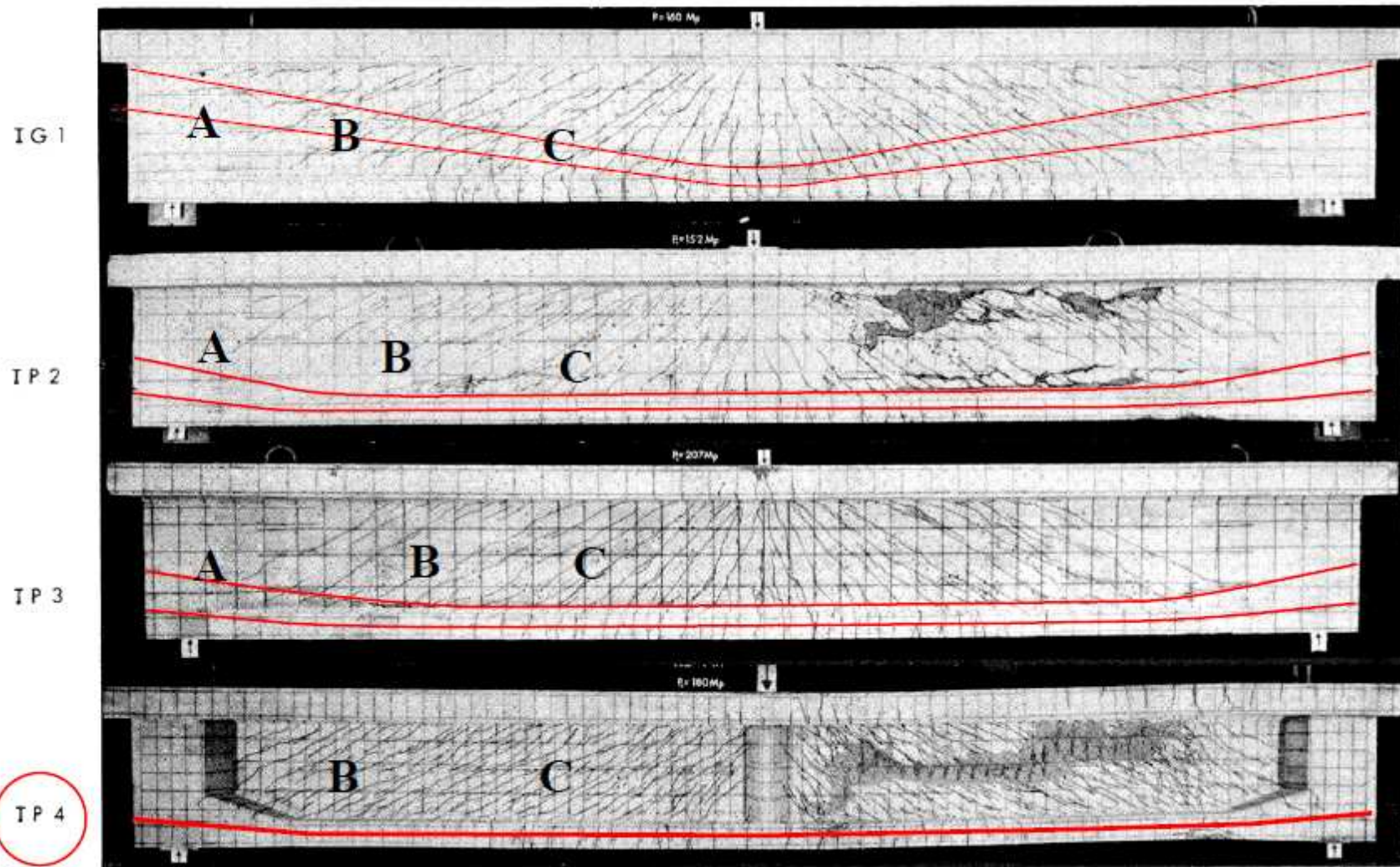




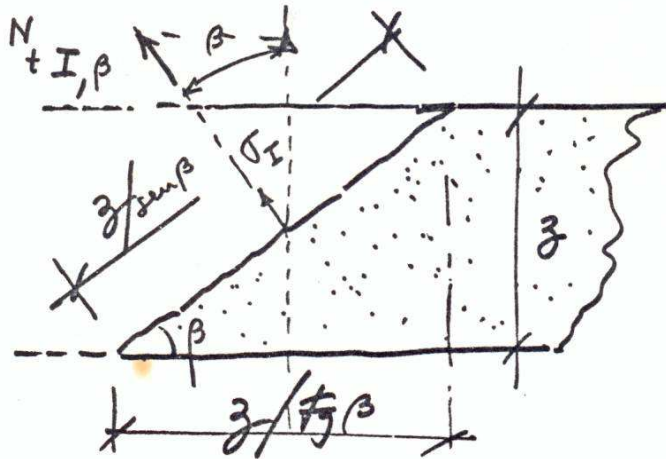
## Estribos CEB : Regiões A , B e C



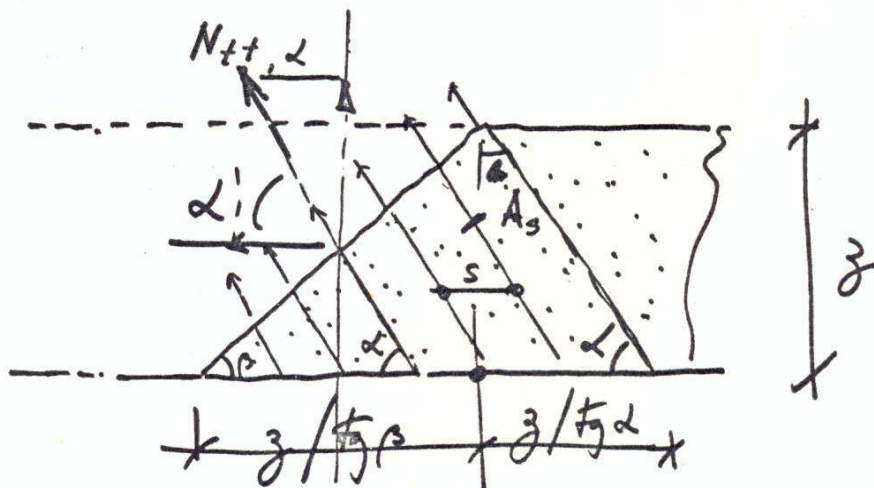
Ruptura : Zona A sem qualquer fissura, Zona B sem fissura no bordo inferior , Zona C com fissura no bordo inferior



Armadura calculada pelas  
tensões principais (CASO A, B)



$$N_{tI, \beta} = b_w \cdot \frac{z}{\sin \beta} \cdot \sigma_I$$



$$N_{tt, \alpha} = \left( \frac{z/\sin \beta + z/\sin \alpha}{s} \right) \cdot A_s \cdot \sigma_{s, \alpha}$$



$$N_{tI, \beta} \times \cos \beta = N_{tt, \alpha} \cdot \sin \alpha$$

$$\left( b_w \cdot \frac{z}{s \sin \beta} \cdot \sigma_I \right) \cos \beta = \left[ \frac{z}{s} \left( \frac{1}{\sin \beta} + \frac{1}{\sin \alpha} \right) A_s \cdot \sigma_{sd} \right] \cdot \sin \alpha$$

Chaveando

$$\rho_w \alpha = \frac{A_s}{b_w \cdot s \cdot \sin \alpha}$$

$$b_w \cdot \frac{\sigma_I \cdot \cos \beta}{s \sin \beta} = \frac{A_s \cdot \sigma_{sd}}{s} \left( \frac{\cos \beta}{s \sin \beta} + \frac{\cos \alpha}{s \sin \alpha} \right) \cdot \sin \alpha$$

$$b_w \cdot \frac{\sigma_I \cdot \cos \beta}{s \sin \beta} = \frac{A_s \cdot \sigma_{sd}}{s} \left( \frac{\sin \alpha \cos \beta + s \sin \beta \cos \alpha}{s \sin \beta \sin \alpha} \right) \sin \alpha$$

$$\sigma_I \cdot \cos \beta = \frac{A_s \cdot \sigma_{sd}}{b \cdot s} \cdot \sin(\alpha + \beta)$$

$$\frac{\sigma_I \cdot \cos \beta}{\sin \alpha \cdot \sin(\alpha + \beta)} = \left( \frac{A_s}{b \cdot s \cdot \sin \alpha} \right) \cdot \sigma_{sd}$$

