



INTRODUÇÃO

1 - Tema : Flambagem lateral de vigas pré-moldadas. Ver link :

http://aquarius.ime.eb.br/~webde2/prof/ethomaz/conc_protend/24_flambagem_lateral_vigas_premoldadas.pdf

2 - Ludwig Prandtl foi quem primeiro estudou a flambagem lateral de vigas em sua dissertação de 1900 em Munique.

3- Analisando a flambagem lateral de vigas **I**, Stephen Timoshenko , em 1932, fazia referência a um desastre em uma ponte ferroviária na França em 1897.

<http://doi.org/10.5169/seals-443> página 129 - Nota 2 de rodapé

4 - Em seu livro Theory of Elastic Stability - 1936 (1ª edição) e 1961 (2ª edição) , Timoshenko criou formulação para a verificação da estabilidade lateral de vigas **I** .

" The critical value of the load at which such a buckling may occur depends not only on the lateral rigidity of flanges but also on the torsional rigidity of the beam and on the ratio of the span length to the depth of the beam."

5 - Com base na formulação de Timoshenko pode-se fazer a verificação da estabilidade lateral de uma viga pré-moldada, com dimensões pré-definidas.

Essa verificação é feita para a situação de "transporte" da viga.

7- A formulação de Timoshenko trata de material homogêneo ao verificar a estabilidade lateral da viga.

8- Como as vigas pré-moldadas são de concreto protendido, ou de concreto armado, é necessário calcular também a armadura na mesa superior comprimida da viga.

Segundo Timoshenko :

" The usual approximate method of calculating critical stresses for laterally unsupported beams consists in considering the upper compressed flange of the beam as a column."

9 - Pode-se usar, para isso, a teoria do " Pilar-Padrão" apresentada por P.B.Fusco em seu Livro " Estruturas de Concreto - Solicitações Normais." Página 182. Foi feito um exemplo .

10 - Foram anexadas fotos para ilustrar o assunto.

11- Curiosidade : Na época do acidente ferroviário na France, 1897, muitos jornais não sabiam como imprimir fotos. <http://gares.loucrup65.fr/accidenttarbes.htm>

*"Les photographies ont été prises par M. Lacaze, notaire à Vic-Bigorre, qui disposait d'un **appareil photographique**. Les différents journaux ont fait des dessins à partir des photographies originales, car beaucoup de journaux de l'époque ne savaient pas imprimer les photographies."*

LUDWIG PRANDTL (1875 - 1953)

Foi quem primeiro estudou a flambagem lateral de vigas em sua dissertação de 1900 na Universidade de Munique

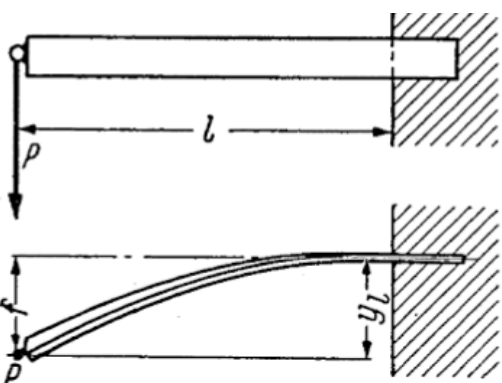
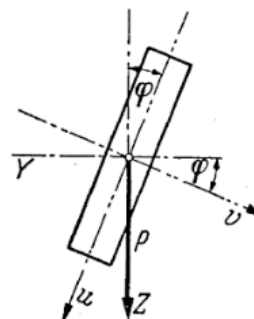
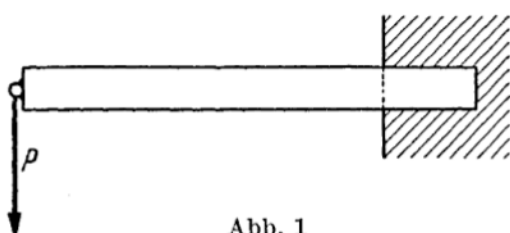
Dissertation der Universität München 1900, S. 1—75

Kipperscheinungen

Ein Fall von instabilem elastischem Gleichgewicht

Lateral torsional buckling

A case of unstable elastic equilibrium



$$M_B = \frac{B}{\varrho} \quad \text{Flexão}$$

$$M_T = C \frac{d\varphi}{dx} \quad \text{Torção}$$

Carga de Flambagem Lateral

$$P_k = \frac{4,0126}{l^2} \sqrt{B C}$$

Ludwig Prandtl

Gesammelte Abhandlungen

zur angewandten Mechanik, Hydro- und Aerodynamik

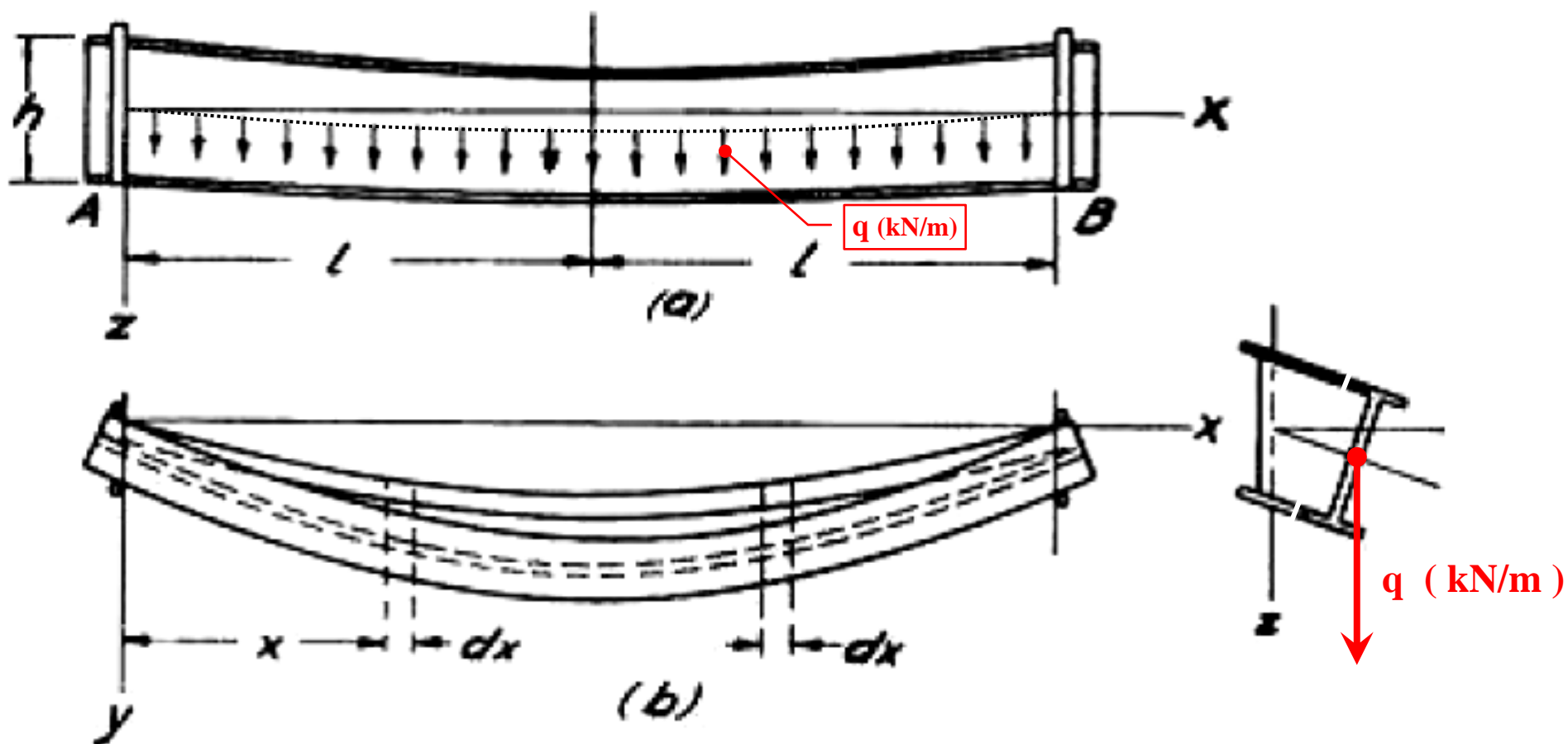
https://link.springer.com/chapter/10.1007/978-3-662-11836-8_2

AS VIGAS PRÉ- MOLDADAS **I** PODEM FLAMBAR LATERALMENTE DURANTE O TRANSPORTE

STEPHEN P. TIMOSHENKO foi dos primeiros a estudar essa flambagem lateral.

<http://doi.org/10.5169/seals-443> página 129

Stability of plate girders subjected to bending



The usual approximate method of calculating critical stresses for laterally unsupported beams consists in considering the upper compressed flange of the beam as a column.

Stability of plate girders subjected to bending

<https://www.e-periodica.ch/digbib/view?pid=bse-cr-001:1932:1::13> página 129

Autor(en): Timoshenko, S.

Zeitschrift: IABSE congress report = Rapport du congrès AIPC = IVBH
Kongressbericht

Band (Jahr): 1 (1932)

Introduction.

In the design of a plate girder not only the stresses but also the elastic stability of the structure should be considered. It is well known¹ that an I-beam bent in the plane of the web may prove to be insufficiently stable and buckle sidewise. The critical value of the load at which such a buckling may occur depends not only on the lateral rigidity of flanges but also on the torsional rigidity of the beam and on the ratio of the span length to the depth of the beam. To eliminate this kind of instability an adequate system of lateral bracing is necessary².

The usual approximate method of calculating critical stresses for laterally unsupported beams consists in considering the upper compressed flange of the beam as a column.

¹ - Ludwig Prandtl - The title of the dissertation was: *Kipperscheinungen, ein Fall vom instabilem elastischem Gleichgewicht* (Lateral torsional buckling : A case of unstable elastic equilibrium), 1899.

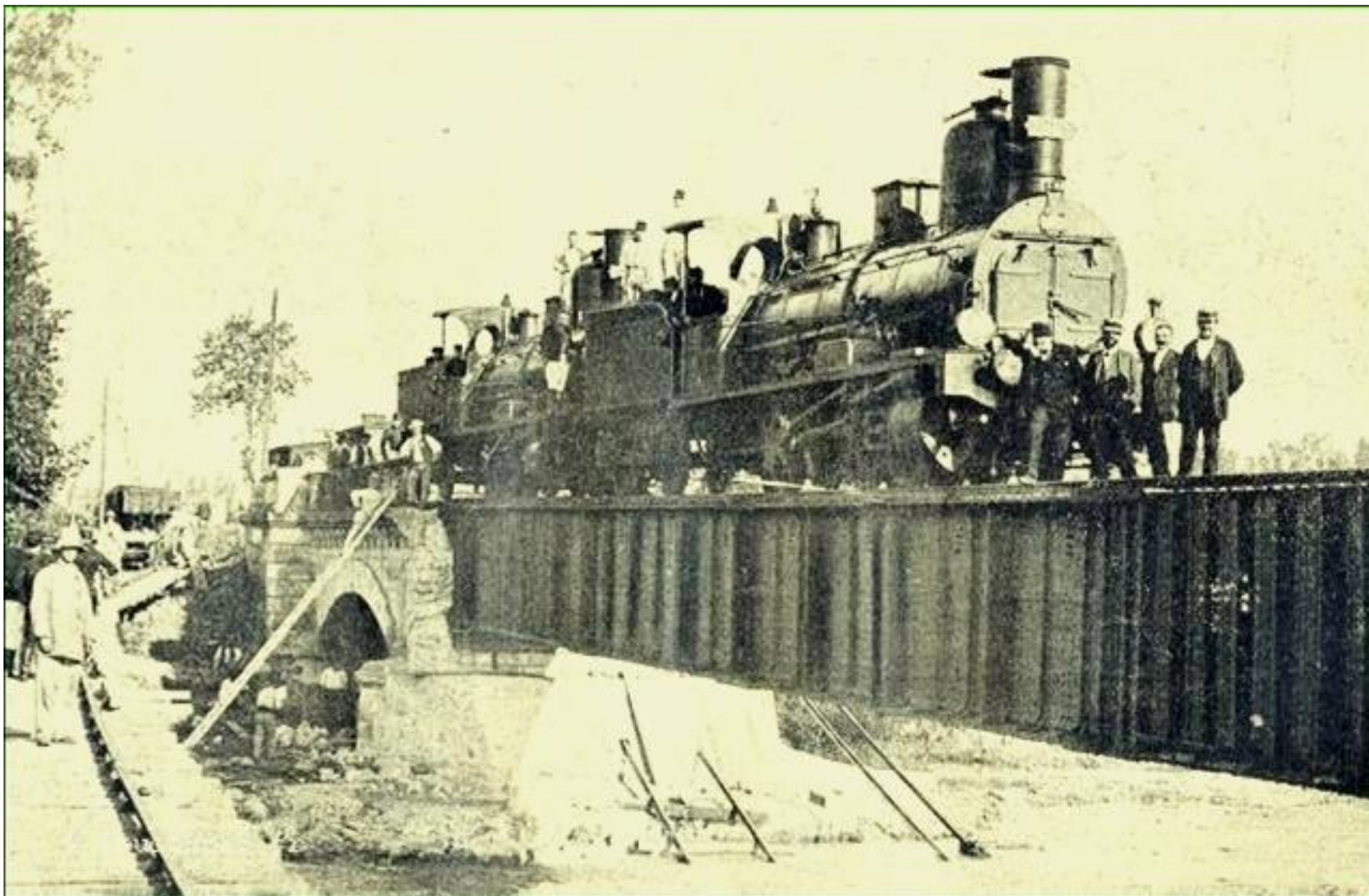
 ² - An example of the failure of girders in consequence of sidewise buckling is given by the bridge disaster near Tarbes, La Revue Technique, November 15, 1897.

L'accident de Tarbes en 1897

<http://gares.loucrup65.fr/accidenttarbes.htm>

Le 17 juillet, tout est prêt pour la dernière épreuve de test.

