of size and shape of the columns as well as of the casting position is therefore included in the factor 0.85.

It is believed that the initial, curved part of the stress-strain diagram in Fig. 10 is fairly similar to the relation in direct compression. Since the important factors in a flexural analysis related to the stress-strain

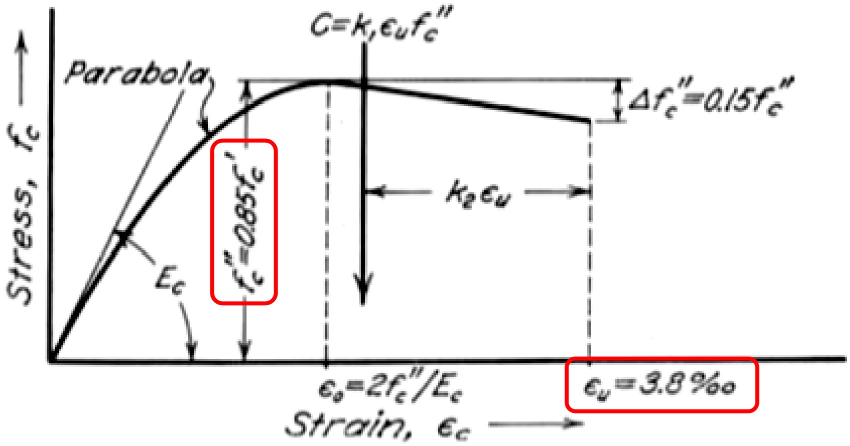


Fig. 10. Assumed Stress-Strain Diagram in Flexure

relation are k_1, k_2 and ϵ_u, small variations in the initial part of the stress-strain diagram are of minor importance. Auxiliary tests of 6- by 12-in. cylinders showed that Ritter's parabola is a good approximation when expressed in the following form

$$f_c = f_c''\left[2\frac{\epsilon}{\epsilon_o} - \left(\frac{\epsilon}{\epsilon_o}\right)^2\right]. \tag{32}$$