$\rho_w \cdot \alpha$



$$\rho_w \cdot \alpha \cdot \sigma_s \alpha = \frac{\sigma_I \cdot \cos \beta}{\sin \alpha \cdot \sin (\alpha + \beta)}$$

$$\rho_w \cdot f_y d = \frac{\sigma_I d \cdot \cos \beta}{\sin \alpha \cdot \sin (\alpha + \beta)}$$

se  $\alpha = 90^\circ$

$$\rho_w \cdot 90 \cdot f_y d = \frac{\sigma_I \cdot d \cdot \cos \beta}{1 \cdot \sin (90 + \beta)} = \frac{\sigma_I d}{1}$$

$$\rho_w 90 = \frac{\sigma_I d}{f_y d}$$

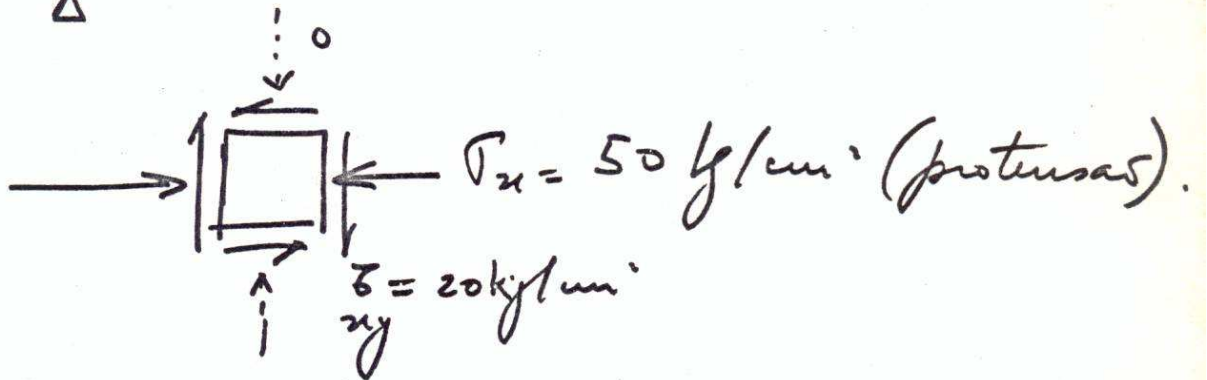
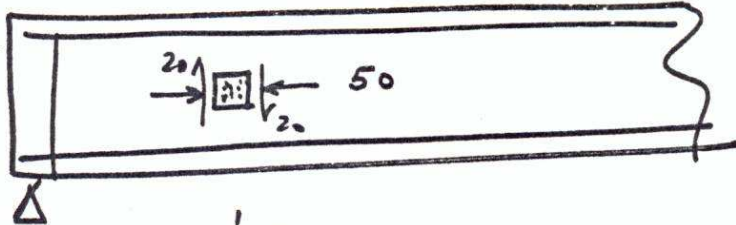
levando em conta a participação do concreto.

$$\rho_w \cdot \alpha = \frac{1}{f_y d} \cdot \frac{(\sigma_I d - 0,5 \sqrt{f_{ck}}) \cdot \cos \beta}{\sin \alpha \cdot \sin (\alpha + \beta)}$$

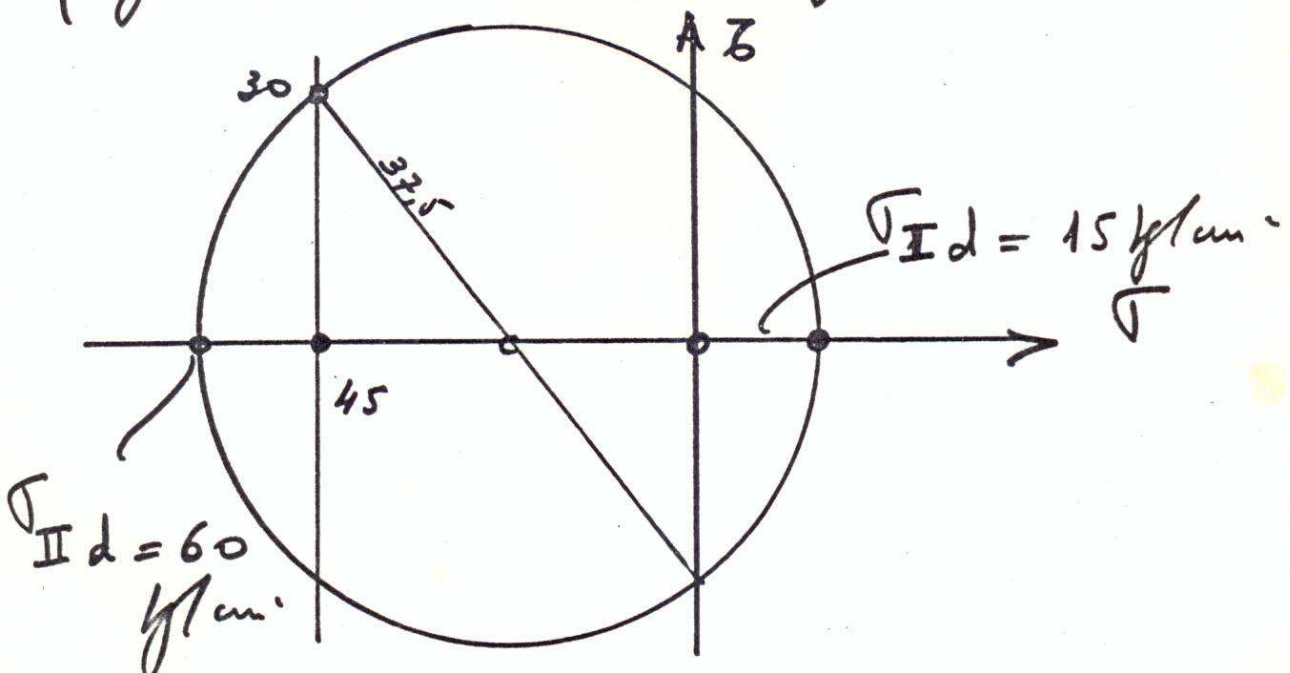
$$\alpha = 90^\circ \quad \rho_w \cdot \alpha = \frac{1}{f_y d} \cdot (\sigma_I d - 0,5 \sqrt{f_{ck}}) \quad (kg/cm^2)$$



## Exemplo



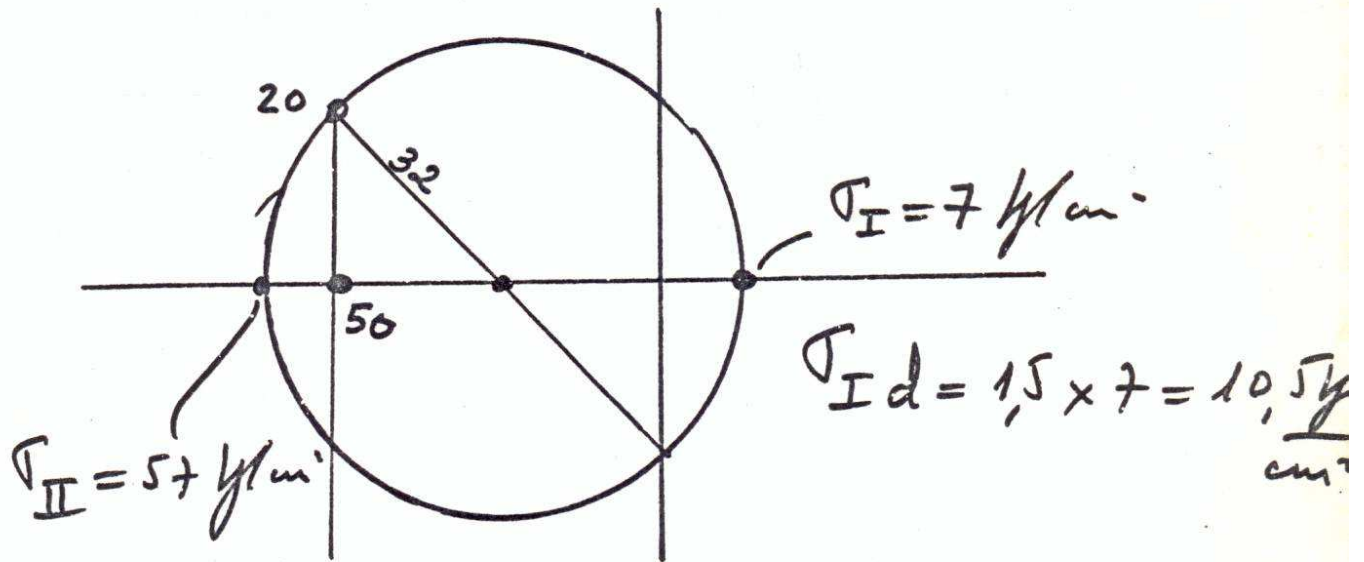
$$\left\{ \begin{array}{l} \tau_{xd} = 0,9 \times 50 = 45 \text{ kg/cm}^2 \\ \tau_{ygd} = 1,5 \times 20 = 30 \text{ kg/cm}^2 \end{array} \right.$$





Como observar:

Se inicialmente fore achado  $\sigma_I$  e  $\sigma_{II}$   
e posteriormente majorados para  
 $\sigma_{Id}$  e  $\sigma_{IId}$  teriamos.