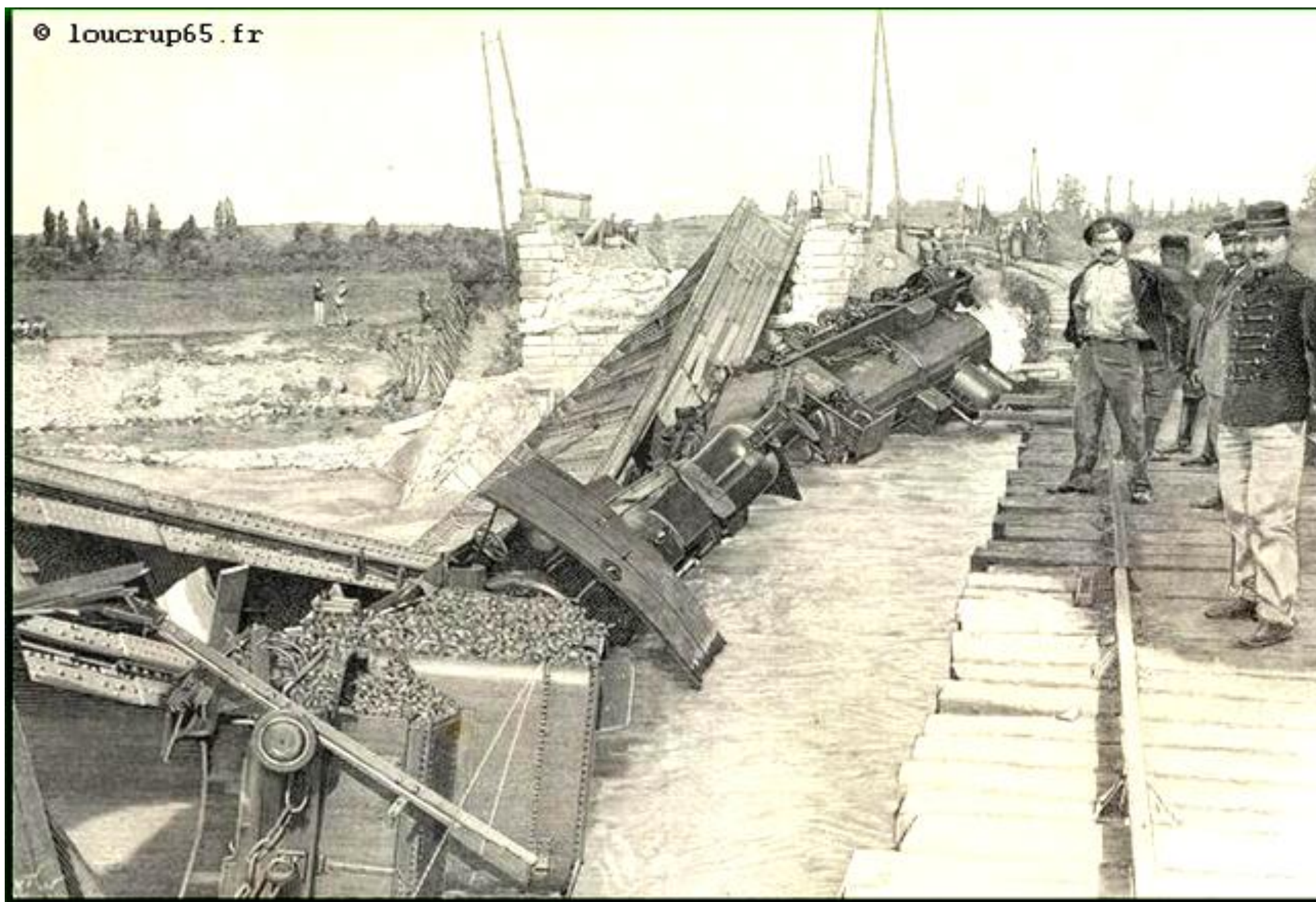
Deux locomotives chargées d'ingénieurs, officiers et agents de la compagnie s'apprêtent à franchir ce nouveau pont.



L'accident de Tarbes en 1897

<http://gares.loucrup65.fr/accidenttarbes.htm>

Malheureusement, le test est loin d'être concluant. Le pont s'affaisse et tout le convoi se couche sur le côté.



L'accident de Tarbes en 1897

<http://gares.loucrup65.fr/accidenttarbes.htm>



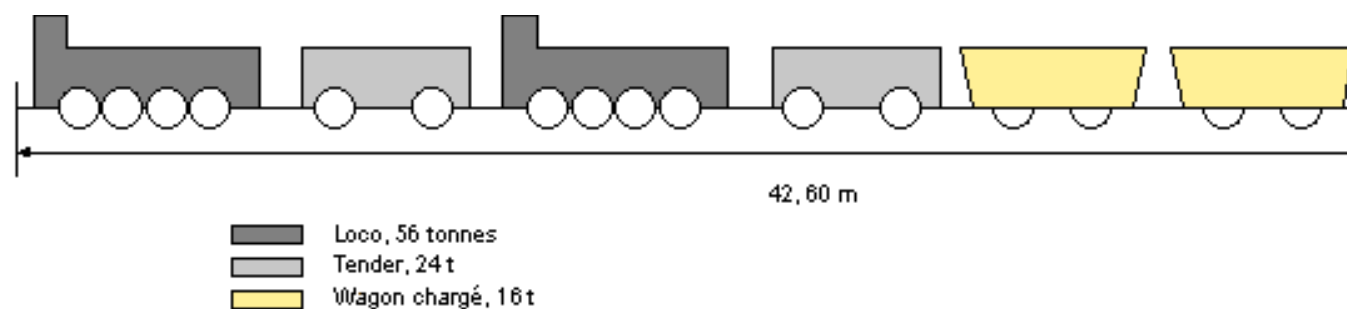
Cet ouvrage atteignant 45 mètres de portée, et dont les poutres, espacées seulement de 1, 50 m d'axe en axe et mesurant 2, 20 m de hauteur, supportent directement les rails.

La première locomotive avait presque entièrement franchi le pont lorsque cet ouvrage a fléchi et précipité dans l'Adour le matériel qu'il supportait. La charge était, à ce moment, d'environ 160 tonnes.

http://gares.loucrup65.fr/wp_generated/wp4afed08a_05_06.jpg ; http://agmauran.pagesperso-orange.fr/breves/pont_fer_tarbes0.htm



Premier test : Epreuve par poids mort, consistant à laisser séjourner, pendant environ une demi-heure, sur le milieu du pont, une locomotive avec son tender, pesant ensemble 72 tonnes, et l'on n'avait constaté qu'une flèche dépassant à peine la flèche calculée et de nature à n'inspirer aucune inquiétude. Second Test : Consistait à faire stationner sur le pont un train composé de deux locomotives pesant chacune 54 tonnes, et 72 tonnes avec leur tender, et d'un certain nombre de wagons pesant de 15 à 16 tonnes. La charge était d'environ 160 tonnes.





Assinalada na figura a flambagem local da mesa comprimida, numa seção com grande momento fletor, sob as rodas da locomotiva.

PÁGINA EM BRANCO

As Vigas pré-moldadas devem ter dimensões que evitem a flambagem lateral durante o transporte.

A armadura da mesa superior deve ser calculada para esse fim.



https://www.eec.ufg.br/up/140/o/VERIFICA%C3%87%C3%83O_DA_INSTABILIDADE_LATERAL_DE_VIGAS_PR%C3%89-MOLDADAS_DE_CONCRETO_EM_FASES_TRANSIT%C3%93RIAS.pdf

https://upcommons.upc.edu/bitstream/handle/2099.1/25533/Victor_Herrando_Lateral_stability_analysis_of_long_precast_prestressed_concrete_beams.pdf



Viga com instabilidade lateral

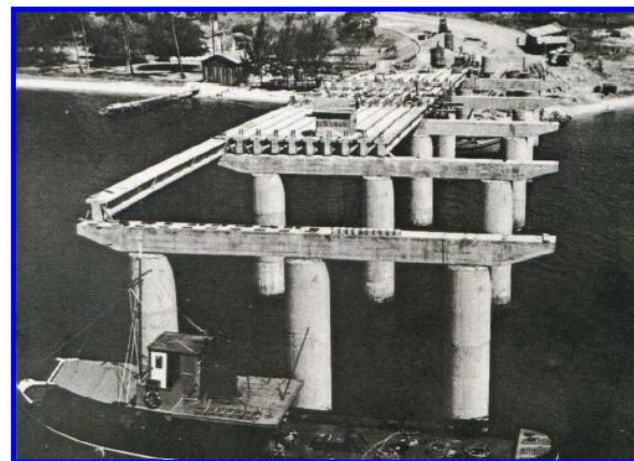
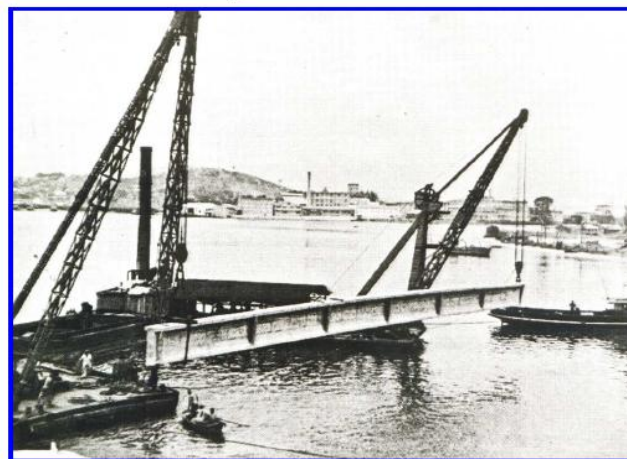
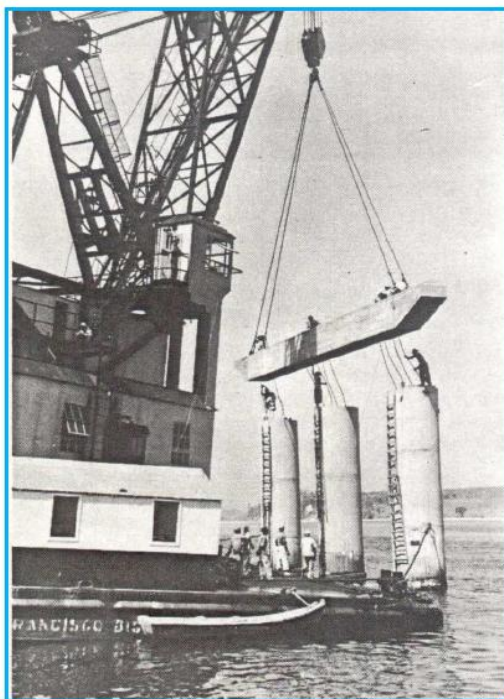
PONTE DO GALEÃO / RJ - 1948

PRIMEIRA PONTE COM VIGA PRÉ- MOLDADA PROTENDIDA NO BRASIL

1948 - PONTE DO GALEÃO - FREYSSINET- <http://dx.doi.org/10.5169/seals-4102> página 405 / 418

" Enfin, il faut mentionner un pont à poutres actuellement en construction au Brésil, de 329 mètres de longueur et 20 mètres de largeur, comportant treize travées indépendantes : deux de 28^m30, deux de 37^m20, huit de 19^m40 et une de 43^m40. "

Fotos do livro "O CONCRETO NO BRASIL" – VOL 1 do Prof. Augusto Carlos de Vasconcelos.

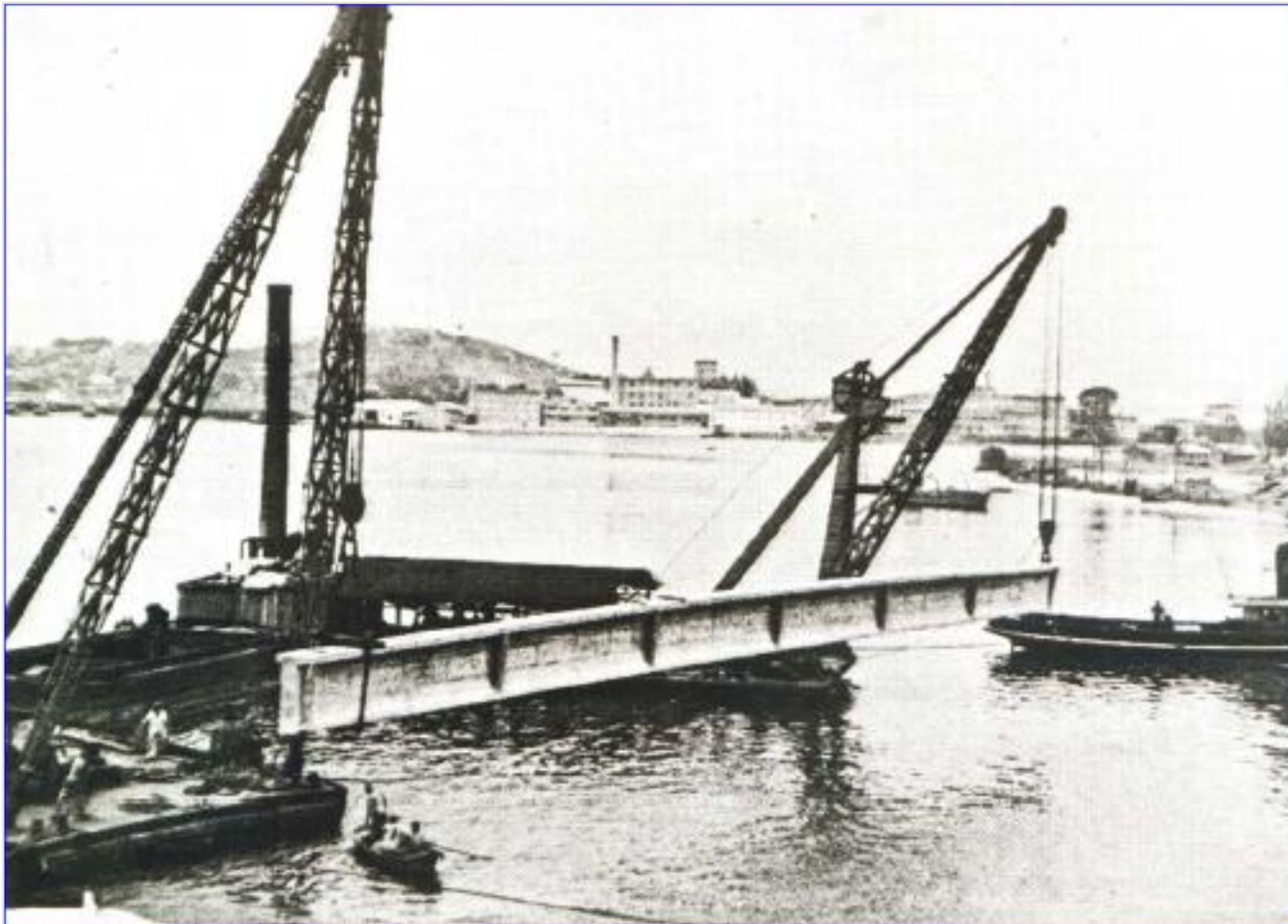


- Tubulões a ar comprimido assentes na rocha ≈ a 20m de profundidade.
- Travessas de apoio prêmoldadas.
- Vigas longitudinais prêmoldadas protendidas e justapostas transversalmente. Protensão total, isto é, sem tensões de tração na flexão.
- Vãos de 19.40m, 28.30m, 37.20m e 43.40m