With $\epsilon_o = 2f_c''/E_c$ this may be written as

$$f_c = \epsilon E_c\left(1 - \frac{\epsilon E_c}{4f_c''}\right). \tag{33}$$

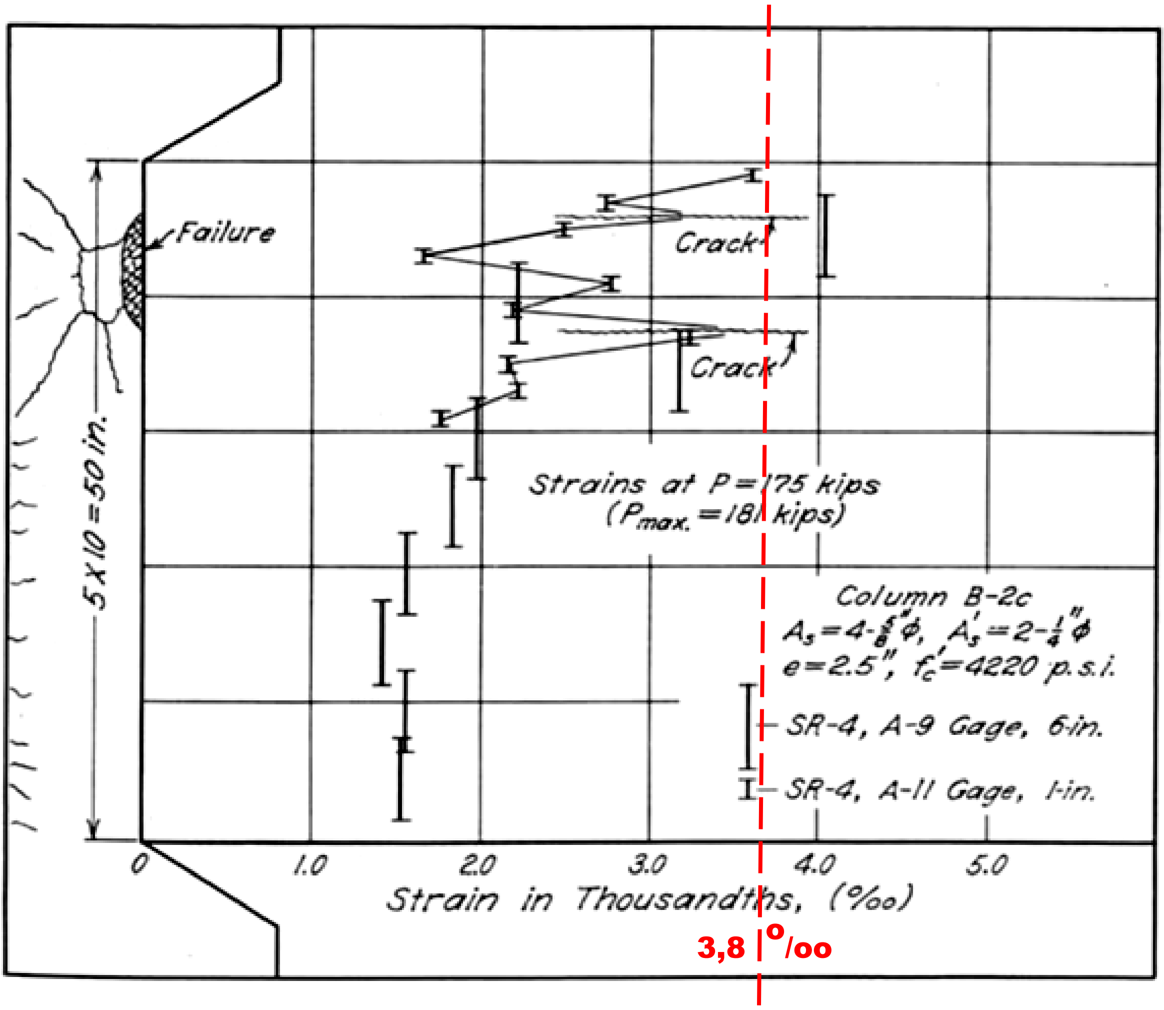


Fig. 12. Distribution of Strains over Height of Column

In order to throw some light on the distribution of strains along the length of the column shaft, an extra column specimen was prepared and tested with a large number of SR-4 gages attached to the compression face. The distribution of strains near the ultimate load is given in Fig. 12. It should be noticed that a general trend of increasing strains exists from the bottom towards the top of the column shaft. Near failure two major compression cracks extending horizontally across the compression face were developed as indicated in Fig. 12. As the final failure took place, the shaded area shown in the figure and marked "Failure" was extruded. Unfortunately, both major compression cracks occurred between two 1-in. gages. Nevertheless it appears that the strain at a given high load is not a well-defined quantity. At least three different quantities of strain may be considered: (1) The average strain over the entire length of the compression face, (2) A strain measured over a reasonably long gage length (6 in.) in the failure region, and (3) A local strain measured over a short gage length (1 in.) at a compression crack. These three quantities will increase in value in the order listed above, and it is believed that near failure No. 3 will be at least twice as large as No. 1.

Fig. 13 represent an extrapolation of strain to a known ultimate load. Ultimate strains obtained in this manner are presented in Fig. 14 for all square columns failing in compression before or shortly after the tension steel reached yielding.

The experimental findings in Fig. 14 are compared with relations between f_c' and ϵ_u given by previous investigators.