$\sigma_{IId} = 1,5 \times 57 = 85,5 \text{ kg/cm}^2$  (o que  
resulta majorar demais  
a protensão exagerando  
em torno o  $\sigma_{IId}$ ).



Para o cálculo da armadura devemos notar que a força de protensão é pequena e portanto  $f_t$  devemos multiplicar por 0,9 a tensão  $\sigma_{tr}$ .

$$\sigma_{Id} = 15 \text{ kg/cm}^2.$$

$$\rho_{w90} = \frac{\sigma_{Id} - 0,5 \sqrt{f_{ck}}}{f_{yd}} \quad (\text{g/cm}^2)$$

$$f_{yd} \leq \frac{f_{yk}}{1,15} \quad \text{em } f_{yk} \leq \frac{4500}{5}.$$
$$\leq \frac{4500}{1,15} = 3913 \text{ kg/cm}^2.$$

$$f_{ck} = 260 \text{ kg/cm}^2$$

$$0,5 \sqrt{f_{ck}} = 0,5 \sqrt{260} = 8,1 \text{ kg/cm}^2$$



$$P_{w90} = \frac{15 - 8,1}{3913} = 1,8 \times 10^{-3} = 0,0018 = 0,18\%$$

$$P_{wmin} = 0,14\%$$

Usar  $P_{w90} = \underline{0,18\%}$ .

— + —

### Verificação do concreto

Levando-se em conta o estado duplo de tensão.

$$\left\{ \begin{array}{l} \text{Se } \sigma_{Id} \leq 0,125 f_{cd} \\ \text{então } \sigma_{II d} \leq f_{cd} = 4 \sigma_{Id} \end{array} \right.$$

$$\text{Se } \sigma_{Id} > 0,125 f_{cd} \\ \sigma_{II d} \leq 0,0625 \frac{f_{cd}}{\sigma_{Id}} \cdot f_{cd}$$





No novo caso .

1.ª hipótese de majoração dos tensores :

$$\sigma_{Id} = 60 \text{ kg/cm}^2$$

$$\sigma_{Id} = \underline{15 \text{ kg/cm}^2}$$

$$f_{ck} = 260 \text{ kg/cm}^2 \rightarrow f_{cd} = \frac{260}{1,5} = 173,3 \frac{\text{kg}}{\text{cm}^2}$$

$$0,125 f_{cd} = 0,125 \times 173,3 = \underline{21,7 \text{ kg/cm}^2}$$

$$\boxed{\sigma_{Id} < 0,125 f_{cd}}$$

$$15 < 21,7 \text{ kg/cm}^2$$

$$\sigma_{Id} \leq f_{cd} - 4\sigma_{Id} \quad \checkmark$$

$$\underline{60} \leq 173,3 - 4 \times 15 = \underline{113,3 \text{ kg/cm}^2}$$

2.ª hipótese de majoração dos tensores

$$\sigma_{Id} = 10,5 < 0,125 f_{cd} = 21,7$$

$$\sigma_{Id} = 85,5 \leq f_{cd} - 4\sigma_{Id} =$$

$$\underline{85,5} \leq 173,3 - 4 \times 10,5 = \underline{131,3 \text{ kg/cm}^2}$$

Exatadamente grande OK.



Se não existir protensão transversal (o que normalmente é o caso), pode-se usar a verificação resumida.

$$\tau_{d} \leq 0,25 f_{cd}$$

$$\tau_{d} = \frac{V \cdot E_{pt}}{b \cdot I}$$

$\tau_{d}$  sendo a tensão tangente na seção transversal no centro de gravidade de aço. (calculada pela fórmula de Cirst)

No novo exemplo.

$$\tau_{d} = 1,5 \times 20 = 30 \text{ kg/cm}^2$$

$$0,25 f_{cd} = 0,25 \times \left( \frac{260}{1,5} \right) = 0,25 \times 173,3 = 43,3 \text{ kg/cm}^2$$

$$30 < 43,3 \text{ kg/cm}^2$$

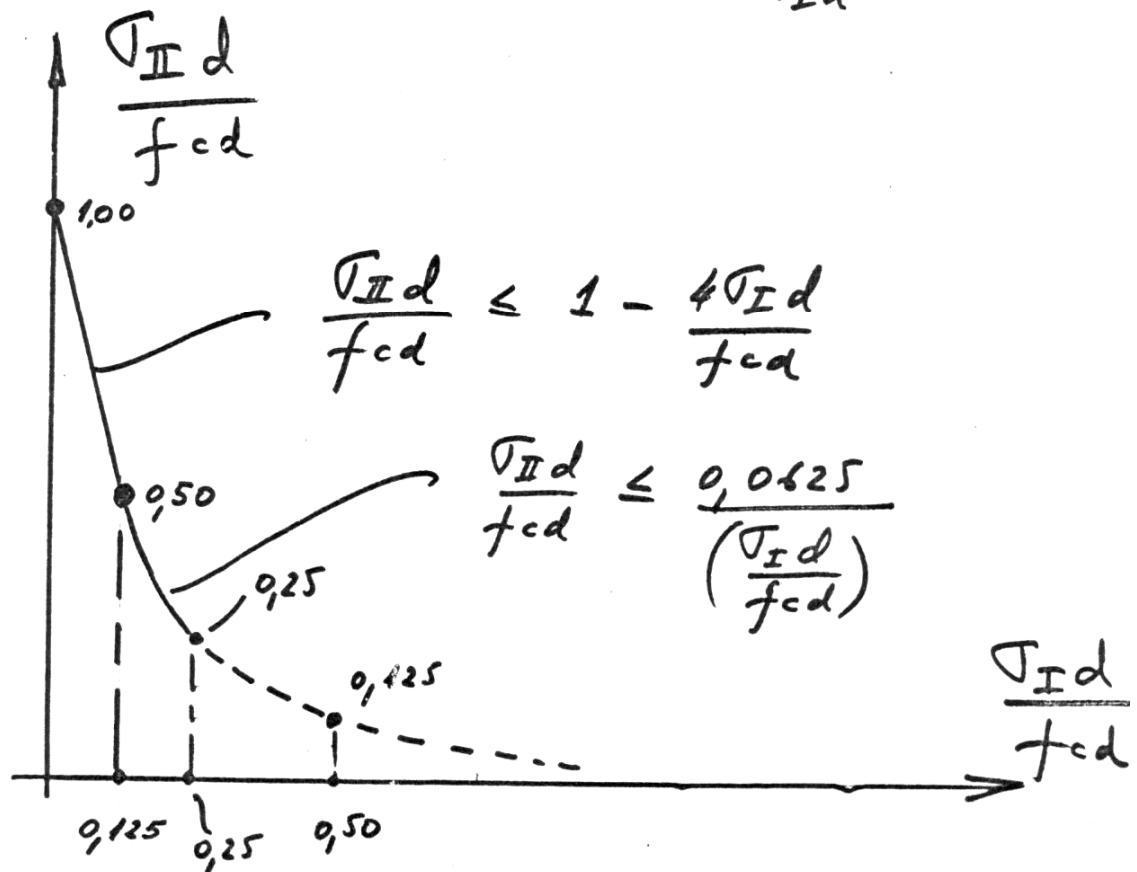


O CEB nas fórmulas indicadas correlaciona  $\sigma_I$  e  $\sigma_{II}$ , sem apresentar um limite para  $\sigma_I$ .