Fotos S.T.U.P. de 1948 / 1949

1948 - STUP

DURANTE O TRANSPORTE A VIGA PRÉ MOLDADA PODE FLAMBAR LATERALMENTE



STUP

1948 - B. ENSAIO DA VIGA DA PONTE DO GALEÃO

http://aquarius.ime.eb.br/~webde2/prof/ethomaz/lobocarneiro/ponte_galeao.pdf

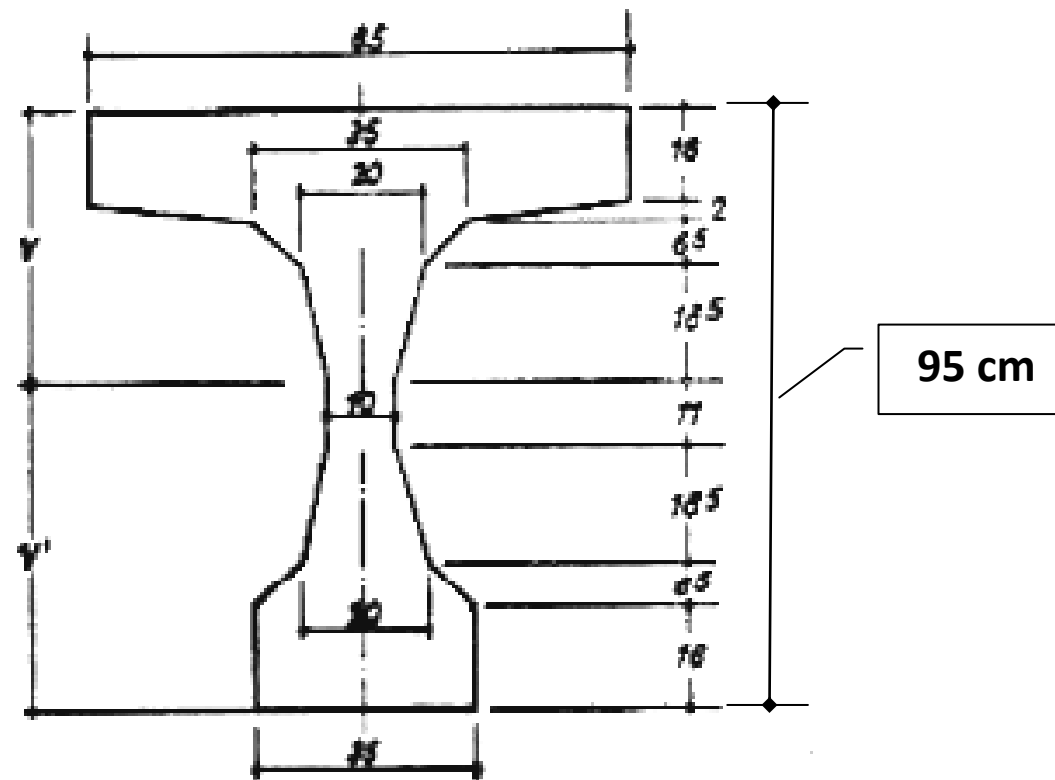


Fig. 9

STUP

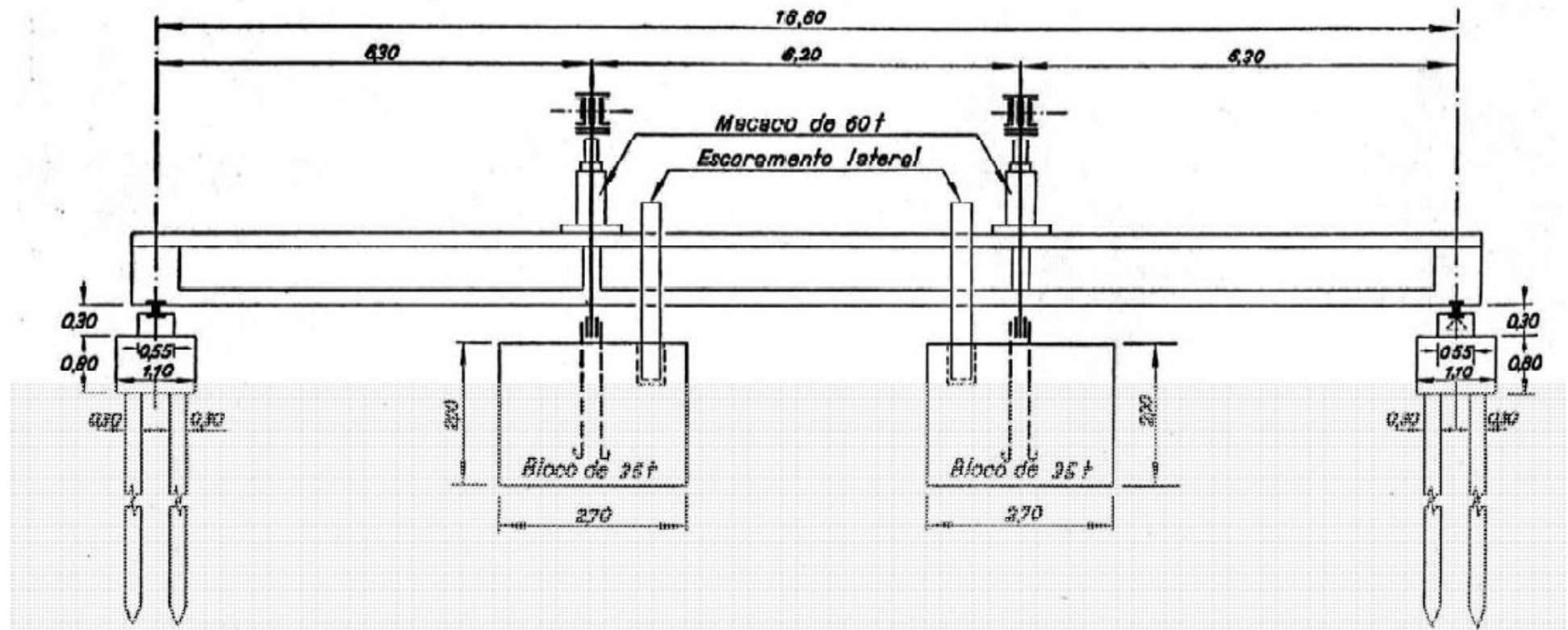
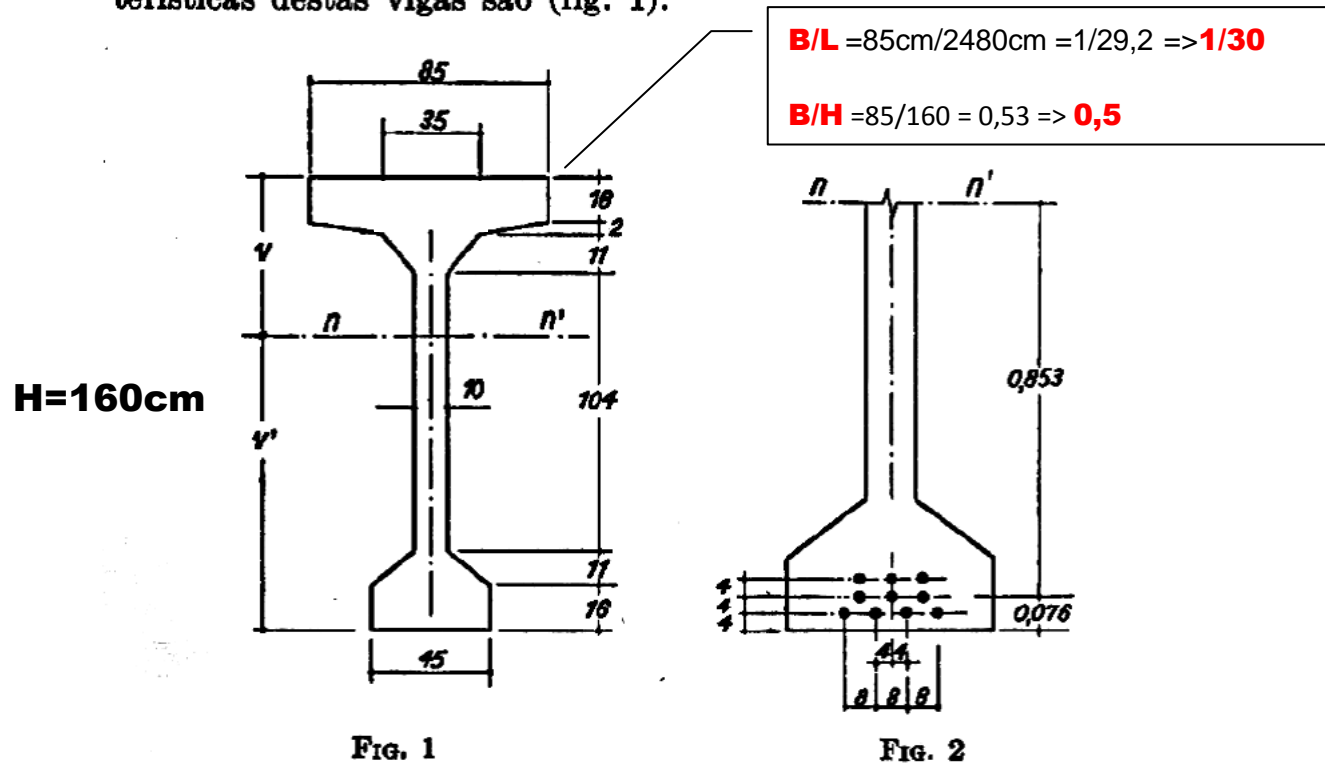


Fig. 10

STUP

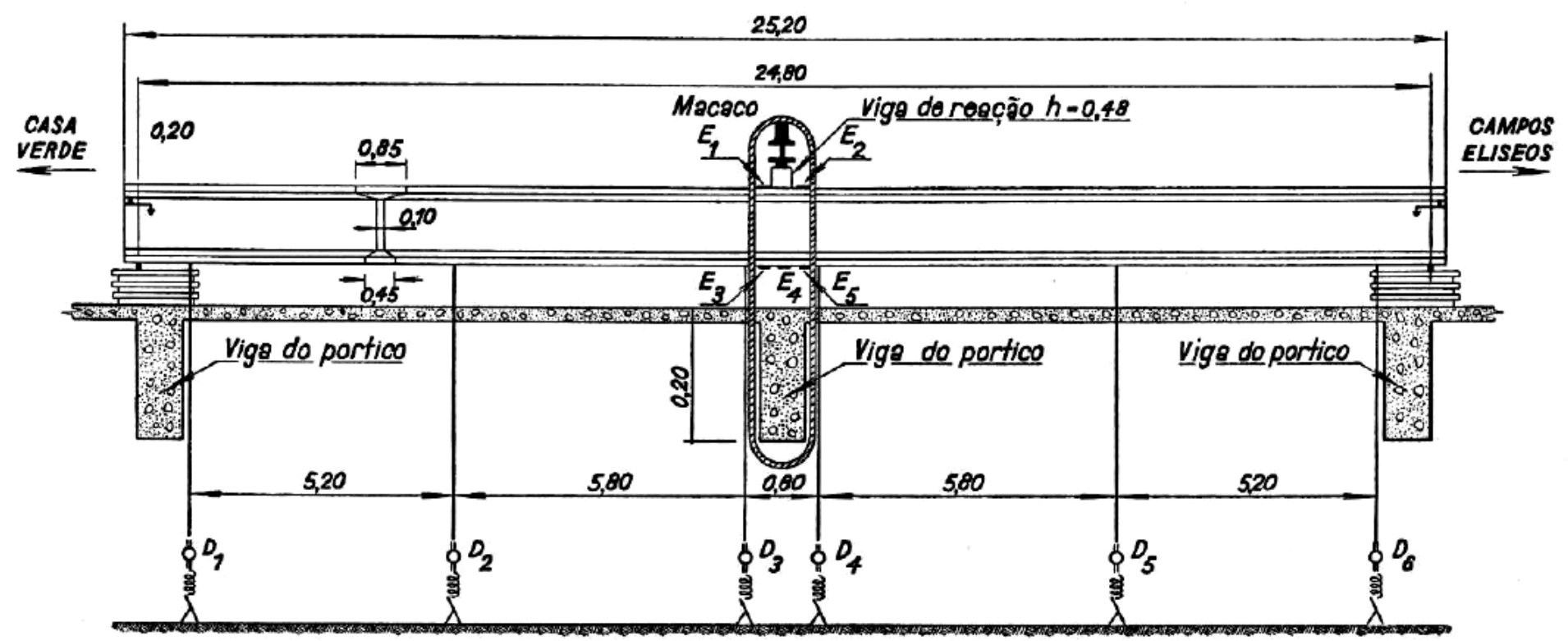
A. Ensaio da viga do Viaduto Rio Branco 1959

1. O viaduto Rio Branco tem vigas simplesmente apoiadas de 25,20 m de comprimento e 24,80 m de vão teórico. As características destas vigas são (fig. 1).



altura $h = 1,600$ m
 área secção $s = 0,379$ m²
 centro gravidade

STUP



Aparelhagem utilizada
D ϕ Defletometro
C \rightarrow Clinometro
E - Extensometro

Fig. 4

PÁGINA EM BRANCO

VIGAS PRÉ-MOLDADAS

FLÓRIDA

<https://www.google.com.br/urlbvm=bv.147448319,d.Y2I&cad=rja>



FLÓRIDA

<https://www.google.com.br/urlbvm=bv.147448319,d.Y2I&cad=rja>



FLÓRIDA



FLÓRIDA

PÁGINA EM BRANCO

VERIFICAÇÃO DA ESTABILIDADE LATERAL

STEPHEN TIMOSHENKO & JAMES M. GERE

THEORY OF ELASTIC STABILITY

ITEM 6.4 ; PÁGINA 267

LATERAL BUCKLING OF SIMPLY SUPPORTED **I** BEAMS

UNIFORM LOAD

$$(q \cdot L) \text{ critico} = Y_4 \times \left[\frac{\sqrt{(E \times I_y) \times (G \times J)}}{L^2} \right]$$

$E \times I_y$ = Rigidez à Flexão no plano horizontal ;

$C = G \times J$ = Rigidez à Torção

$C_1 = E \times C_w$ = Rigidez ao Empenamento

Valores de Y_4 para vigas I com carga uniforme							
Carga aplicada no ↓	$\left(\frac{L^2 \times C}{C_1} \right)$						
	0,4	4	8	16	24	32	48
Bordo superior	92,9	36,3	30,4	27,5	26,6	26,1	25,9
Centróide	143	53,0	42,6	36,3	33,8	32,6	31,5
Bordo inferior	223	77,4	59,6	48,0	43,6	40,5	37,8

Valores de Y_4 para vigas I com carga uniforme							
Carga aplicada no	$\left(\frac{L^2 \times C}{C_1} \right)$						
	64	80	128	200	280	360	400
Bordo superior	25,9	25,8	26,0	26,4	26,5	26,6	26,7
Centróide	30,5	30,1	29,4	29,0	28,8	28,6	28,6
Bordo inferior	36,4	35,1	33,3	32,1	31,3	31,0	30,7

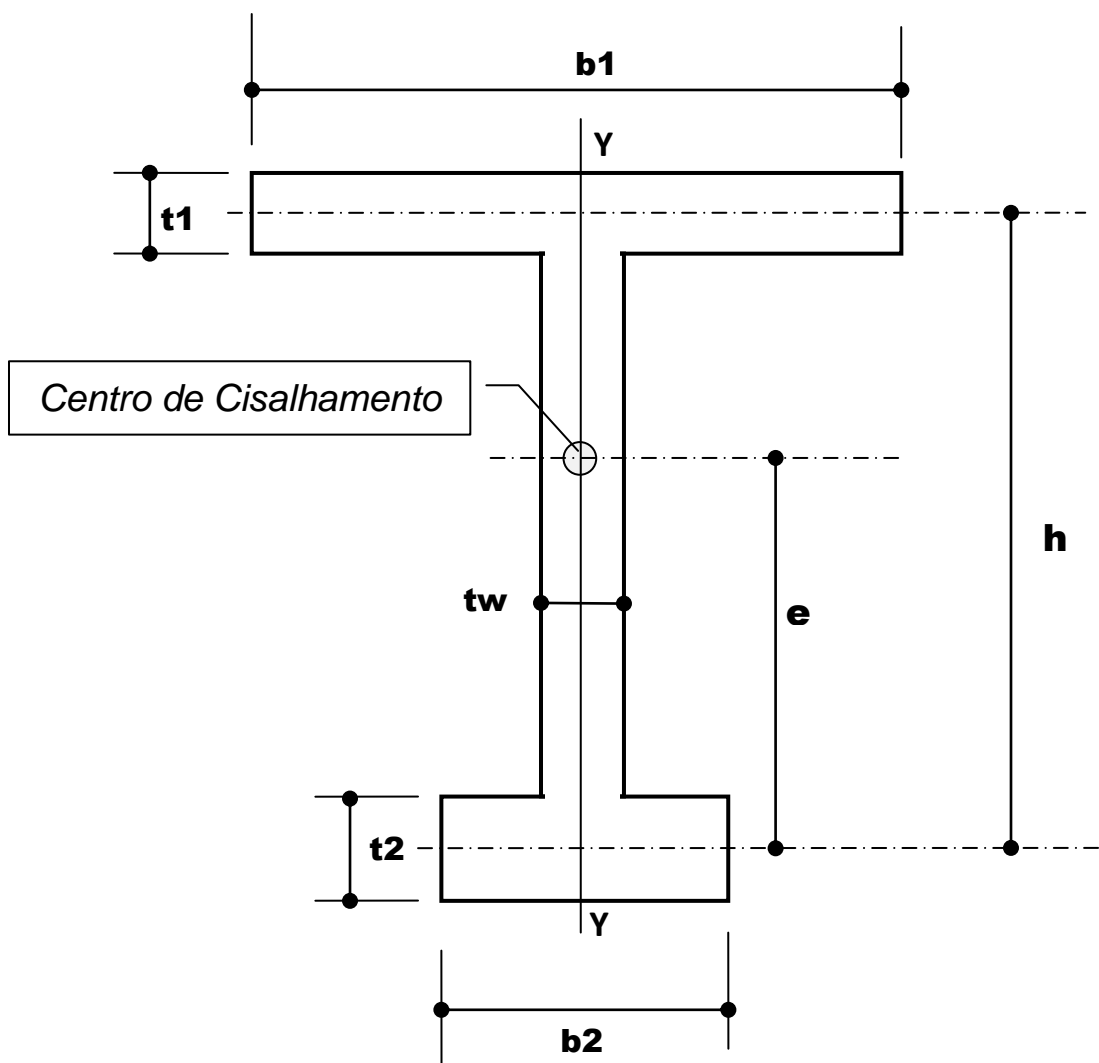
Propriedades da seção transversal -

Livro de S. P. TIMOSHENKO - Elastic Stability - páginas 253 e 530

e Livro de R.J. ROARK - Stress and Strain - página 302

$C = G \times J =$ Rigidez à Torção

$C_1 = E \times C_w =$ Rigidez ao Empenamento



$$e = \frac{t_1 \times b_1^3 \times h}{t_1 \times b_1^3 + t_2 \times b_2^3} \quad ; \quad I_y = \frac{t_1 \times b_1^3}{12} + \frac{t_2 \times b_2^3}{12} + \frac{\left(h - \frac{t_1}{2} - \frac{t_2}{2} \right) \times t_w^3}{12}$$