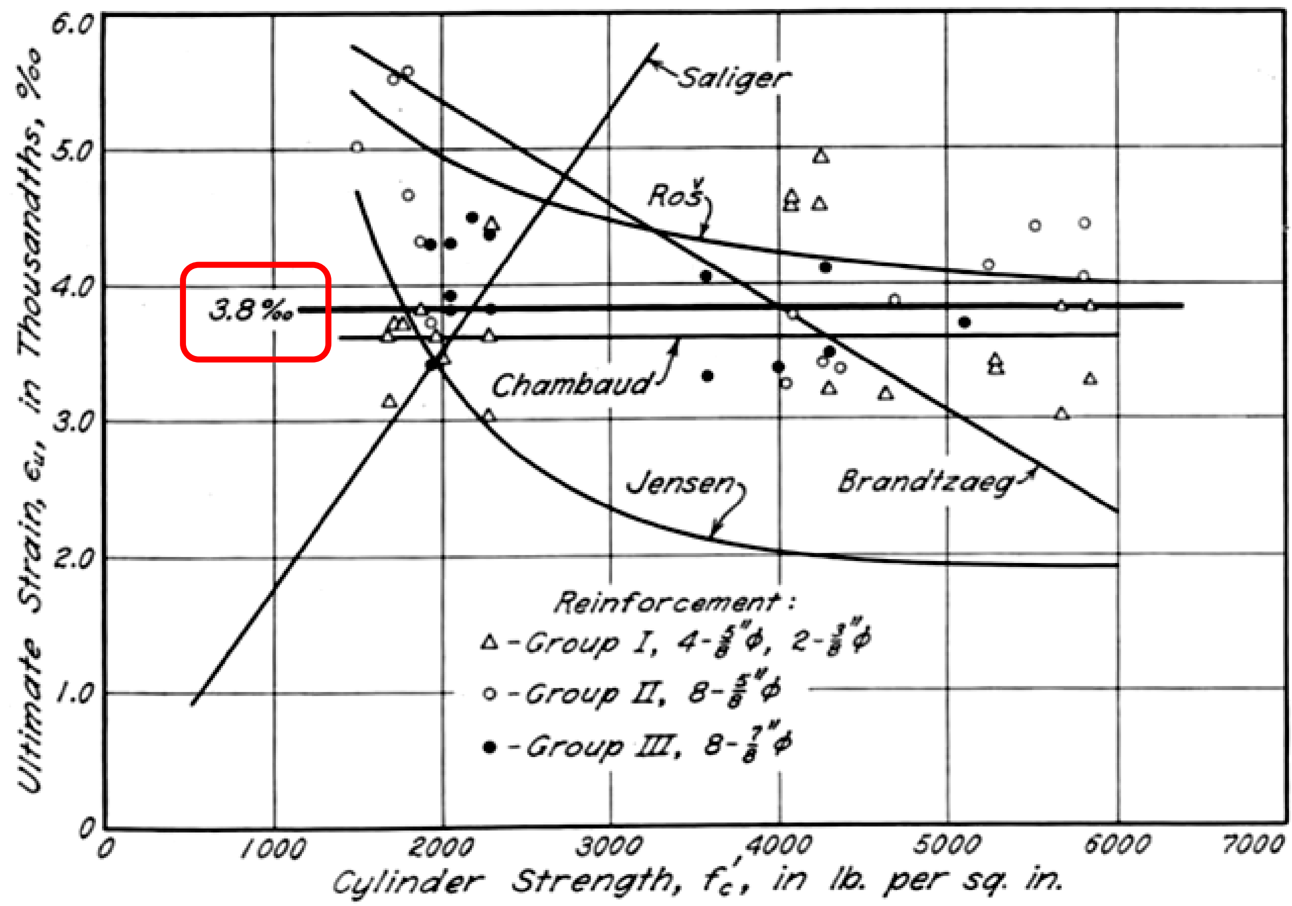


Fig. 14. Ultimate Strains

A. Brandtzæg(51, 56, 57, 61) developed the empirical Eq. (17). R. Saliger(60) reported values of ultimate strain from three to seven per mill and recommended Eq. (18).(99) V. P. Jensen(80, 81) assumed

$$\epsilon_u = \frac{f_c'}{(1 - \beta) E_c}$$

where the plasticity ratio, β, and the modulus of elasticity, E_c, were assumed as functions of f_c' (Eq. 27). M. Roš(78) gave the following equation for the ultimate strain