Tal limite é apresentado mais adiante quando se fala de fôrmas rasas.

$$\sigma_{Id} \leq 0,125 f_{cd} \quad \sigma_{II d} \leq f_{cd} - 4\sigma_{Id}$$

$$\sigma_{Id} \geq 0,125 f_{cd} \quad \sigma_{II d} \leq 0,0625 \frac{f_{cd}}{\sigma_{Id}} \cdot f_{cd}$$





O limite para a DIN 1045  
impõe para concreto protendido  
a seguinte equação

$$\sigma_{IR} \approx 0,25 f_{ck}$$

$$1,75 \sigma_I \leq 0,25 f_{ck}$$

$$\sigma_I \leq 0,143 f_{ck}$$

(serviço)

$$\sigma_{Id} = 1,5 \sigma_I \leq 0,21 f_{ck} = 0,32 f_{cd}$$

$$\sigma_{Id} \approx 0,30 f_{cd}$$

Se fixamos  $0,25 f_{cd}$ .

$$\sigma_{Id} \leq 0,25 f_{cd} \approx 2 f_{td}$$



Classe III (Fimuraçã).

$$\frac{\sigma_I}{1,1 f_{ctk}} \leq 1 - \frac{0,8 \sigma_{IIc}}{1,1 f_{cek}}$$

$$1,5 \sigma_I = \sigma_{Id}$$

$$f_{ctk} \approx 0,10 f_{cek}$$

$$1,5 \sigma_{II} = \sigma_{IId}$$

$$\frac{1,5 \sigma_I}{1,1 (0,1 f_{cek})} \leq 1,5 - \frac{0,8 \times 1,5 \sigma_{IIc}}{1,1 f_{cek}}$$

$$\frac{\sigma_{Id}}{0,11 f_{cek}} \leq 1,5 - 0,73 \frac{\sigma_{IId}}{f_{cek}}$$

$$f_{cek} = 1,5 f_{ced}$$

$$\boxed{6,1 \cdot \frac{\sigma_{Id}}{f_{ced}} \leq 1,5 - 0,5 \frac{\sigma_{IId}}{f_{ced}}}$$



$$\sigma_{Id} = 15 \text{ k/cm}^2 \quad \sigma_{II d} = 60 \text{ k/cm}^2$$
$$f_{cd} = 173,3 \text{ "}$$

$$6,1 \times \frac{15}{173,3} \leq 1,5 - 0,5 \times \frac{60}{173,3}$$

$$0,53 \leq 1,5 - 0,17 = 1,33$$

OK.

— + —

Classe II

$$\frac{\sigma_I}{f_{ctk}} \leq 1 - \frac{0,8 \sigma_{IIc}}{f_{ctk}}$$

$$\frac{\sigma_{Id}}{0,1 f_{ctk}} \leq 1,5 - \frac{0,8 \sigma_{II d}}{f_{ctk}}$$

$$6,67 \frac{\sigma_{Id}}{f_{cd}} \leq 1,5 - 0,53 \frac{\sigma_{II d}}{f_{cd}}$$

$$6,67 \times \frac{15}{173,3} \leq 1,5 - 0,53 \times \frac{60}{173,3}$$
$$0,58 < 1,32 \quad \text{OK.}$$



## Estribos : Região C Ver Anexos 1 e 2

Caso de emenda protendida (Caso C)

$$\bar{\sigma}_d = 1,15 \bar{\sigma}_{wd} - \bar{\sigma}_c$$

$$\bar{\sigma}_c = \psi_1 \sqrt{f_{ck}} \left( 1 + 3 \frac{\sigma_{cmd}}{f_{ck}} \right)$$

$\psi_1$  = interprete o efeito de rigidez de amplitude.

A NB-1 pode diz no caso de protendido.

O CEB - mas nunca usar  $\psi_0 = 1,0$

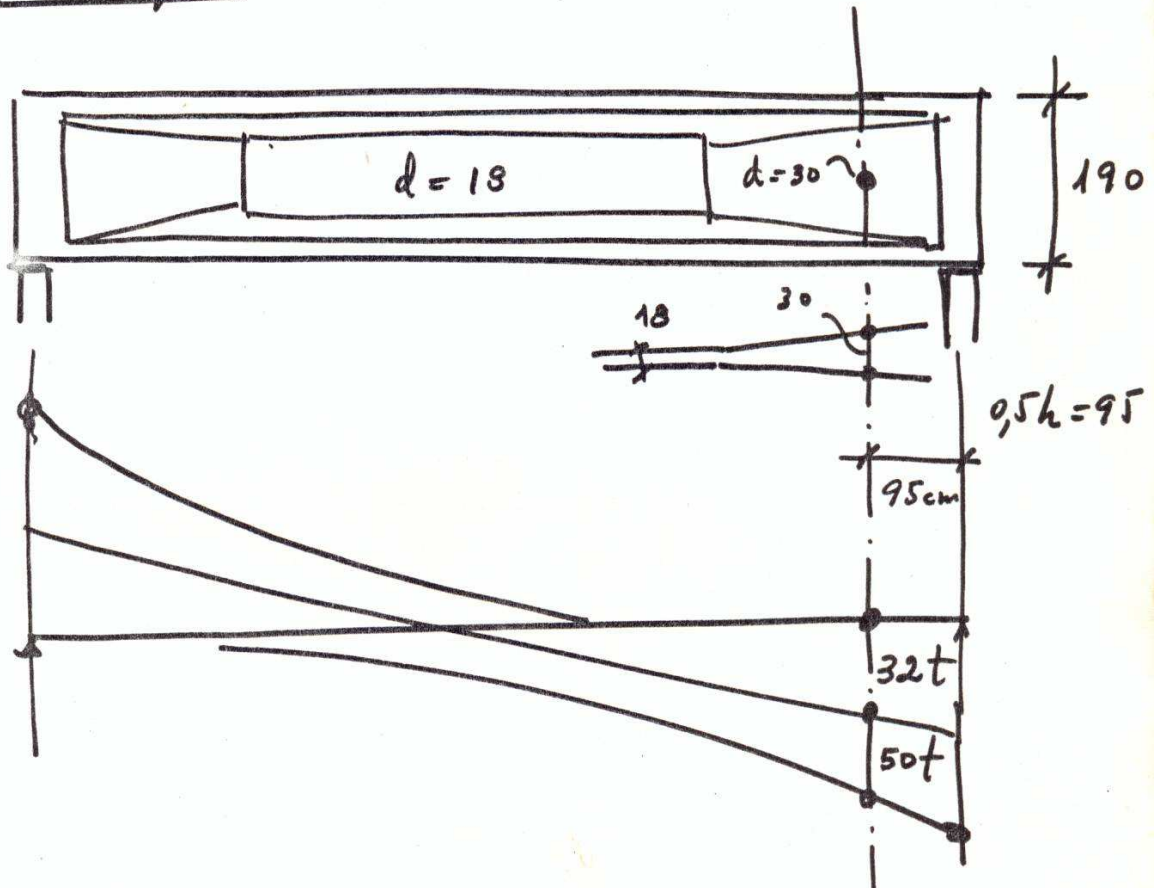
$$\bar{\sigma}_c = 0,46 \sqrt{f_{ck}} \left( 1 + 3 \frac{\sigma_{cmd}}{f_{ck}} \right) \times \psi_0$$

Pela NB1 nunca  $\psi_1 = 0,45$

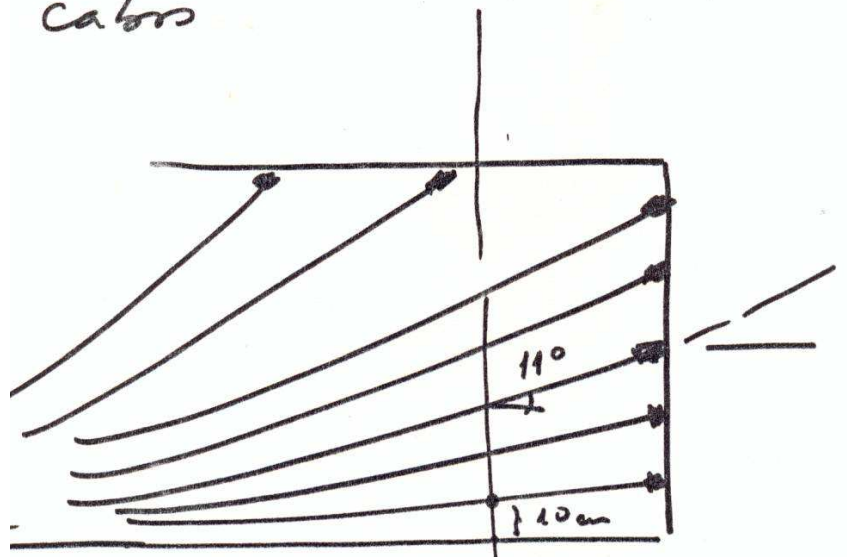
$$\bar{\sigma}_c = 0,45 \sqrt{f_{ck}} \left( 1 + 3 \times \frac{\sigma_{cmd}}{f_{ck}} \right)$$