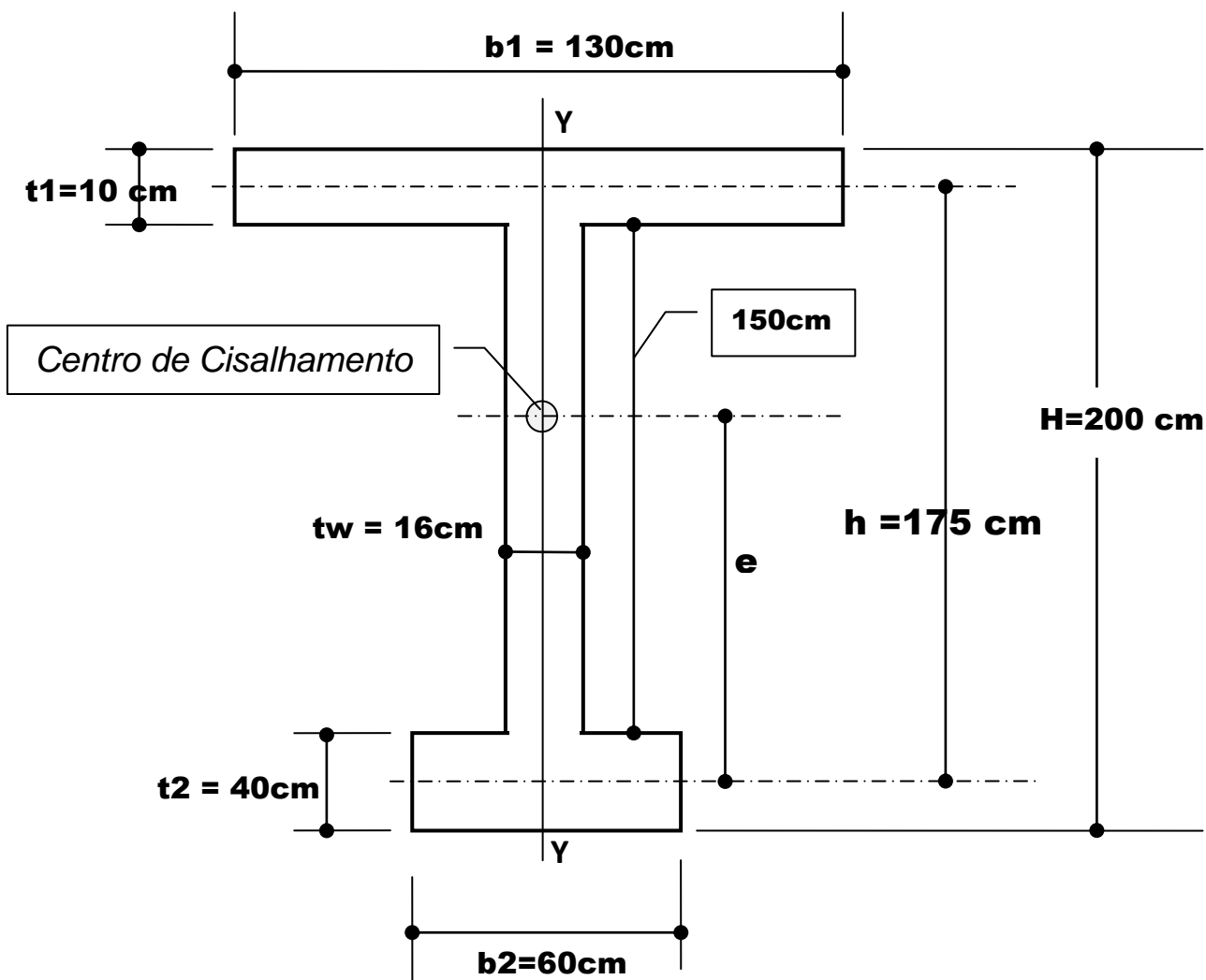
$$J = \left(\frac{1}{3} \right) \times \left(t_1^3 \times b_1 + t_2^3 \times b_2 + t_w^3 \times h \right) \quad ; \quad C_w = \frac{h^2 \times t_1 \times t_2 \times b_1^3 \times b_2^3}{12 \times \left(t_1 \times b_1^3 + t_2 \times b_2^3 \right)}$$

PÁGINA EM BRANCO

EXEMPLO

EXEMPLO

VÃO (de apoio a apoio) = 40 metros



Área / C.G. =

$$1,30\text{m} \times 0,10\text{m} = 0,13\text{ m}^2 \quad \times 1,75\text{m} = 0,2275\text{ m}^3$$

$$0,60\text{m} \times 0,40\text{m} = 0,24\text{ m}^2 \quad \times \text{zero} = \text{zero}$$

$$1,50\text{m} \times 0,16\text{m} = 0,24\text{ m}^2 \quad \times 0,95\text{m} = 0,228\text{ m}^3$$

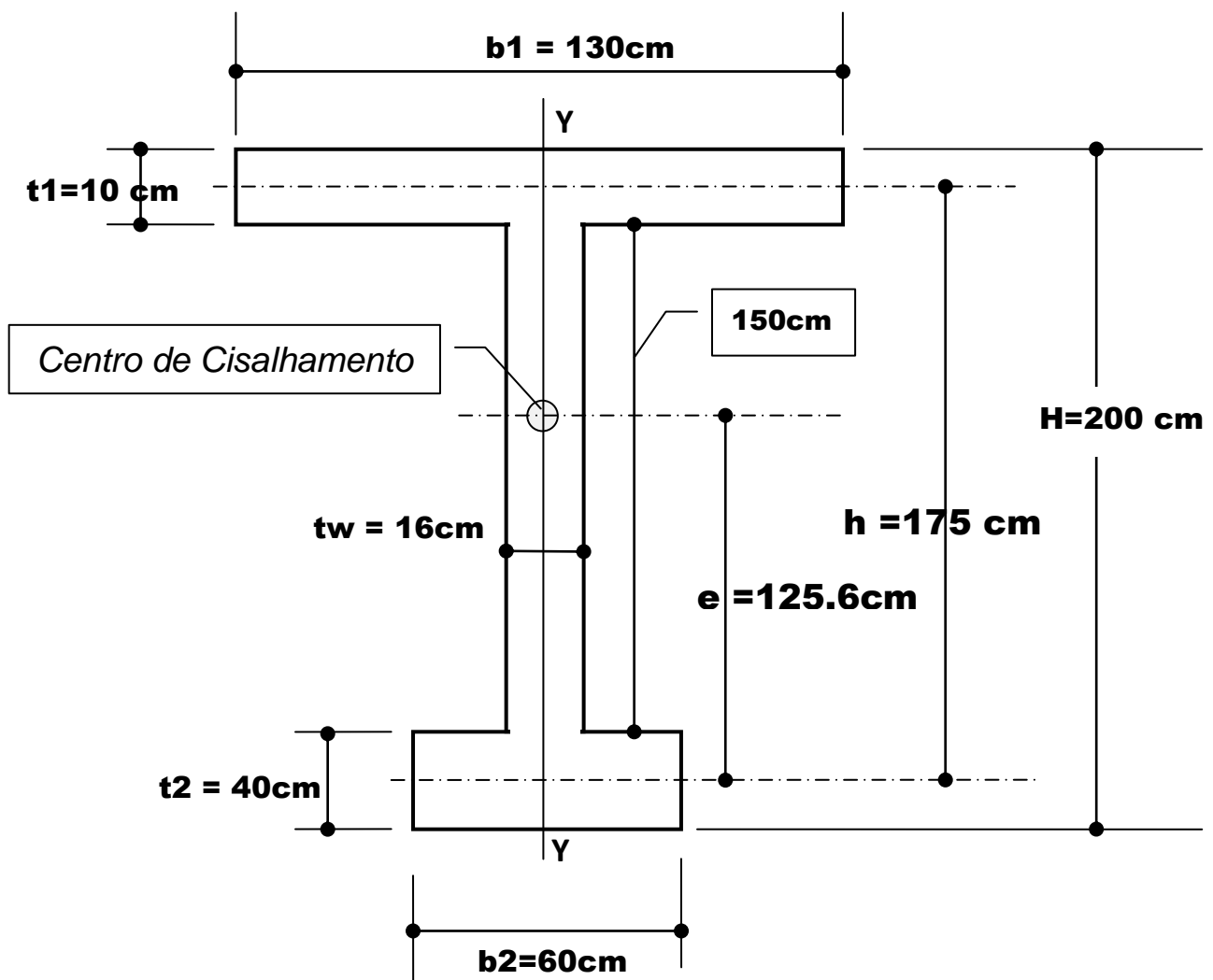
$$\text{Área} = 0,61\text{ m}^2 \quad Y_{\text{c.g.}} = 0,4555\text{m}^3 / 0,61\text{m}^2 = 0,7467\text{ m}$$

$$\text{peso próprio} = g = 0,61 \times 25\text{ kN/m}^3 = 15,25\text{ kN/m}$$

$$e = \frac{t_1 \times b_1^3}{t_1 \times b_1^3 + t_2 \times b_2^3} \times h =$$

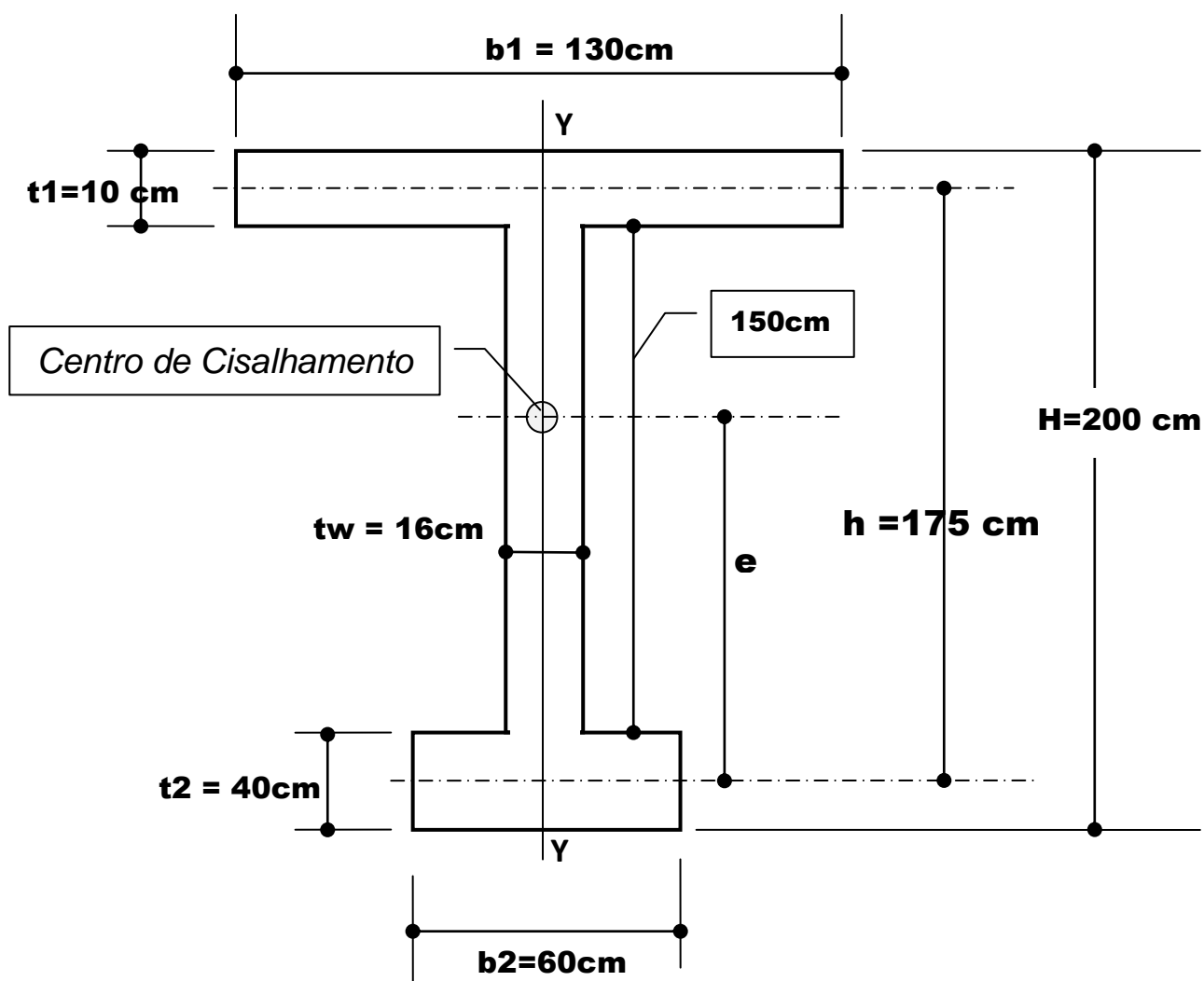
$$e = \frac{0,10\text{m} \times (1,30\text{m})^3}{0,10 \times (1,30\text{m})^3 + 0,40\text{m} \times (0,60\text{m})^3} \times 1,75\text{m} = \frac{0.2197}{0.2197 + 0.0864} \times 1.75\text{m} =$$

$$e = \frac{0.2197}{0.3061} \times 1,75\text{m} = 1.256..m$$



$$I_y = \frac{0,10 \times 1,30^3}{12} + \frac{0,40 \times 0,60^3}{12} + \frac{1,50 \times 0,16^3}{12}$$