$$\epsilon_u = (3.5 + 2860/f_c') 10^{-3}. \qquad (35)$$

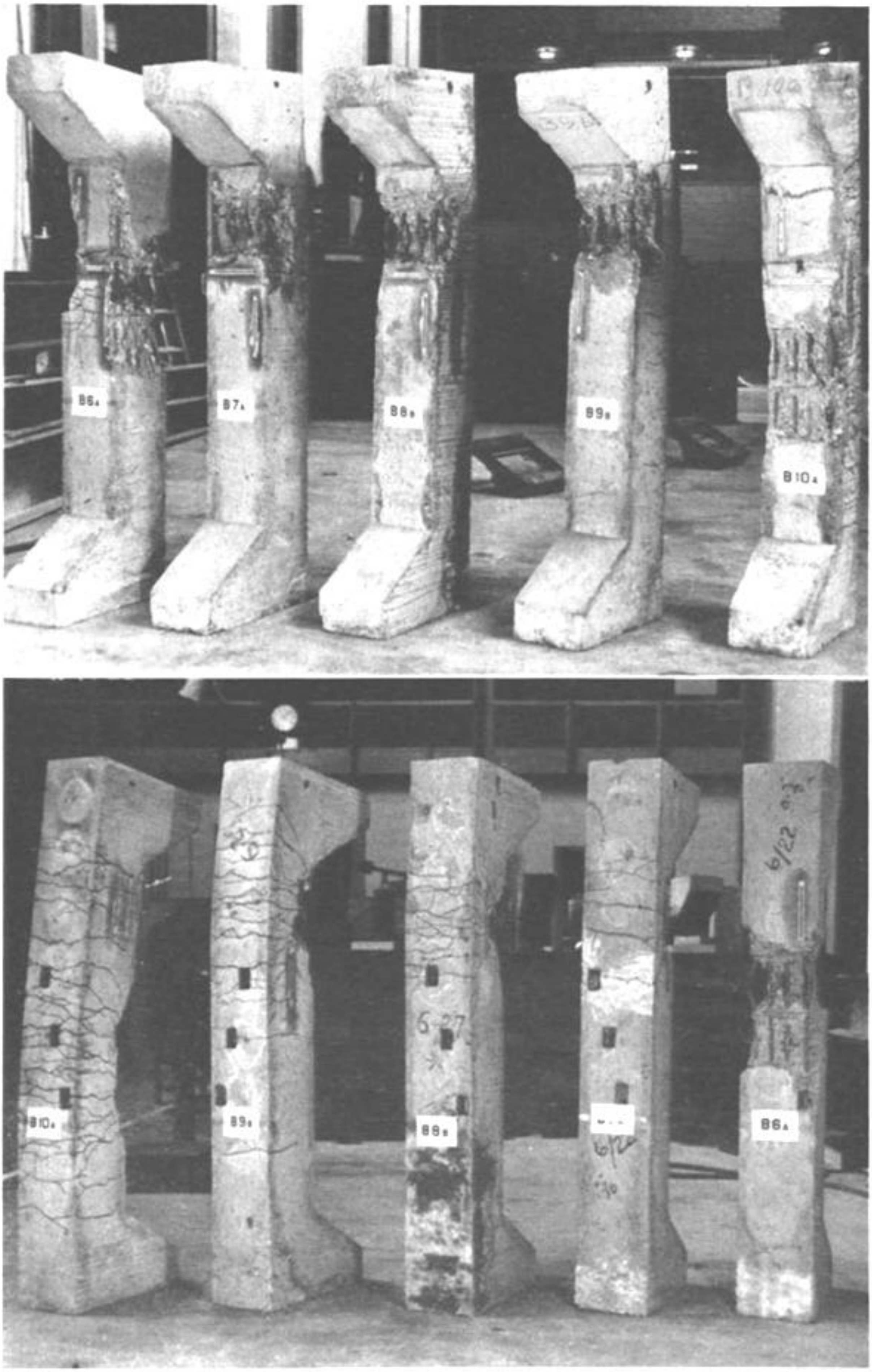


Fig. 23. Columns B-6 to B-10 after Failure

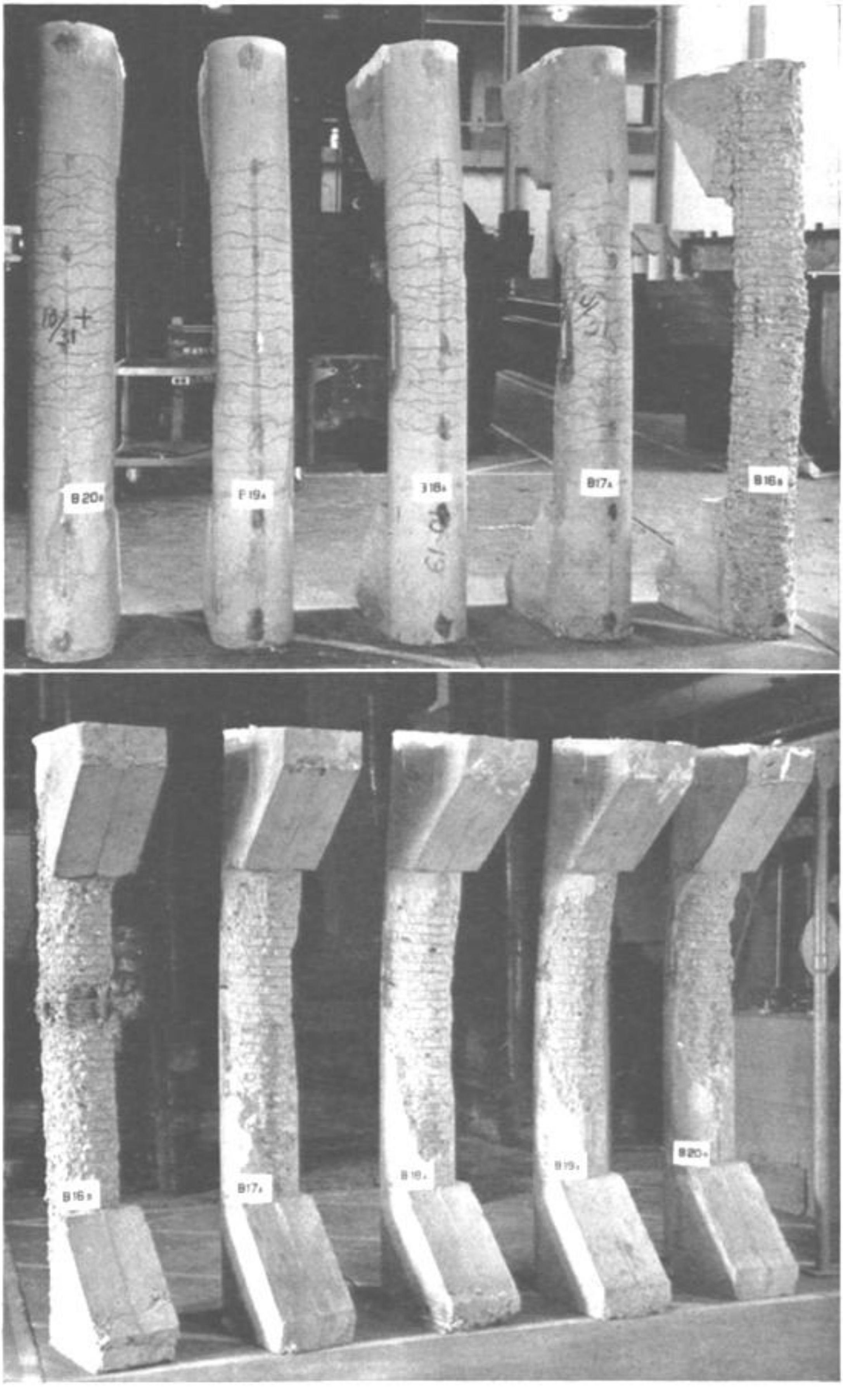


Fig. 39. Columns B-16 to B-20 after Failure

The spirally reinforced columns appeared to reach a stage of semi-neutral equilibrium very similar to that described for tied columns, before the shell failed. The ultimate strains of the shell (Table 10), however, appear to be somewhat smaller than the values found for tied square columns. Thus, the ultimate strain, ϵ_u, appears to be dependent on the