Exemplo:



cabos

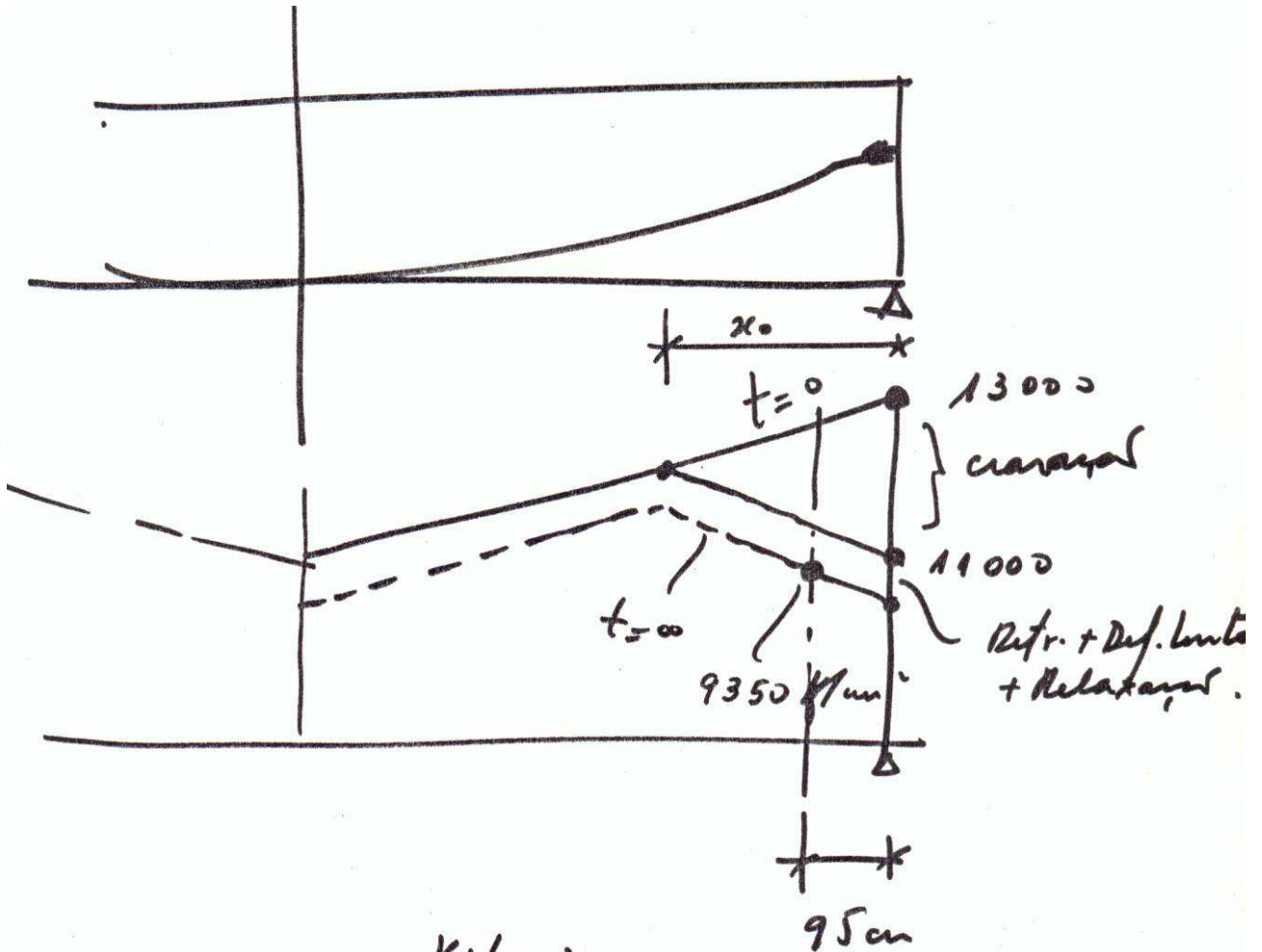


7 cabos  $6\phi 1/2$  RN 150/176





# Tensão no cabo: (cabo médio)



$$x_0 = \sqrt{\frac{\Delta_0 \times E_{ara}}{f_{y,d} \times \frac{K_f}{m}}} = \dots \text{ m.}$$



$$A_s \text{ (1 cabo)} = 5,57 \text{ cm}^2$$

$$N_p = 5 \times 5,57 \times 9350 = 260 \text{ t.}$$

(t=00)

$$N_{pd} = 0,9 \times 260 = 234 \text{ t}$$

(Reduções)

$$\alpha_{\text{médio}} = 11^\circ \quad \text{sen} \alpha = 0,19$$

$$N_{pd} \times \text{sen} \alpha = 234 \times 0,19 = 44,5 \text{ t.}$$

$$V_{rd} = 1,4(32 + 50) - 44,5 =$$
$$114,8 - 44,5 = 70,3 \text{ t.}$$

Verificação do concreto:  
Se não houveresse furos de bainha

$$\sigma_{wd} = \frac{70.300}{180 \times 30} = 13,0 \text{ kg/cm}^2.$$



limites de  $\tau_{wd}$ .

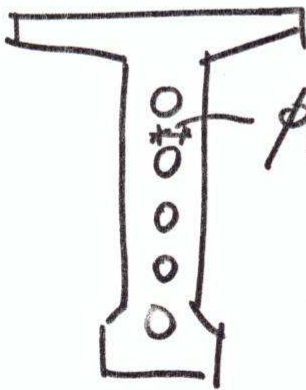
$$f_{ck} = 260$$

$$0,20 f_{cd} = 0,20 \times \frac{260}{1,4} = 37,1 \text{ kN/cm}^2$$
$$36 \text{ kN/cm}^2$$

$$13,0 < 36 \text{ kN/cm}^2 \quad \text{OK.}$$



Descontando os furos da bainha.



$$\phi_{\text{bainha}} = 5,5 \text{ cm.}$$

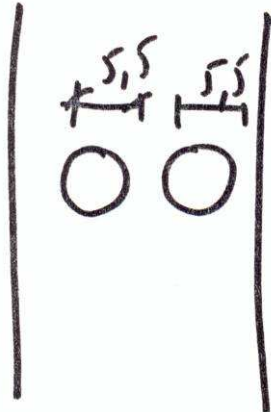
$$b_w = 30 - 5,5 = 24,5 \text{ cm.}$$

$$\tau_{wd} = \frac{70.300}{24,5 \times 180} = 15,9 \text{ kN/cm}^2$$
$$< 36 \text{ kN/cm}^2$$

OK.



Se fosse usado 2 cabos de  
uma vez.



$$b_w = 30 - 11 = 19 \text{ cm.}$$

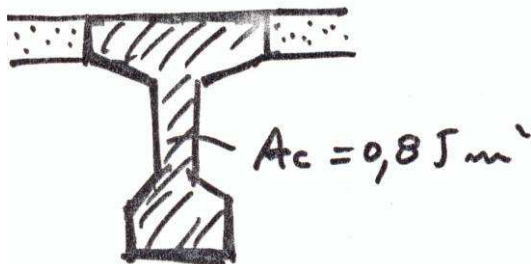
$$\sigma_{wd} = \frac{70300}{19 \times 180} = 23 \text{ kg/cm}^2.$$

$$c = 36 \text{ kg/cm}^2.$$

— + —

Cálculo de amedure transversal.

$$V_{cmd} = \frac{\mu_{pd} \cdot N_p \cdot 0,98}{0,85 \text{ m}^2} = 271 \frac{\text{t}}{\text{m}^2}$$



$$= 27,1 \text{ kg/cm}^2$$



$$\tau_c = 0,45 \times \sqrt{f_{ck}} \left( 1 + 3 \frac{\sigma_{cmd}}{f_{ck}} \right)$$

$$= 0,45 \times \sqrt{260} \left( 1 + 3 \frac{27,1}{260} \right)$$

$$= 7,26 \times 1,313 = 9,5 \text{ kg/cm}^2$$

$$\tau_{ed} = 1,15 \tau_{wd} - \tau_c$$

$$= 1,15 \times 15,9 - 9,5 = 8,8 \text{ kg/cm}^2$$

$$\rho_w = \frac{A_s}{b_w \cdot s \cdot \text{send}} = \frac{\tau_{ed}}{f_{yd}}$$

$$\rho_w = \frac{8,8}{4000} = 0,0022 = 0,22\%$$

$$\rho_{w \min} \geq 0,14\%$$

$$\text{Usar } \rho_w = 0,22\%$$



$$A_s = b_w \cdot s \cdot \text{sen } \alpha \times 0,22\%$$

Estribos a  $90^\circ$

$$\alpha = 90^\circ \quad \text{sen } \alpha = 1.$$

$$A_s = b_w \cdot s \cdot 0,22\%$$

$$A_s = 24,5 \times 100 \times 0,22\% = 5,4 \frac{\text{cm}^2}{\text{m}}$$

\* (descontado o furo) (1m)

Espaçamentos entre estribos.

$$\frac{\tau_{wd}}{\tau_{wm}} = \frac{15,9}{36,0} = 0,44$$

$$s \leq \left\{ \begin{array}{l} 30 \text{ cm} \\ (45 - 30 \times \frac{\tau_{wd}}{\tau_{wm}}) = \\ = 45 - 30 \times 0,44 = 31 \text{ cm} \end{array} \right\} s \approx 30 \text{ cm}$$



$$1 \phi 3/8 - A_s = 2 \times 0,71 = 1,42 \text{ cm}^2$$

$$n = \frac{5,4}{1,42} = 3,8 \phi / 1 \text{ m} \rightarrow \text{espaçamentos} = 25 \text{ cm}$$

$\phi 3/8 \text{ c } 25$



Exemplos para comparações:

Desprezando o efeito de protensão

1) Não há desconto do cortante do calor.

$$V_d = 1,4(32 + 50) = 114,8 \text{ t.}$$

2) Verificação do momento nos pontos de apoio e balanço.

$$\tau_{wd} = \frac{114.800}{180 \times 30} = 21,3 \text{ kg/cm}^2 < \frac{31}{2}$$

limite do  $\tau_{wd}$  continua o mesmo

$$\left. \begin{array}{l} 0,20 f_{cd} = 37 \text{ kg/cm}^2 \\ 36 \end{array} \right\}$$

3) Armadura.