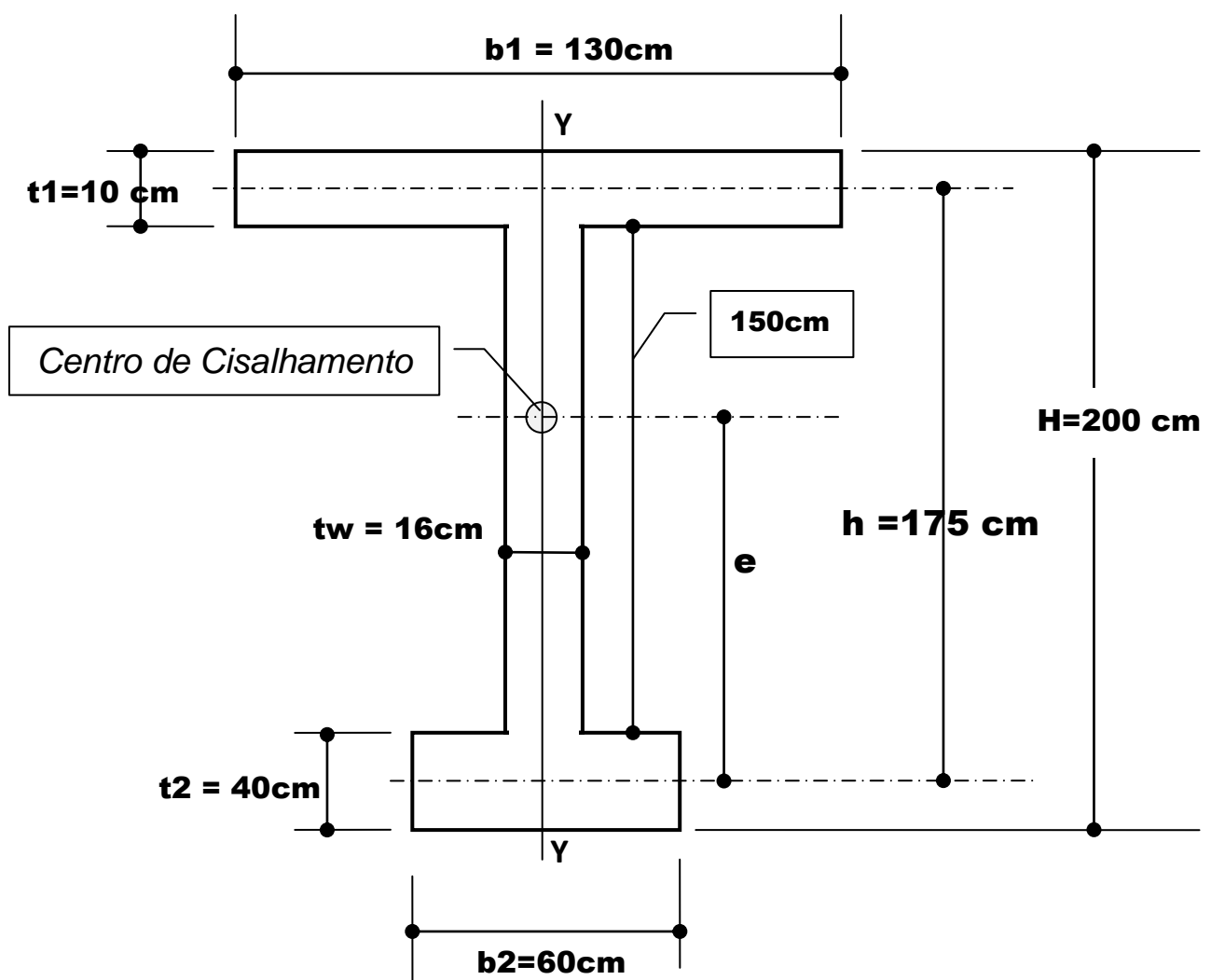
$$I_y = 0.018308 + 0.0072 + 0.000512 = 0.026020 \text{ m}^4$$



$$J = \left(\frac{1}{3}\right) \times (t_1^3 \times b_1 + t_2^3 \times b_2 + t_w^3 \times h)$$

$$J = \left(\frac{1}{3}\right) \times (0,10^3 \times 1,30 + 0,40^3 \times 0,60 + 0,16^3 \times 1,50) =$$

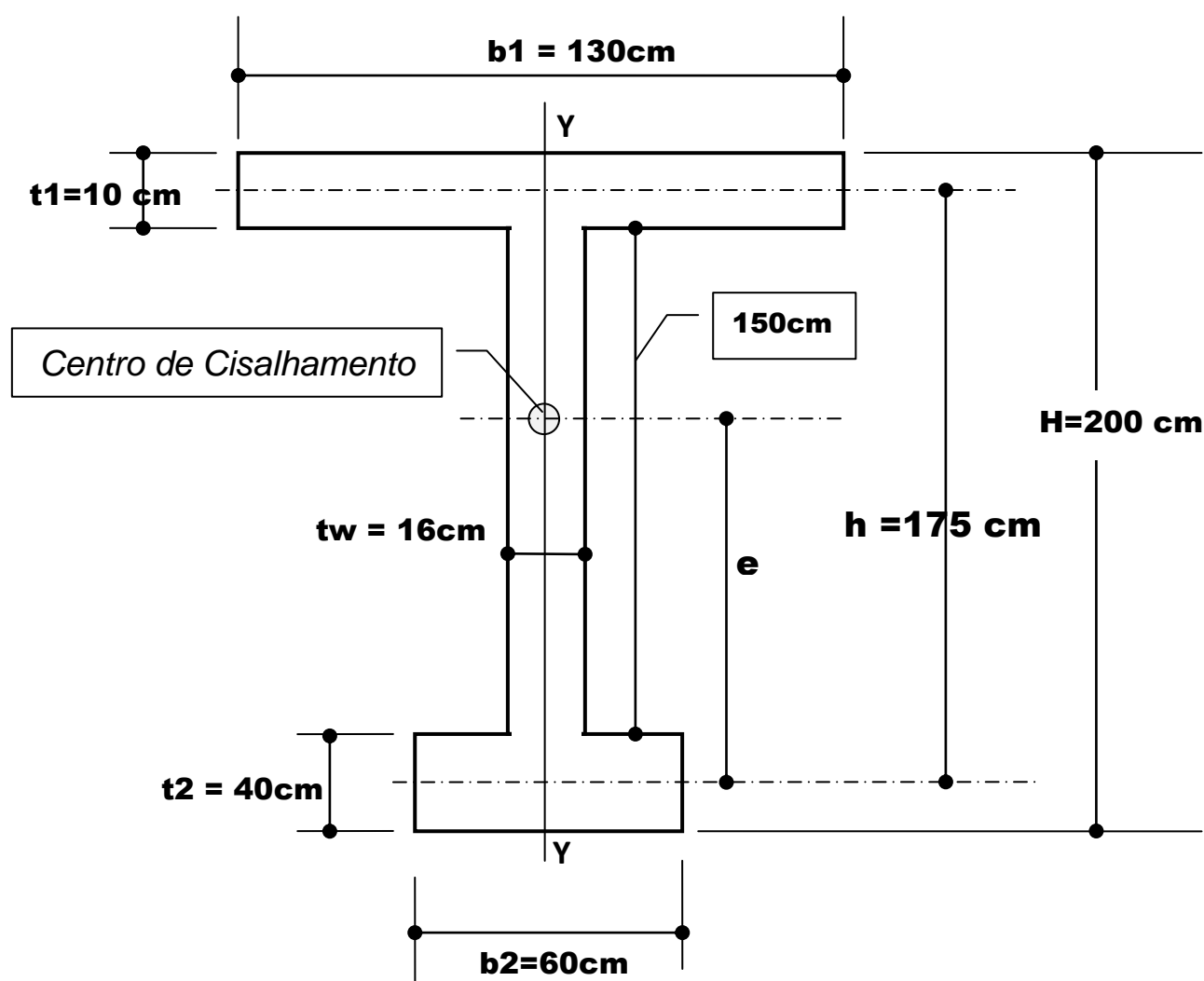
$$J = \left(\frac{1}{3}\right) \times (0,0013 + 0,0384 + 0,006144) = 0,045844 \text{ m}^4$$



$$C_w = \frac{h^2 \times t_1 \times t_2 \times b_1^3 \times b_2^3}{12 \times (t_1 \times b_1^3 + t_2 \times b_2^3)} = \frac{(t_1 \times b_1^3) \times (t_2 \times b_2^3)}{12 \times (t_1 \times b_1^3 + t_2 \times b_2^3)} \times h^2$$

$$C_w = \frac{(0,10 \times 1,30^3) \times (0,40 \times 0,60^3)}{12 \times (0,10 \times 1,30^3 + 0,40 \times 0,60^3)} \times 1,75^2 =$$

$$C_w = \frac{0,01898}{12 \times (0,2197 + 0,0864)} \times 1,75^2 = 0,015824 \text{ m}^6$$



$$(q \cdot L) \text{ critico} = Y4 \times \left[\frac{\sqrt{(E \times I_y) \times (G \times J)}}{L^2} \right]$$

Módulo de Elasticidade = 250 000 kgf/cm² = 25 000 MPa = 25 GPa

$$E = 25 \text{ GPa} = 25 \text{ GN/m}^2$$

Módulo de Cisalhamento $G = 0,4 \times E = 10 \text{ GN/m}^2$

Rigidez à Torção = $C = G \times J = (10 \text{ GN/m}^2) \times (J = 0.045844 \text{ m}^4) = 0.45844 \text{ GN.m}^2$

Rigidez ao Empenamento = $C_1 = E \times C_w = (25 \text{ GN/m}^2) \times (C_w = 0.015824 \text{ m}^6) = 0.3956 \text{ GN.m}^4$

$$\left(\frac{L^2 \times C}{C_1} \right) = \frac{(40\text{m})^2 \times 0,45844 \text{ GN.m}^2}{0,3956 \text{ GN.m}^4} = 1854$$

Valores de Y4 para vigas I com carga uniforme							
Carga aplicada no	$\left(\frac{L^2 \times C}{C_1} \right)$						
	64	80	128	200	280	360	400
Bordo superior	25,9	25,8	26,0	26,4	26,5	26,6	26,7
Centróide	30,5	30,1	29,4	29,0	28,8	28,6	28,6 - limite ==> 24
Bordo inferior	36,4	35,1	33,3	32,1	31,3	31,0	30,7

Dai resulta : $Y4 = 28,6$;

$$(q.L)_{\text{critico}} = Y4 \times \left[\frac{\sqrt{E \times I_y \times C}}{L^2} \right] = 28,6 \times \frac{\sqrt{\left(25 \frac{\text{GN}}{\text{m}^2}\right) \times \left(0,026020 \text{ m}^4\right) \times \left(0,45844 \text{ GN} \cdot \text{m}^2\right)}}{(40\text{m})^2} = 1950 \text{ kN}$$

q crítico = 1950 kN / 40m = 48.75 kN / m

Como o peso próprio é 15.25 kN/m

O coeficiente de segurança é $48,75 / 15,25 = 3,20$ **OK**

Esse valor do coeficiente de segurança à flambagem lateral deve ser maior que 3,0.

Aí incluído o impacto vertical devido ao transporte $1,4 \times 1,3 = 1,82$

NBR 9062 - Projeto e execução de estruturas de concreto pré-moldado

5.2.3.6 Manuseio no canteiro e na montagem

5.2.3.6.1 Durante o manuseio e a montagem, os elementos são submetidos a ações dinâmicas.

5.2.3.6.2 Na fixação da carga estática equivalente, deve ser usado o coeficiente $\beta_a = 1,3$

CÁLCULO DA ARMADURA

Dimensionamento da armadura da mesa superior - NBR6118 :

15.8.3.3.2 Método do pilar-padrão com curvatura aproximada

Pode ser empregado apenas no cálculo de pilares com $\lambda < 90$, com seção constante e armadura simétrica e constante ao longo de seu eixo.

$$\lambda = \frac{L \text{ flambagem}}{i} = \frac{40m}{\left(\frac{1,30m}{\sqrt{12}}\right)} = 106$$

aceitável.

A não linearidade geométrica é considerada de forma aproximada, supondo-se que a deformação da barra seja senoidal.

A não linearidade física é considerada através de uma expressão aproximada da curvatura na seção crítica.

O momento total máximo no pilar deve ser calculado pela expressão:

$$M_{d,tot} = \alpha_b M_{1d,A} + N_d \frac{\ell_e^2}{10 r} \geq M_{1d,A}$$

sendo $1/r$ a curvatura na seção crítica, que pode ser avaliada pela expressão aproximada:

$$\frac{1}{r} = \frac{0,005}{h(v+0,5)} \leq \frac{0,005}{h}$$

onde

$$v = N_d / (A_c f_{cd})$$

onde

h é a altura da seção na direção considerada;

v é a força normal adimensional;

Considerando a carga que conduz à flambagem lateral = q crítico = 48,75 kN/m ,
o momento fletor no meio do vão será :

$$M = q \text{ crit.} \times L^2 / 8 = 48,75 \text{ kN/m} \times (40\text{m})^2 / 8 = 9750 \text{ kN.m}$$

Como o peso próprio é 15.25 kN/m e com impacto vertical de manuseio de 1,30

NBR 9062 - Projeto e execução de estruturas de concreto pré-moldado

5.2.3.6.1 Durante o manuseio e a montagem, os elementos são submetidos a ações dinâmicas.

5.2.3.6.2 Na fixação da carga estática equivalente, deve ser usado o coeficiente $\beta_a = 1,3$

$$q \text{ impacto de manuseio} = 1,3 \times 15,25 \text{ kN/m} = 19,83 \text{ kN/m}$$

$$M_d \text{ plano vertical} = 19,83 \text{ kN/m} \times (40\text{m})^2 / 8 = 3966 \text{ kN.m} \quad (\text{já majorado de 1,3 do impacto vertical})$$

O braço de alavanca interno vale :

$$z = 2,0\text{m} - (16\text{cm} / 2 = \text{laje superior}) - 20\text{cm} \text{ (C.G. } \approx \text{ dos cabos no talão inferior)} = 172 \text{ cm}$$

$$R_d \text{ tração no talão inferior} = \text{Força de tração nos cabos} = 3966 \text{ kN.m} / 1,72\text{m} = 2305 \text{ kN}$$

Área de cabos necessária =

$$= (2305 \text{ kN} \times 1,4) / (f_{yk} = 170 \text{ kN/cm}^2 \text{ CP190RB} / 1,15) = 21.84\text{cm}^2 \Rightarrow$$

$$\gggg \text{ 2 cabos 12 cordoalhas } 12,5\text{mm} = 24 \text{ cm}^2 \text{ OK}$$

Força de compressão no laje superior da viga N impacto = 2305 kN (já com impacto de manuseio)

Tensão de cálculo no concreto = $N_d / b_1 \times t_1 = 2305 \text{ kN} \times 1,4 / (1,3\text{m} \times 0,10\text{m}) = 24,8\text{MPa}$

Considerando um concreto com $f_{ck} = 40\text{MPa}$

$$v = 24,8\text{MPa} / (40\text{MPa}/1,4) = 0,87$$

Considerado que a carga na mesa não é concentrada na extremidade, usaremos o critério do Prof. P.B.Fusco , considerando a carga triangular , i.e. similar à variação da força cortante da viga e portanto da força na interface da mesa superior com a alma da viga :

VER PÁGINA 57 E SEGUINTE

$$e_2 = 0,20 \times (20\text{m})^2 \times \frac{\left(\frac{5}{1000} \right)}{1,30\text{m} \times (0,87 + 0,5)} = 0,225\text{m}$$

Momento fletor 2^a ordem no plano da laje de compressão $M_d = (2305 \text{ kN} \times 1,4) \times 0,225 \text{ m} = 726,075 \text{ kN.m}$