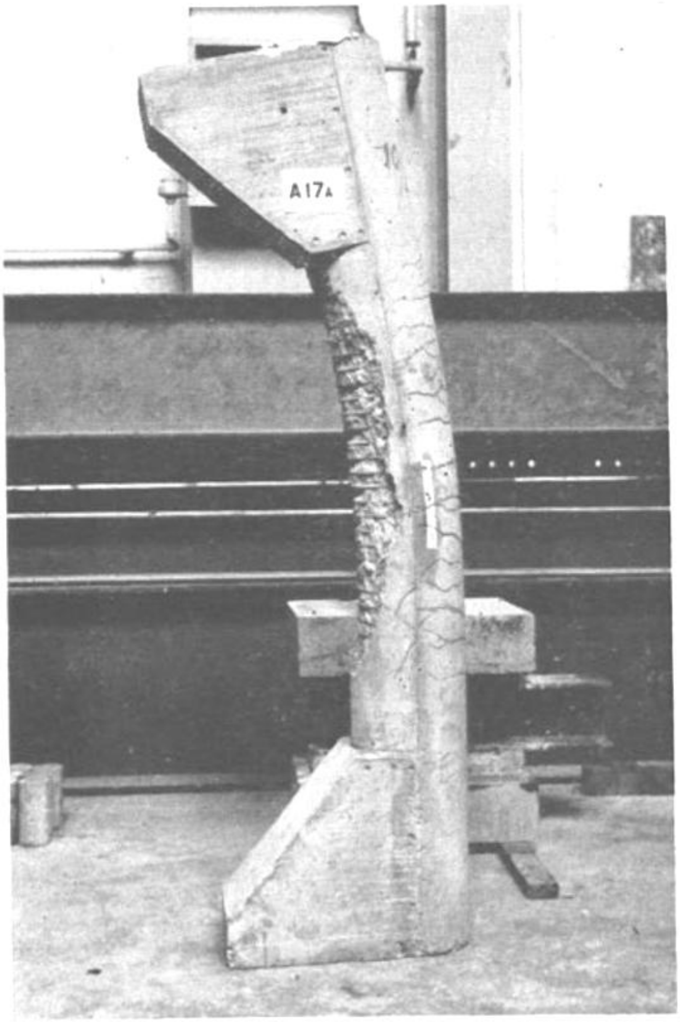


Fig. 40. Column A-17a after Failure

shape of the section. The difference is small, however, and since the constants k_1 and k_2 are rather insensitive to changes in ϵ_u, it is believed that the use of $\epsilon_u = 3.8$ per mill as determined for square columns will not cause important errors in the estimate of ultimate loads.

The behavior of tied columns at high loads was found to depend on the mode of failure. Members failing in tension were able to deform considerably after the tension steel reached yielding and before the concrete at the compression edge failed. For compression failures, such large deformations were not present. Regardless of whether the concrete in the compression zone failed before or after the tension steel reached in which f_c, f_t, A and Z are compressive stress, tensile stress, area and section modulus, respectively. Introducing

$$P_o = Af_{co}; \qquad M_o = Zf_{co} \tag{83}$$

gives at failure

$$\frac{P}{P_o} = 1 - \frac{M}{M_o}$$
$$\frac{P}{P_o} = \frac{f_{to}}{f_{co}} + \frac{M}{M_o} \tag{82a}$$

where f_{co} and f_{to} are compressive and tensile strength, respectively. An interaction diagram based on these two equations is drawn in Fig. 41.

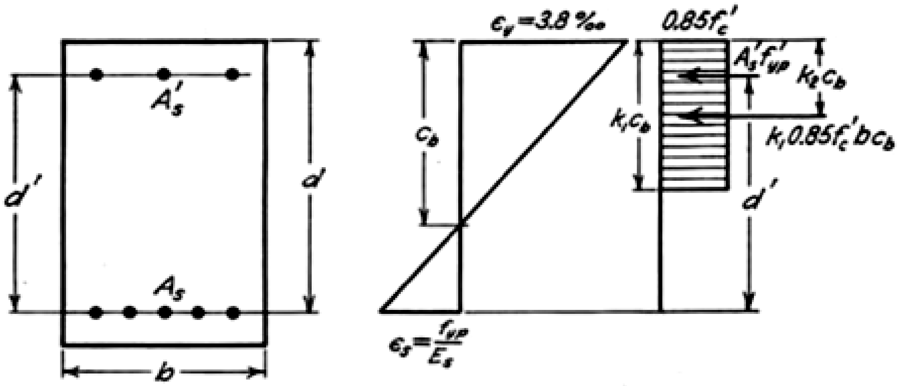


Fig. 42. Computation of M_o

For an isotropic material, $f_{co} = -f_{to}$, and the diagram is completed with one line only from $P/P_o = 1$ to $M/M_o = 1$. If the tensile strength is less than the compressive strength, however, a diagram consisting of two straight lines is obtained, as indicated in the figure for $f_{to}/f_{co} = -0.4$.