Superfície  $\psi_1 = 0,45$

$$\tau_c = 0,45 \sqrt{f_{ck}} = 0,45 \sqrt{260} = 7,3 \text{ kg/cm}^2$$



$$P_{ws} = \frac{1,15 \times 21,3 - 7,3}{4000} = 0,43\%$$

$$\frac{A_{sw}}{(90^\circ)} = \frac{0,43}{100} \times 30 \times 100 = 12,9 \text{ cm}^2/\text{m.}$$

Espaçamento entre os estribos.

$$s \leq \left\{ \begin{array}{l} 30 \text{ cm} \\ 45 - 30 \times \frac{21,3}{36} = 27 \text{ cm} \end{array} \right\}$$

$$s \leq 27 \text{ cm}$$

$$\phi_{estibos} \leq \left\{ \begin{array}{l} \frac{b}{16} = \frac{30}{16} = 1,8 \text{ cm.} \\ 16 \text{ mm} \end{array} \right\}$$



$$1,27 \text{ cm} \quad \phi 1/2 \quad - \quad A_{si} = 2 \times 1,27 = 2,54 \text{ cm}$$

$$n = \frac{12,9}{2,54} = 5,08 \text{ estibos/m}$$

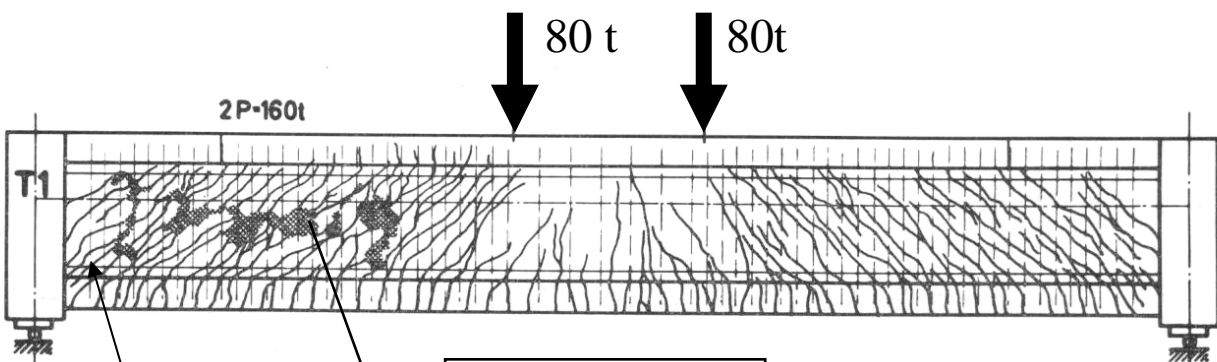
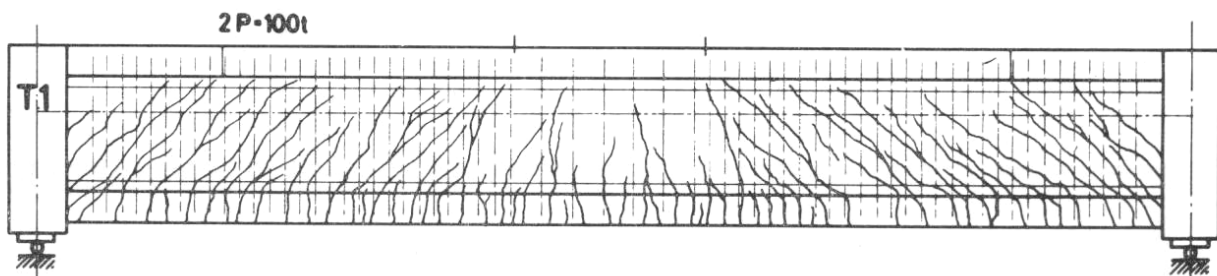
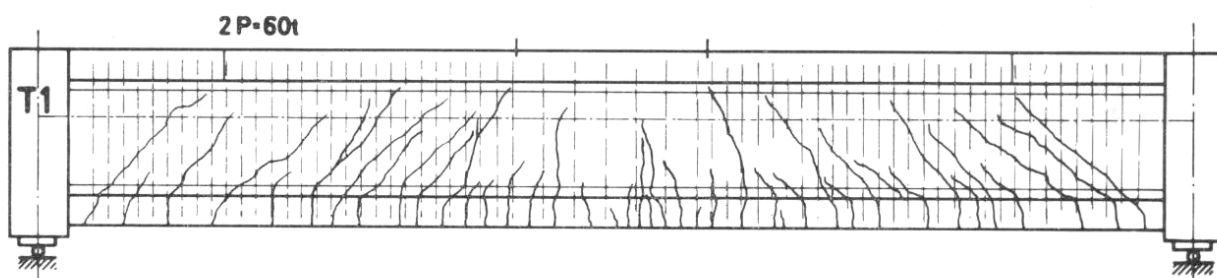
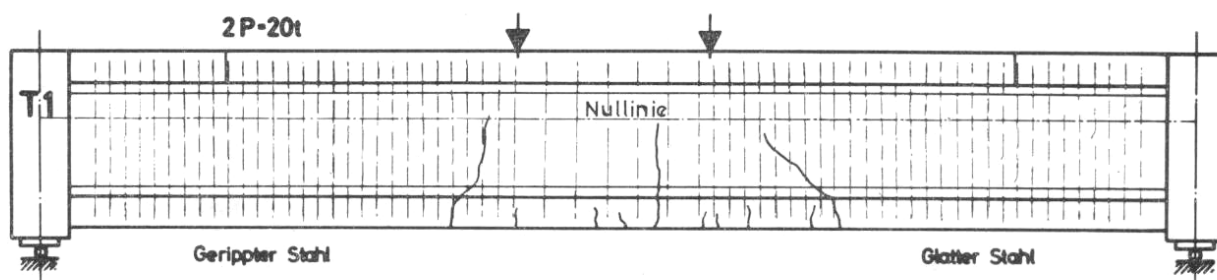
$$\text{espaçamento} = 20 \text{ cm.}$$