EXCENTRICIDADE DA FORÇA NORMAL

PROF. P. B. FUSCO - ESTRUTURAS DE CONCRETO - SOLICITAÇÕES NORMAIS - PÁGINA 182

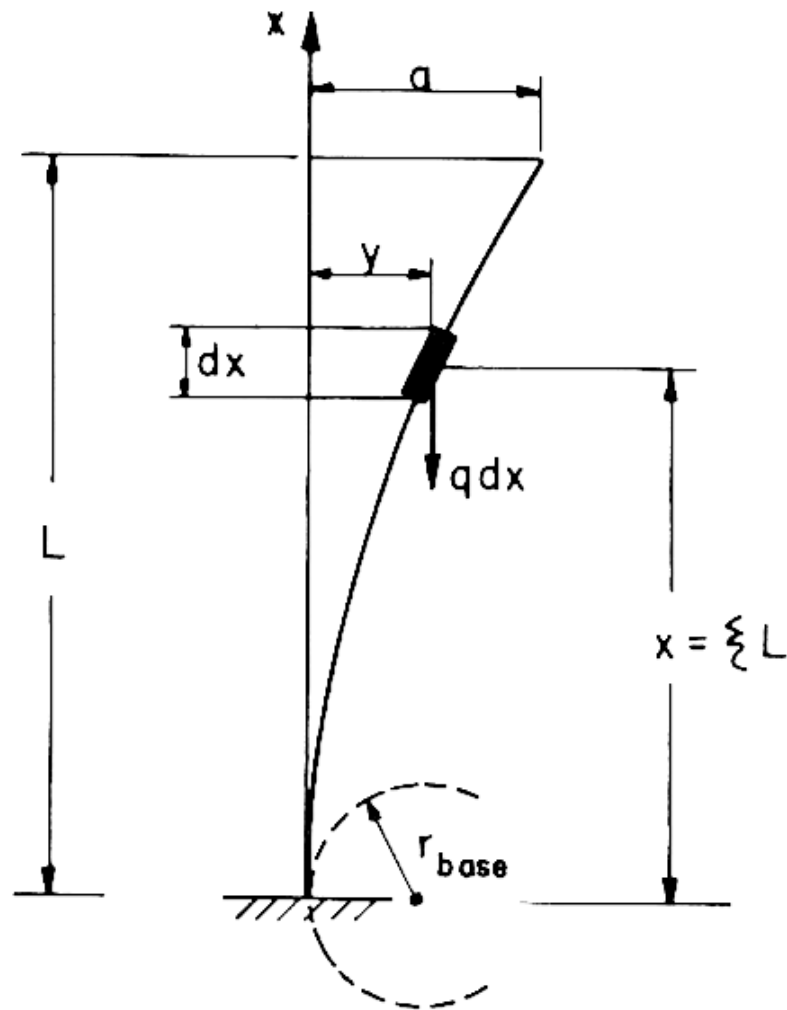


Fig. 9.4.6-1 Cargas axiais uniformemente distribuídas.

Sendo $(1/r)_{base}$ a curvatura da seção da base do pilar, no caso do pilar padrão, carregado na extremidade e de linha elástica senoidal, é obtida a flecha

$$a = \frac{\ell_e^2}{10} \left(\frac{1}{r}\right)_{base}$$

que também pode ser escrita

$$a = 0,4 L^2 \left(\frac{1}{r}\right)_{base}$$

uma vez que $\ell_e = 2 L$.

No caso presente, sendo o carregamento distribuído ao longo do comprimento do pilar, admite-se que a linha elástica seja *parabólica*, com a mesma flecha a que o pilar padrão e com a mesma curvatura da seção da base, resultando então a equação

$$y = 0,4 x^2 \left(\frac{1}{r}\right)_{base} \quad (9.4.6-1)$$

Com essa hipótese, obtém-se o seguinte valor para o momento de 2.^a ordem na seção da base:

$$M_{2, base} = \int_0^L q_d y \, dx$$

logo

$$M_{2, base} = 0,4 \left(\frac{1}{r}\right)_{base} q_d \int_0^L x^2 \, dx = 0,4 \left(\frac{1}{r}\right)_{base} q_d \frac{L^3}{3}$$

Chamando-se de R_{vd} a carga total que produz efeitos de 2.^a ordem, tem-se

$$q_d = \frac{R_{vd}}{L}$$

resultando

$$M_{2, base} = R_{vd} \frac{L^2}{7,5} \left(\frac{1}{r}\right)_{base}$$

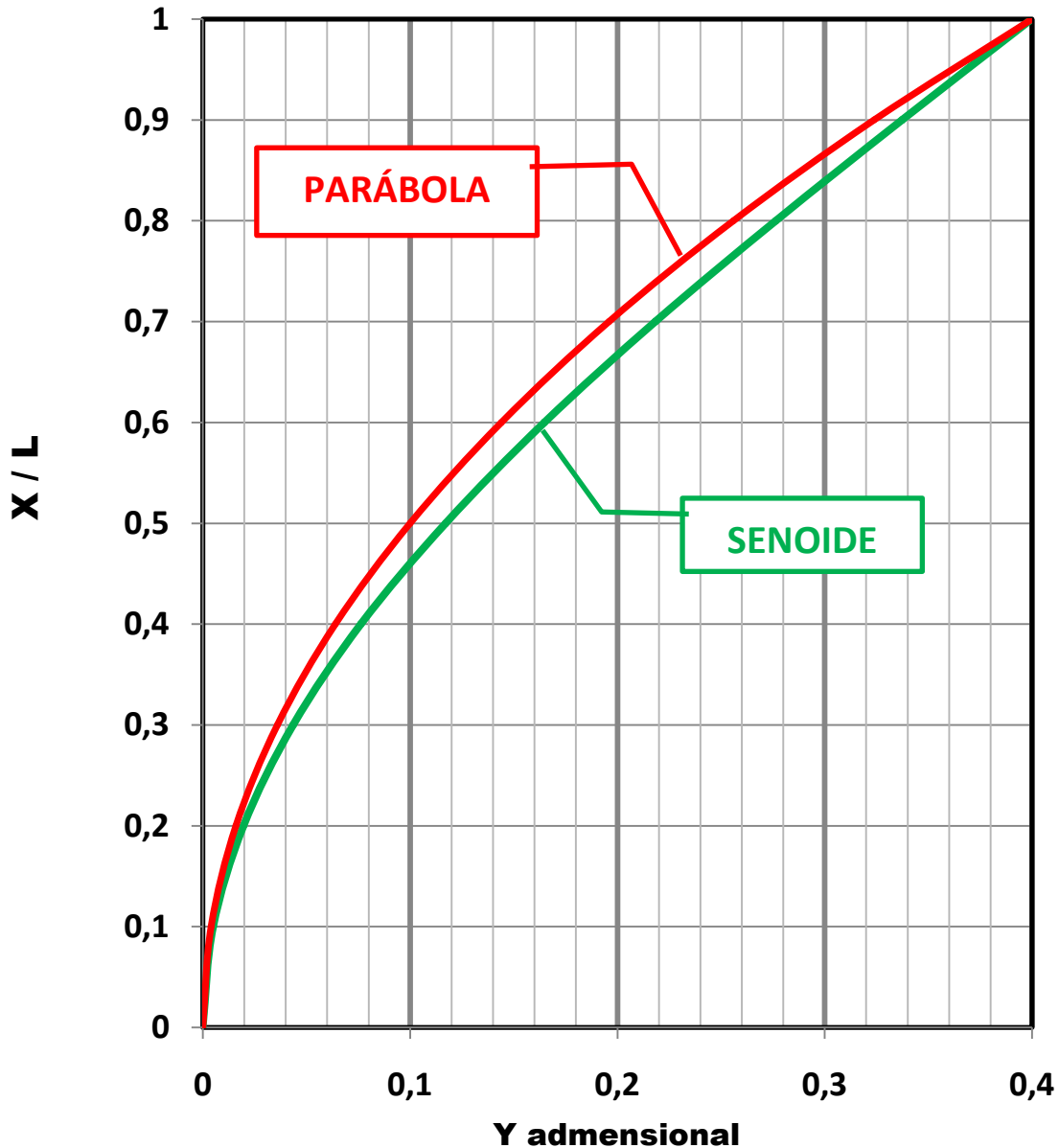
$$\text{PARÁBOLA... } M = R_{vd} \times \left(\frac{1}{7,5} = 0,1333\right) \times \left[L^2 \times \left(\frac{1}{r}\right)_{base}\right]$$

$$\text{SENOIDE... } M = R_{vd} \times (0,1454) \times \left[L^2 \times \left(\frac{1}{r}\right)_{base}\right]$$

APROXIMAÇÃO DA ELÁSTICA DA BARRA DEFORMADA PELA FLAMBAGEM

PROF. P. B. FUSCO - ESTRUTURAS DE CONCRETO - SOLICITAÇÕES NORMAIS -
PÁGINA 182

SENOIDE X PARÁBOLA



$$Parábola = y = 0,4 \times \left[L^2 \times \left(\frac{1}{r} \right)_{base} \right] \times \left(\frac{x}{L} \right)^2$$

$$Senóide = y = 0,4 \times \left[L^2 \times \left(\frac{1}{r} \right)_{base} \right] \times \left(1 - \cos \left(\frac{\pi}{2} \times \left(\frac{x}{L} \right) \right) \right)$$

$$Y \text{ adimensional} = \frac{y}{\left[L^2 \times \left(\frac{1}{r} \right)_{base} \right]}$$

MOMENTO FLETOR (VER PÁGINA 57 E SEGUINTE)

1 - Para carga na extremidade

A resultante da carga está em L ; $YR/L = 1,0$

$$M \text{ base} = Rvd \times 0,400 \times L^2 \times (1/r)_{\text{base}}$$

2 - Para carga uniformemente distribuída

A resultante da carga distribuída está em L/2 ; $YR/L = 0,5$

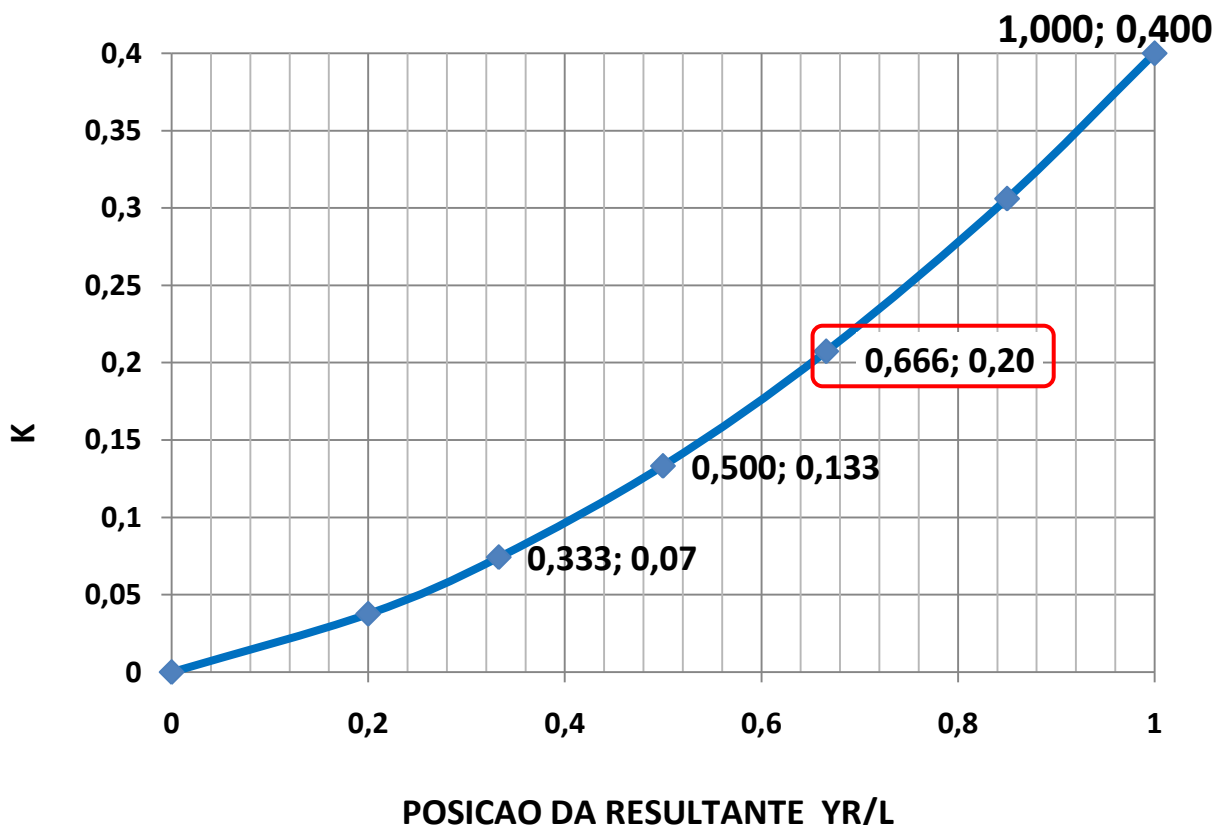
$$M \text{ base} = Rvd \times 0,1333 \times L^2 \times (1/r)_{\text{base}}$$

3 - Para carga triangular (distribuição do cortante) na mesa da viga T

A resultante da carga está em 2/3 L ; $YR/L = 2/3 = 0,666$

$$M \text{ base} = Rvd \times 0,20 \times L^2 \times (1/r)_{\text{base}}$$

$$\text{Excentricidade} = K \times L^2 \times (1/r)$$



$$e^2_{\text{no plano da laje}} = 0,2 \times (20m)^2 \times \frac{\left(\frac{5}{1000} \right)}{1,30m \times (0,87 + 0,5)} = 0,225m$$

Momento Fletor transversal na mesa superior :

Força de compressão no laje superior da viga (já com impacto de manuseio)

$$Nd = 2305 \text{ kN} \times 1,4 = 3227 \text{ kN}$$

$$\text{Excentricidade } e = 0,225 \text{ m}$$

Considerando uma excentricidade devida a imperfeições sendo igual a :

- 2cm ou
- $h/30 = 130/30 = 4,3\text{cm}$ ou
- $L/1000 = 40 \text{ m} / 1000 = 4\text{cm}$ NBR 9062 ITEM 5.2.2.4 ou
- $L / 960 = 40\text{m} / 960 = 4,2\text{cm}$ (P.C.I.)