Combined bending and axial load in reinforced concrete members has been studied by similar methods by Thomas[68] and Whitney.[74] It has been claimed that Whitney's equation for compression failure of rectangular sections (Eq. 21) represents a straight line when plotted in a P versus Pe diagram.[74] The writer has found, however, that this is strictly the case only when

$$\frac{2}{d'} = \frac{3t}{1.178d^2} \text{ or } \frac{d'}{d} = 0.535.$$

It is nevertheless believed that a straight line in an interaction diagram may represent a satisfactory approximation for compression failures of any reinforced concrete section. In order to study the compression failure of the column tests reported herein with the aid of such

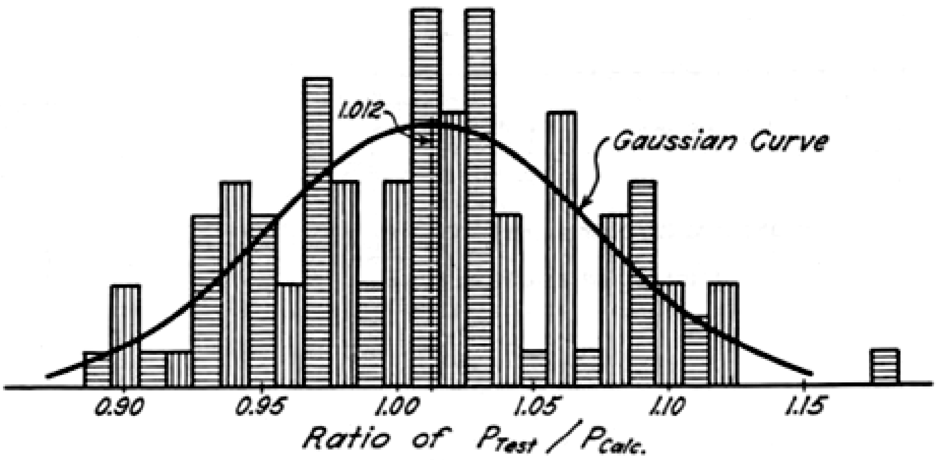


Fig. 45. Frequency Distribution of P_{test}/P_{calc}

The ratios between measured and computed ultimate loads for all columns were therefore plotted in Fig. 44 in the order that the columns were cast. Neighboring blank or shaded blocks indicate columns which were cast as pairs. There are six blocks without such a related neighbor corresponding to the six discarded tests.

It may first be noted that 35 pairs of columns deviate in the same direction while 19 pairs deviate in opposite directions. This may be interpreted as meaning that errors in manufacturing the specimens were more serious than errors in testing. Furthermore a slight trend is present indicating low test values in the period just before May 10. This may have been the effect either of a rainy spring, or of some property of the second lot of aggregates. Otherwise, no systematic trend with time appears to exist. Hence, the improvements in testing skill did not reduce the scatter of the test results, though many time saving methods were developed as the tests proceeded.

Finally Fig. 45 shows the frequency distribution of the load ratios for all 114 columns. The class interval of one percent is rather small compared to the population, and a "saw-tooth" variation results. Nevertheless it appears that the results fall fairly close to the Gaussian curve of a normal distribution.

Within the scope of the tests reported herein it may be concluded that the predicted ultimate loads are, on the average, one percent on the safe side. The test results appear to be distributed at random, since, for all *practical* purposes, no systematic trends of variation were found. The standard deviation of the ratio between measured and computed ultimate loads was 0.058. Hence, 99 percent of the results of tests similar to those reported herein may be expected to fall between 1.16 and 0.86 times the values predicted by a flexural analysis based on the assumptions made in Section 11.