$$\boxed{\phi 1/2'' \text{ c } 20 \text{ cm}}$$





## Anexo 1- Ensaio de Viga de Concreto Armado



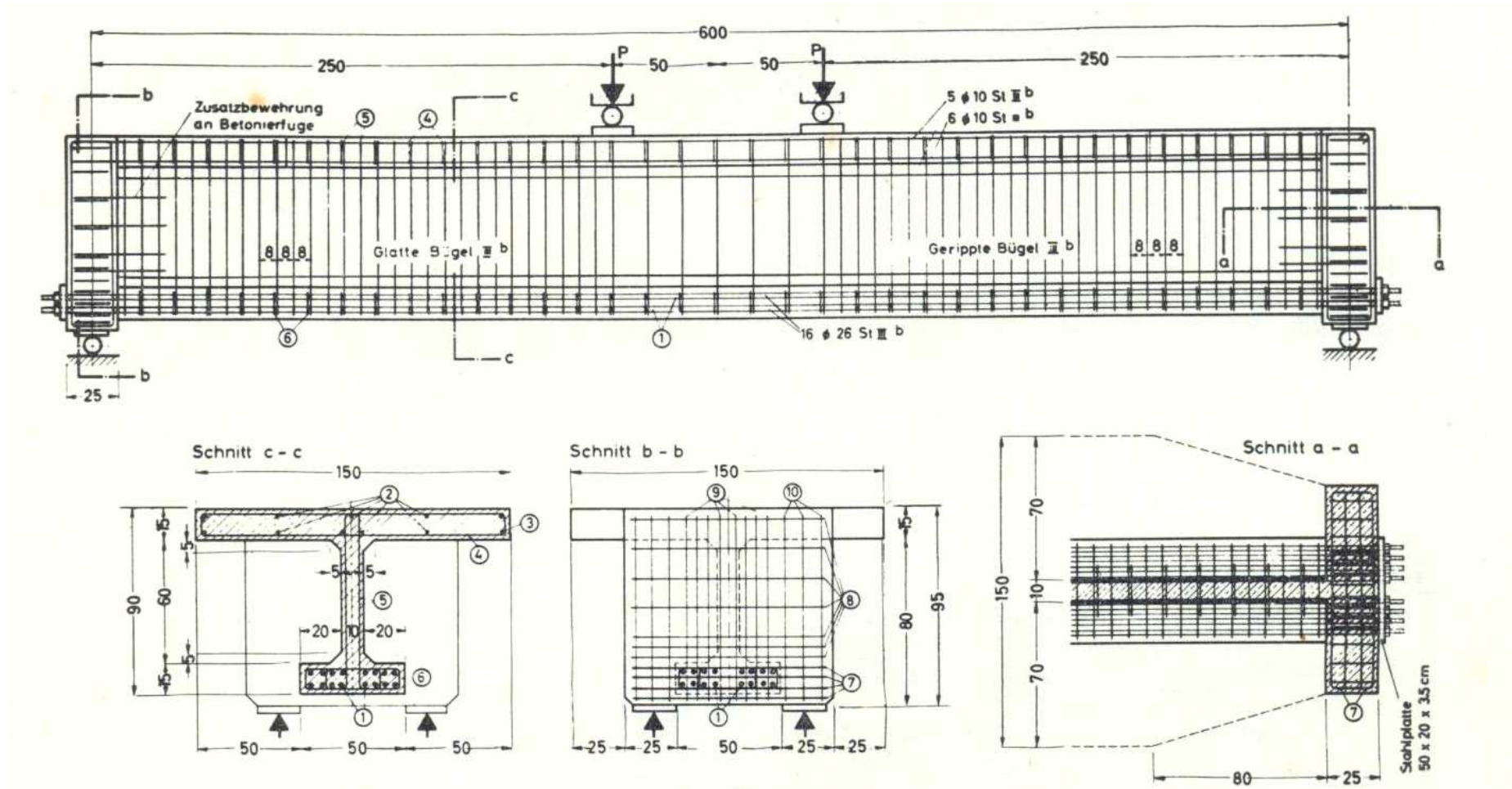
**Esmagamento  
da biela**

**Fissura no bordo inferior  
até o apoio**

*Ensaio feito pelo Prof. Fritz Leonhardt – Stuttgart – Alemanha*



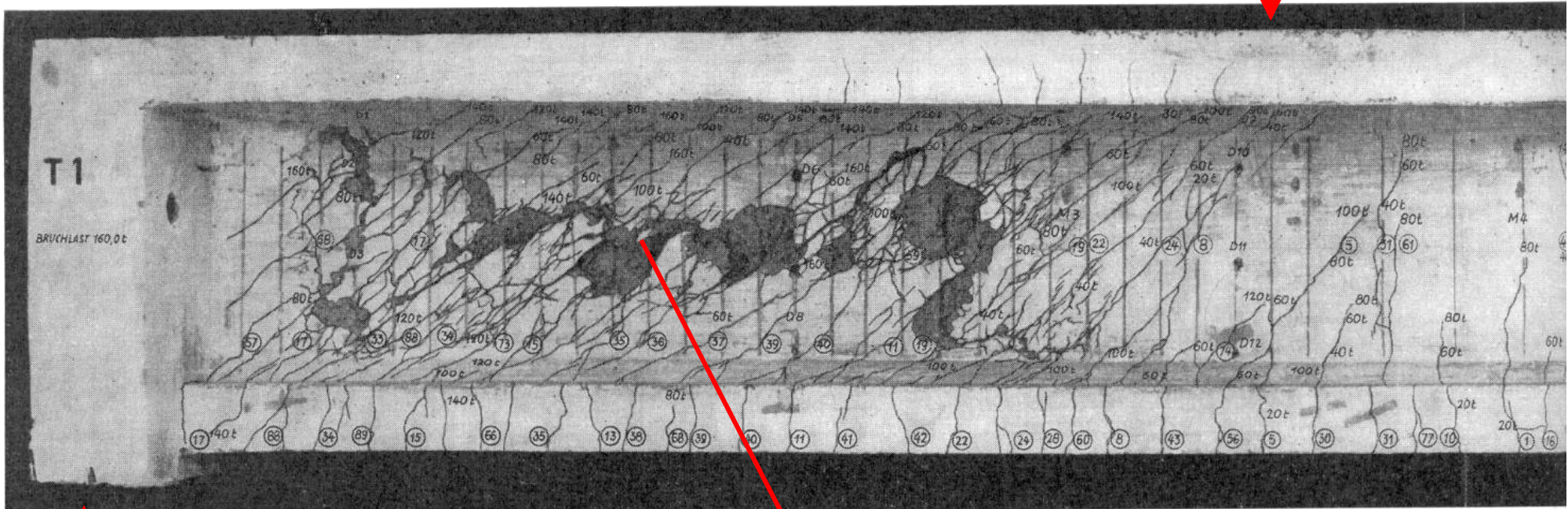
## Ensaio de Viga de Concreto Armado





## Parte esquerda da viga após a ruptura por esmagamento da biela

80 t

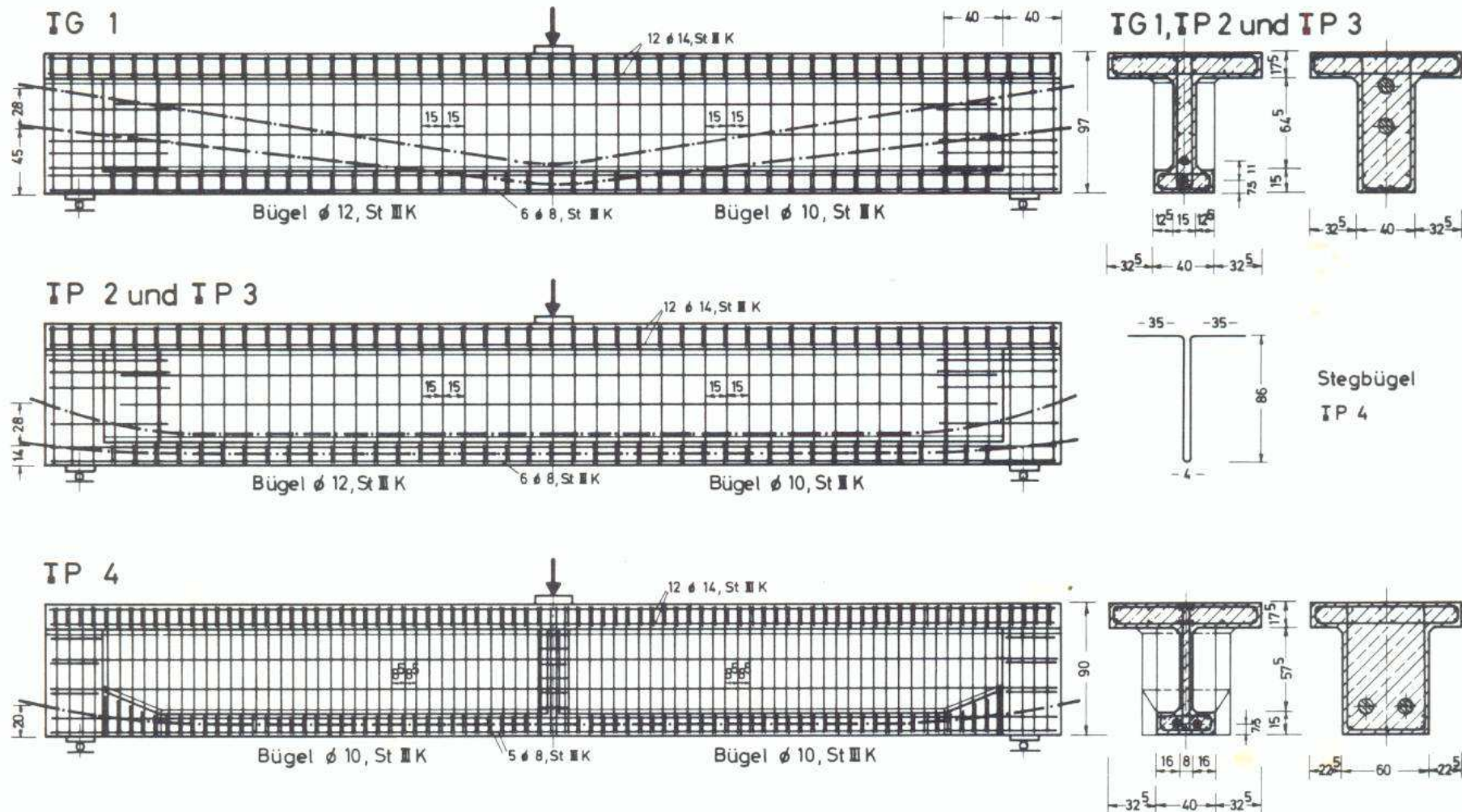