Ver medições em " Lifting Analysis of Precast Prestressed Concrete Beams " autor : Razvan Cojocarú - Virginia Polytechnic Institute - 2012

https://vtechworks.lib.vt.edu/bitstream/handle/10919/32464/Cojocarú_R_T_2012.pdf?sequence=2&isAllowed=y

- e total = $0,225\text{m} + 0,043\text{m} = 0,268\text{m}$

USANDO O GRÁFICO 23 DO MANUAL 141 DO CEB

$$\nu = \frac{Nd}{b \times d \times fcd} = \frac{-3227\text{kN}}{0,10\text{m} \times 1,30\text{m} \times \left(\frac{fck = 40\text{MPa}}{1,4} \right)} = -0,87$$

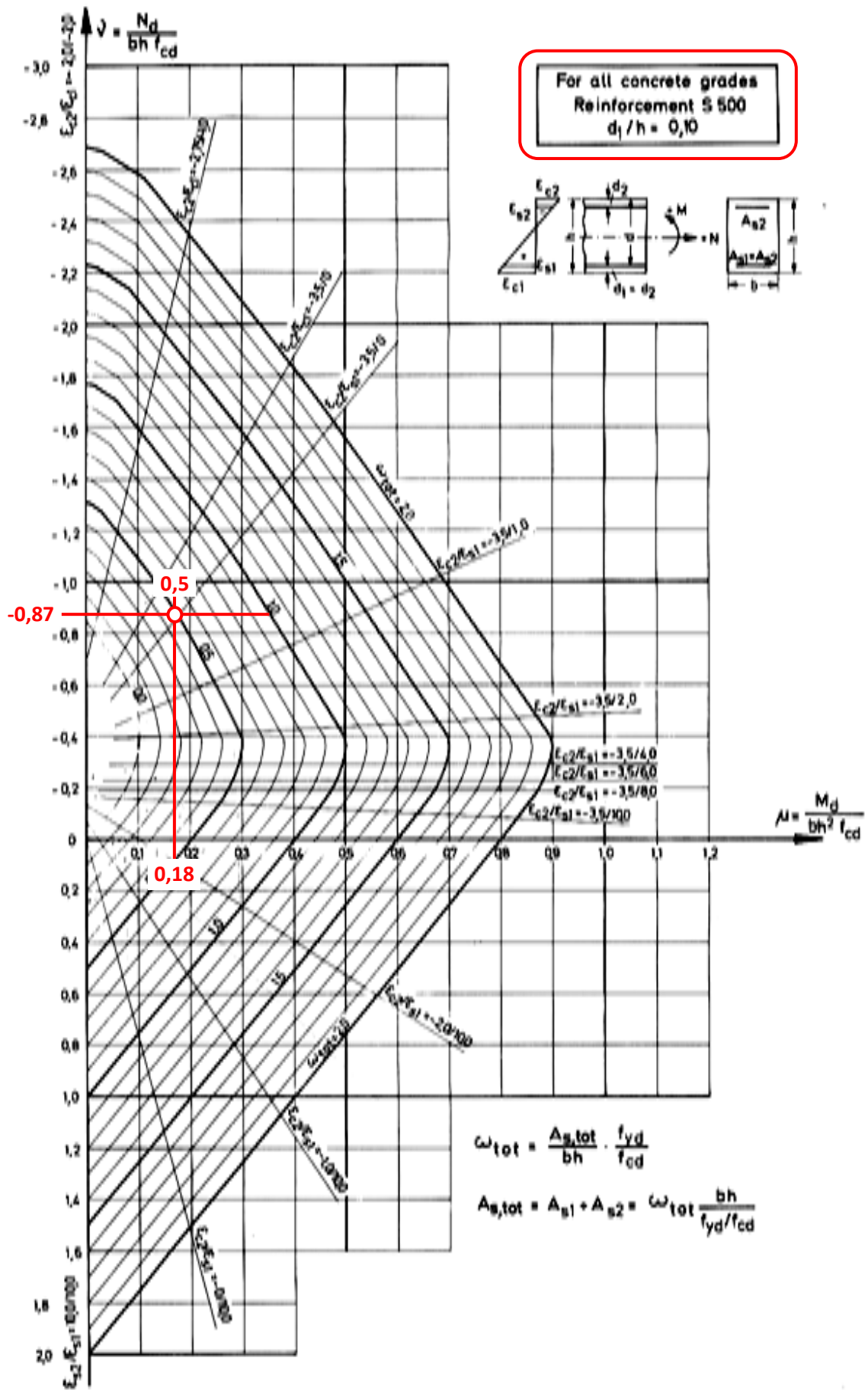
$$\mu = \frac{Md}{b \times d^2 \times fcd} = \frac{Nd}{b \times d \times fcd} \times \frac{e}{d} = \nu \times \frac{e}{d} = 0,87 \times \frac{0,268\text{m}}{1,30\text{m}} = 0,18$$

obtemos $w_{\text{total}} = 0,5$

ϵ_c concreto = $-3,5 \text{ } ^\circ/\text{oo}$ e ϵ_a aço = zero $^\circ/\text{oo}$

Bending and compression

Design Chart 23 Interaction diagram for a symmetrically reinforced rectangular section (S 500; $d_1/h = 0.10$)



$$A_s \text{ total} = (w \text{ total}=0,5) \times [(40\text{MPa}/1,4) / (500\text{MPa}/1,15)] \times 10\text{cm} \times 130\text{cm} =$$

$$A_s \text{ total} = (0,033 =3,3\%) \times 1300 \text{ cm}^2 = 42,7 \text{ cm}^2$$

$$A_{s1} = A_{s2} = 42,7\text{cm}^2 / 2 = 21,4 \text{ cm}^2 = 6 \text{ ferros } 22\text{mm} = 22,8\text{cm}^2$$

Verificação da excentricidade

Excentricidade considerada

$$e_{2 \text{ no plano da laje}} = 0,2 \times (20\text{m})^2 \times \frac{\left(\frac{5}{1000} \right)}{1,30\text{m} \times (0,87+0,5)} = 0,225\text{m}$$

com ϵ_c concreto = $-3,5 \text{ } ^\circ/\text{oo}$ e ϵ_a aço = zero $^\circ/\text{oo}$ obtidos do gráfico 23 do CEB temos:

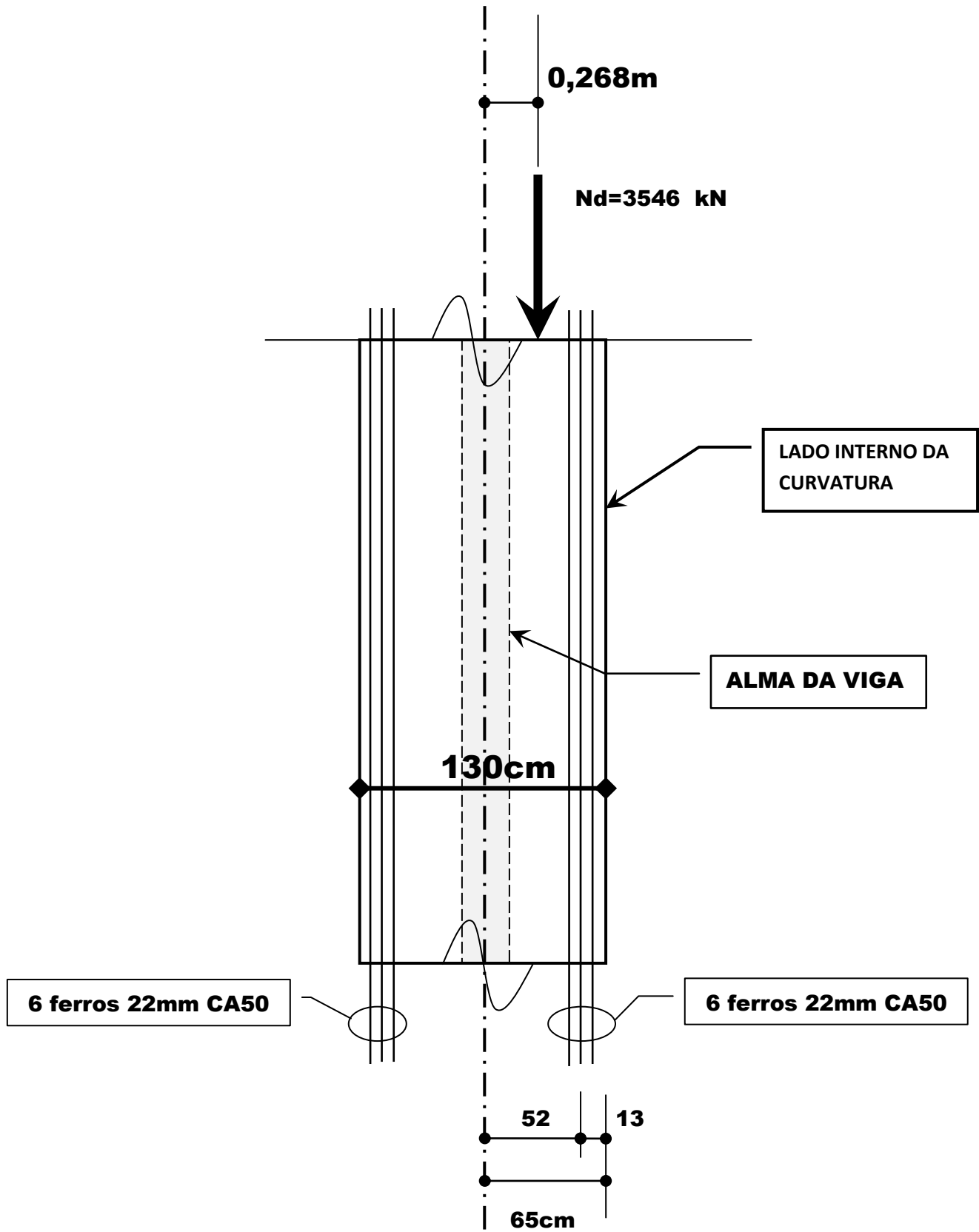
Excentricidade obtida com os epsilons do concreto e do aço, do gráfico :

$$e_2 = 0,20 \times (20\text{m})^2 \times \frac{\left(\left(\epsilon_{aço} = \text{zero } ^\circ/\text{oo} \right) - \left(\epsilon_{concreto} = -3,5 ^\circ/\text{oo} \right) = 3,5 ^\circ/\text{oo} \right)}{1,30\text{m} \times (0,87+0,5)} = 0,165 \text{ m}$$

menor que a excentricidade de 2^a ordem considerada no cálculo da armadura . OK

+ + +

EM PLANTA

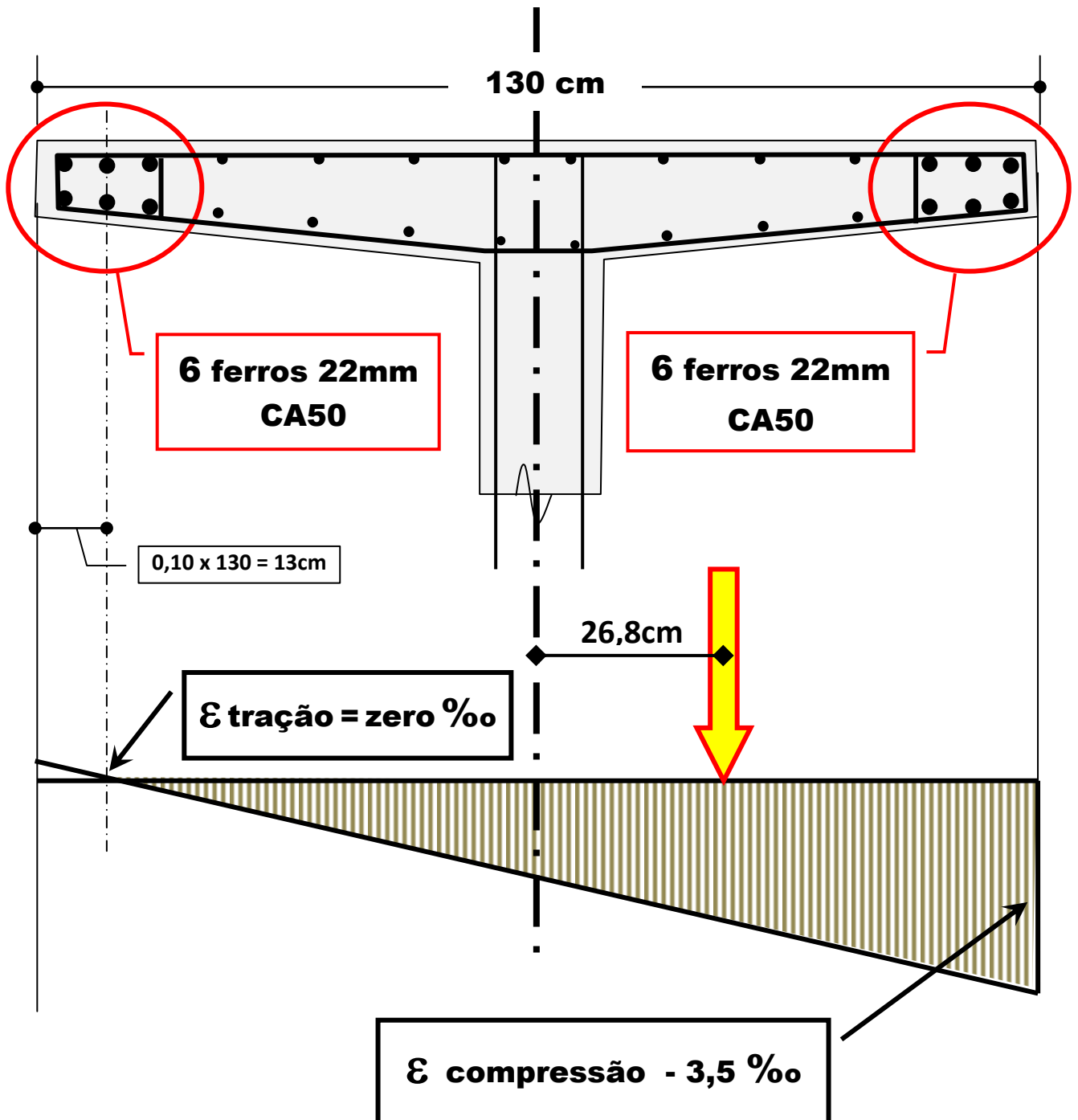


RESUMO

ARMADURA PARA EVITAR FLAMBAGEM LATERAL

VIGA PRÉMOLDADA COM 40 METROS

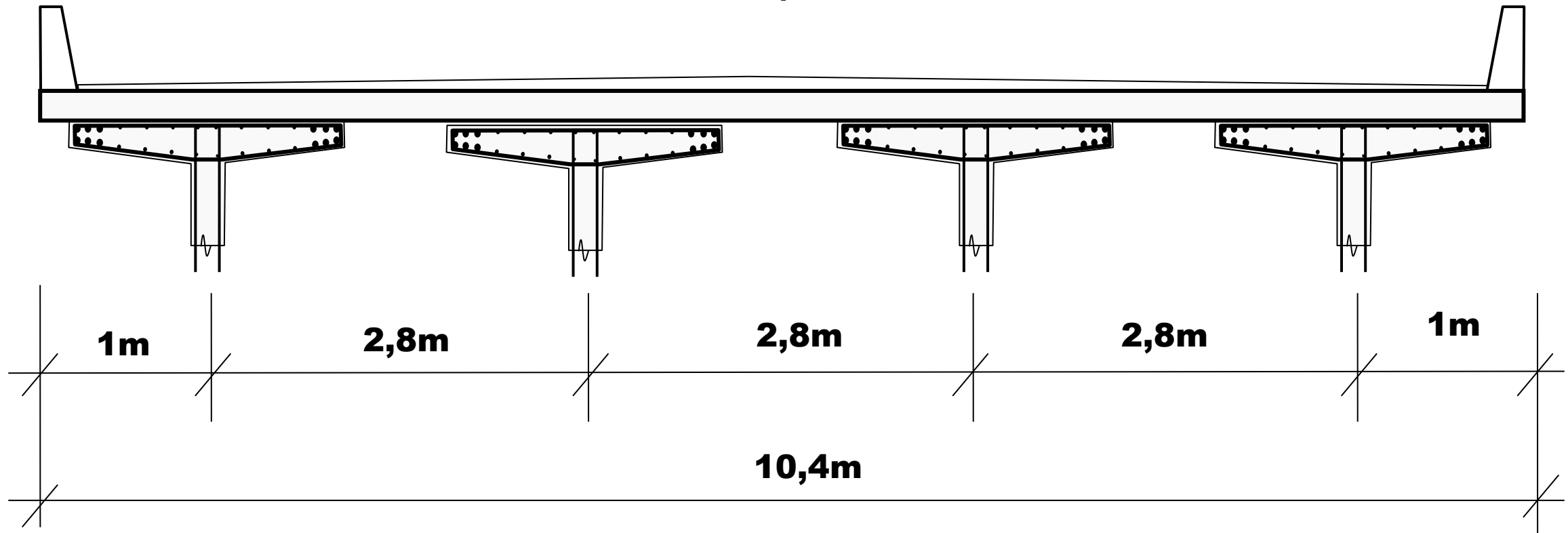
ALTURA = 2,0 METROS ; LARGURA DA MESA SUPERIOR = 130 CM