**Esmagamento  
da biela**

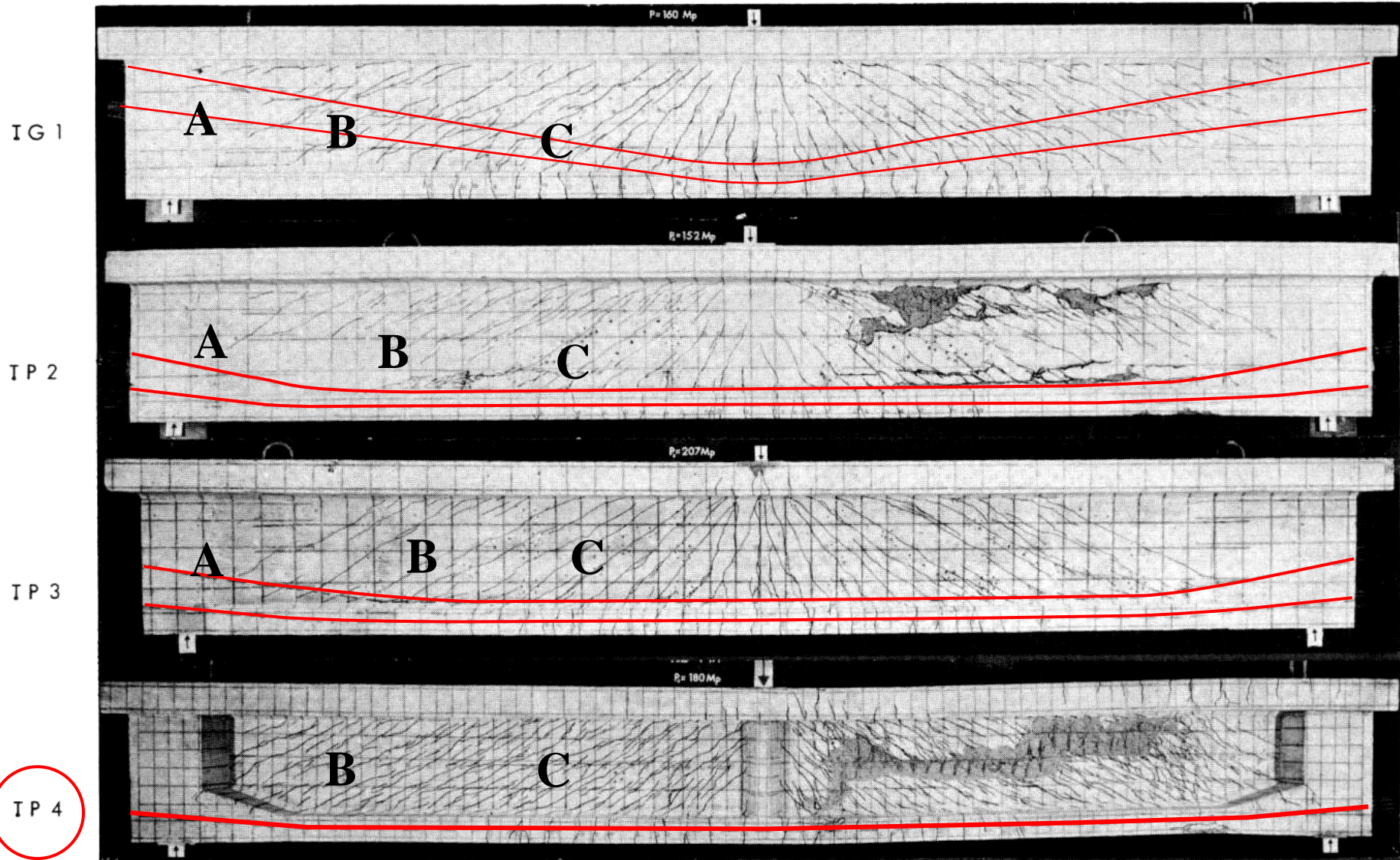
*Ensaio feito pelo Prof. Fritz Leonhardt – Stuttgart – Alemanha*



## Anexo 2 - Ensaios até à ruptura de Vigas Protendidas



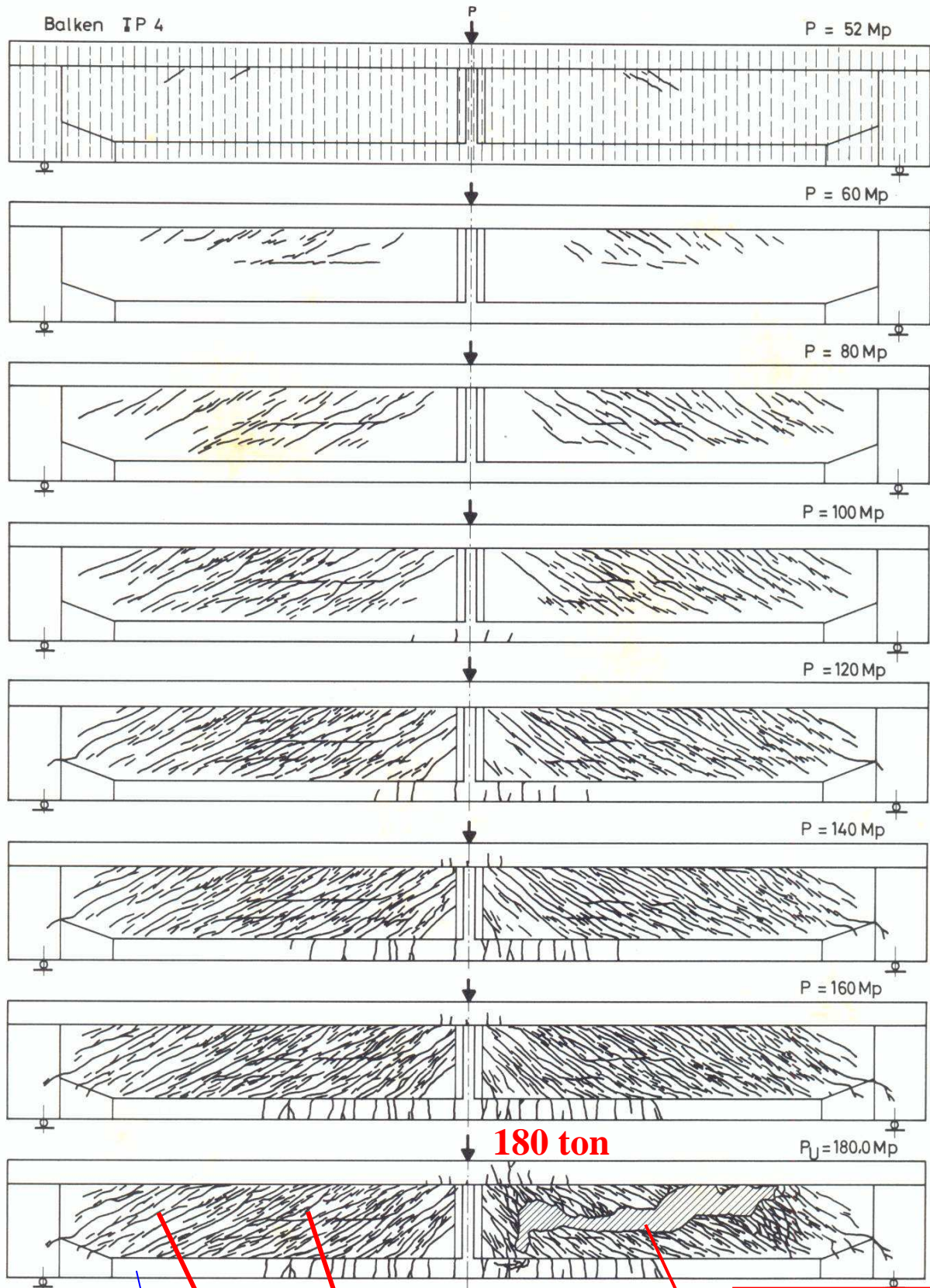
*Ensaios feitos pelo Prof. Fritz Leonhardt – Stuttgart – Alemanha*



Ruptura : Zona A sem qualquer fissura, Zona B sem fissura no bordo inferior , Zona C com fissura no bordo inferior



## Viga IP4



Sem fissuras no bordo

Esmagamento  
da biela