**MONTAGEM DA VIGA
E
CONCLUSÃO DA PONTE**

O tabuleiro da ponte é completado com a laje concretada no local

Exemplo :

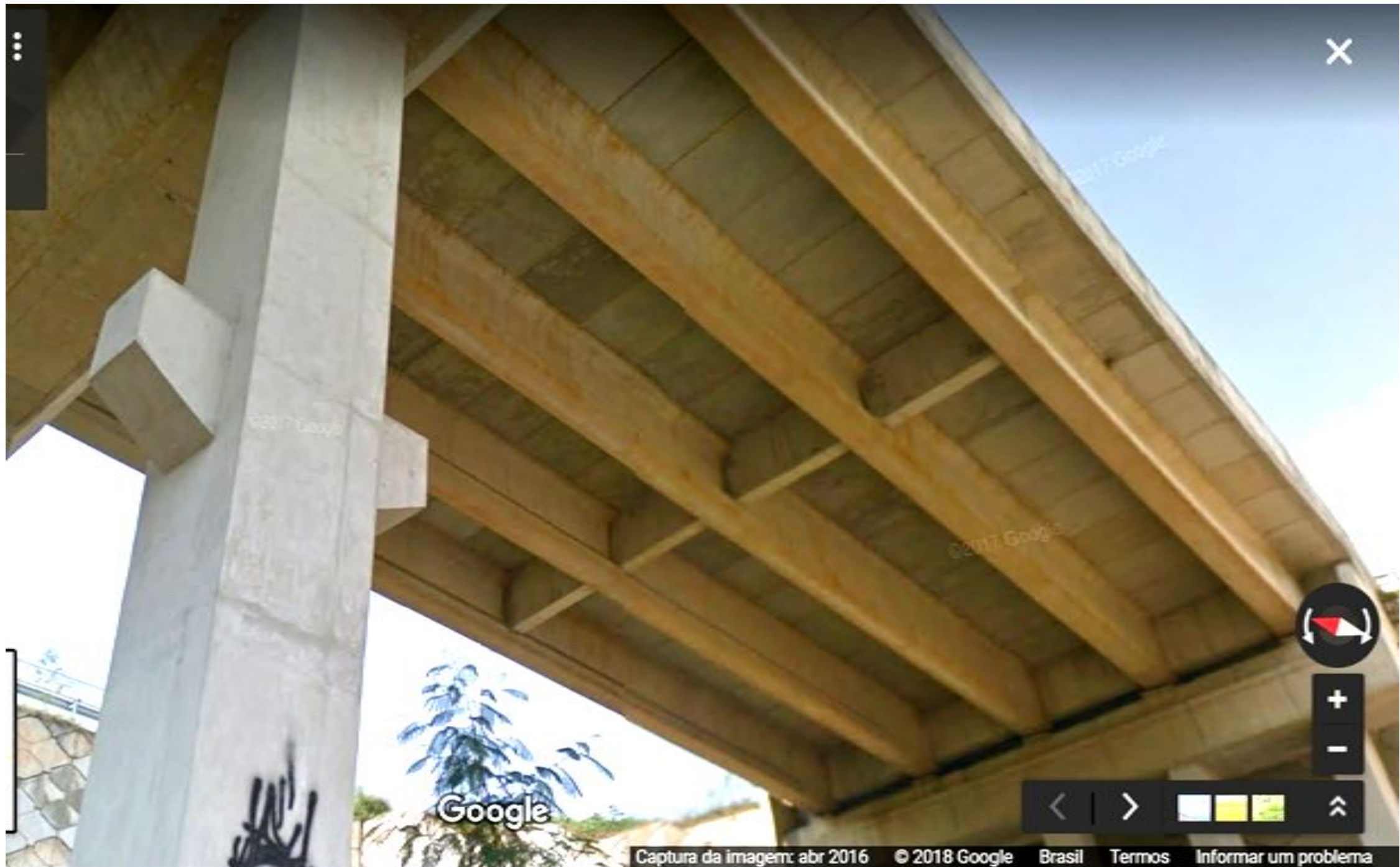


- Na fase final da ponte pronta a mesa superior das vigas pré-moldadas é pouco solicitada devido à presença da laje superior concretada no local.
- Durante o transporte das vigas pré-moldadas a mesa superior dessas vigas pode flambar lateralmente.

MONTAGEM



ARCO METROPOLITANO / RJ - PONTE PRONTA



ARCO METROPOLITANO / RJ - PONTE PRONTA



RO

FOLHA EM BRANCO

ANEXO 01 - Yves Guyon

INSTABILIDADE LATERAL DE VIGAS

Mesmo fazendo verificações atuais, mais elaboradas, sempre obedecer à regra de Yves Guyon , baseada nas obras e projetos da STUP/ Freyssinet.

A recomendação de Yves Guyon , $B > L / 30$, deve predominar.

<http://aquarius.ime.eb.br/~webde2/prof/ethomaz/pontes/premold01.pdf>

Yves Guyon

1- Béton précontraint. Étude théorique et expérimentale, tome 1 e 2 , Éditions Eyrolles, Paris, 1958

2- Constructions em Béton Précontraint - Volumes 1 e 2 - Classes. États Limites – Cours CHEBAP - Eyrolles 1966

As principais dimensões do tabuleiro deverão ter os seguintes valores :

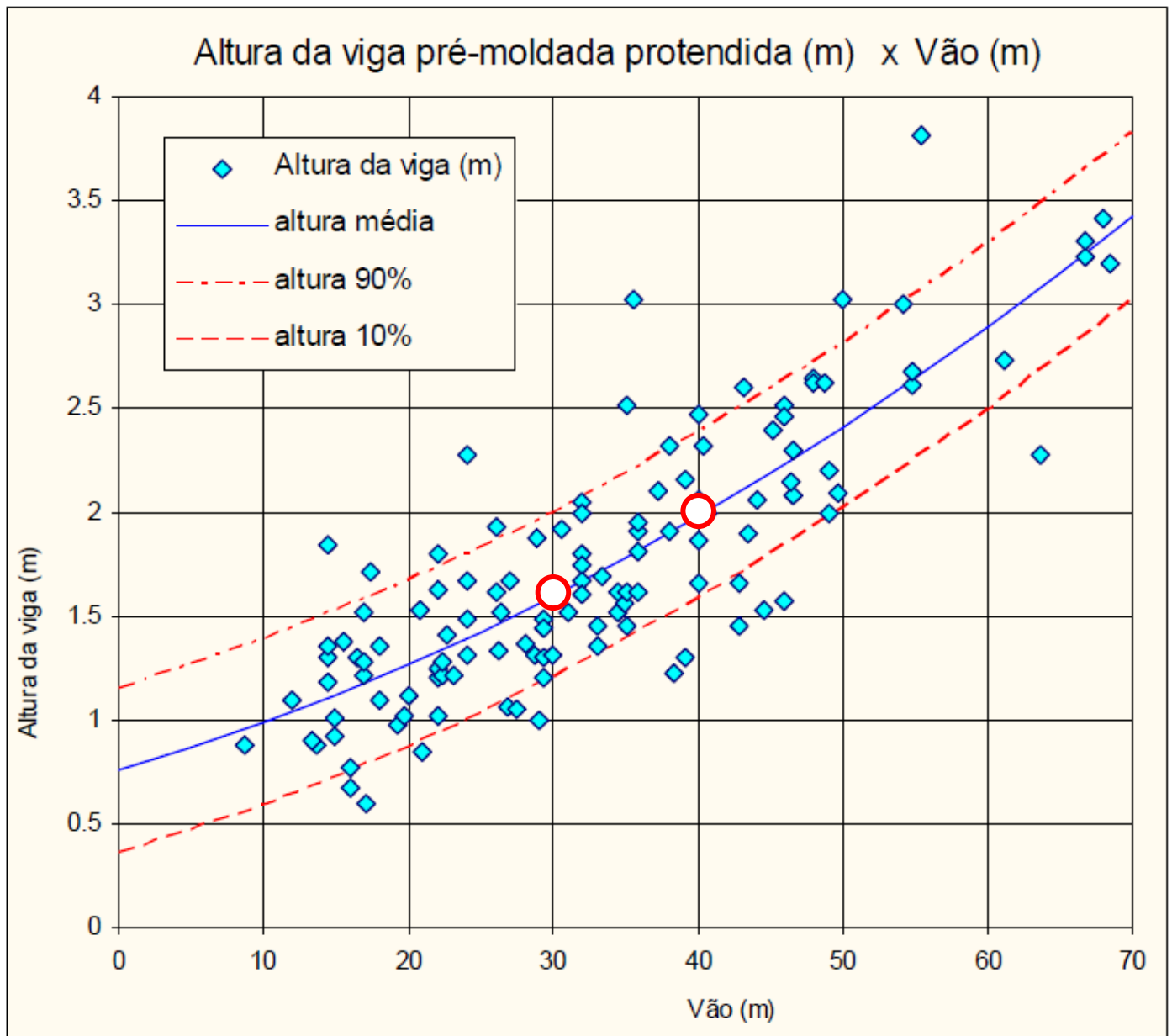
Para evitar a flambagem lateral da viga pré-moldada, a largura "B" da mesa vale :

GUYON	NBR 6118 - item 15.10
$B \geq [L(\text{vão em metros}) / 100\text{metros}] \times h(\text{altura})$	$B \geq 0,40 \times h(\text{altura})$
$B \geq L (\text{vão em metros}) / 30$	$B \geq L (\text{vão em metros}) / 50$

DOIS EXEMPLOS considerando a altura média das vigas :

$L = 30\text{m}$; h média = $1,65\text{m}$

$L = 40\text{m}$; h média = $2,00\text{m}$



EXEMPLO 1 L = 30m h= 1,65m

GUYON	NBR 6118 - item 15.10
$B \geq [L(\text{vão em metros}) / 100\text{metros}] \times h(\text{altura})$ $B \geq [30\text{m}/100\text{m}] \times 1.65\text{m} = 0,30 \text{ h} = \mathbf{0,50\text{m}}$	$B \geq 0,40 \times h(\text{altura})$ $B \geq 0,40 \times 1,65\text{m} = \mathbf{0,66\text{m}}$
$B \geq L (\text{vão em metros}) / 30$ $B \geq L / 30 = 30\text{m}/30 = \mathbf{1,0 \text{ m} \ll}$	$B \geq L (\text{vão em metros}) / 50$ $B \geq L(\text{m}) / 50 = 30\text{m} / 50 = \mathbf{0,60\text{m}}$

EXEMPLO 2 L = 40m h = 2,0m

GUYON	NBR 6118 - item 15.10
$B \geq [L(\text{vão em metros}) / 100\text{metros}] \times h(\text{altura})$ $B \geq [40\text{m}/100\text{m}] \times 2,0\text{m} = 0,40 \text{ h} = \mathbf{0,80\text{m}}$	$B \geq 0,40 \times h(\text{altura})$ $B \geq 0,40 \text{ h}(\text{altura}) = \mathbf{0,80\text{m}}$
$B \geq L (\text{vão em metros}) / 30$ $B \geq L / 30 = 40\text{m}/30 = \mathbf{1,33 \text{ m} \ll}$	$B \geq L (\text{vão em metros}) / 50$ $B \geq L / 50 = 40\text{m}/50 = \mathbf{0,80 \text{ m}}$

A recomendação de Yves Guyon , $B > L / 30$, predomina.

FOI FEITA A VERIFICAÇÃO DA ESTABILIDADE LATERAL DA VIGA COM 40 m DE VÃO, COM 2,0m DE ALTURA E COM 1,30m DE LARGURA DA MESA SUPERIOR

VER PÁGINA 34.

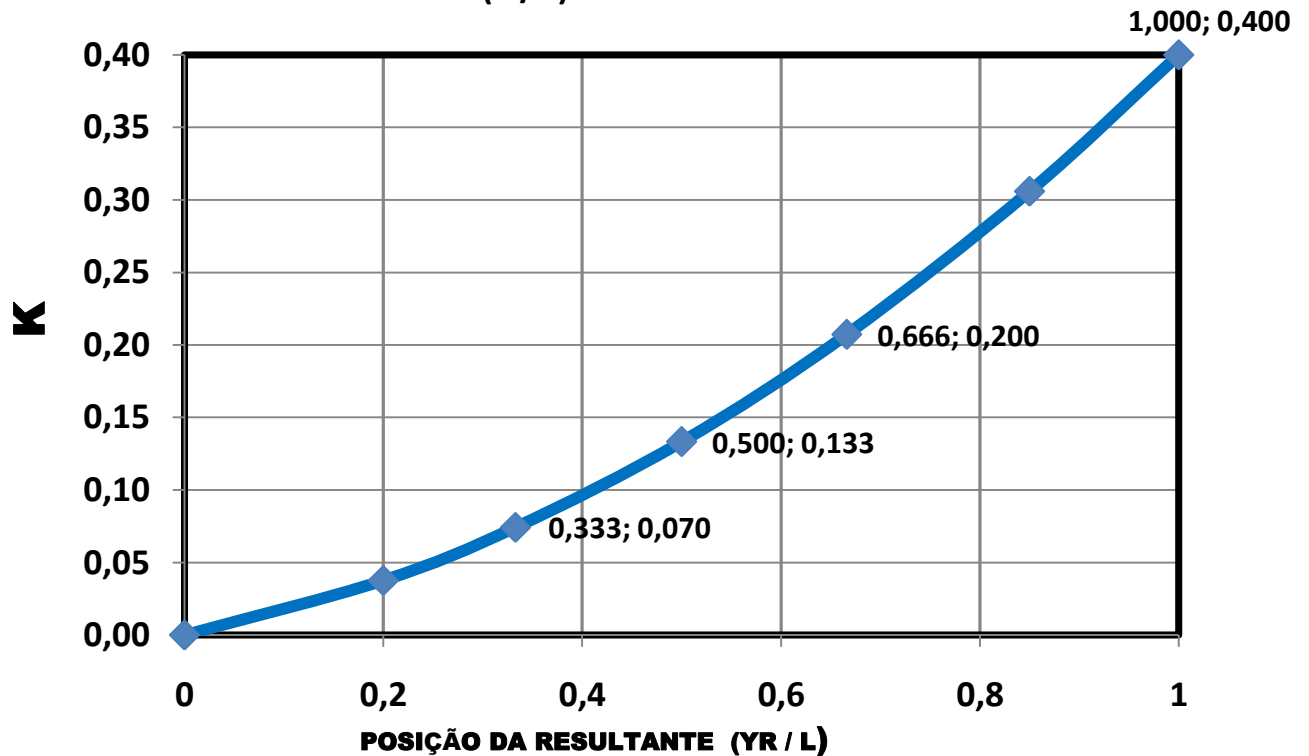
O COEFICIENTE DE SEGURANÇA CONTRA A FLAMBAGEM LATERAL, CALCULADO, FOI 3,20 , SEGUINDO A SUGESTÃO DE GEOMETRIA FEITA POR YVES GUYON / FREYSSINET

**EXCENTRICIDADES DA FORÇA NORMAL
NA MESA DA VIGA, NO MEIO DO VÃO.**

ANEXO 02 - EXCENTRICIDADES

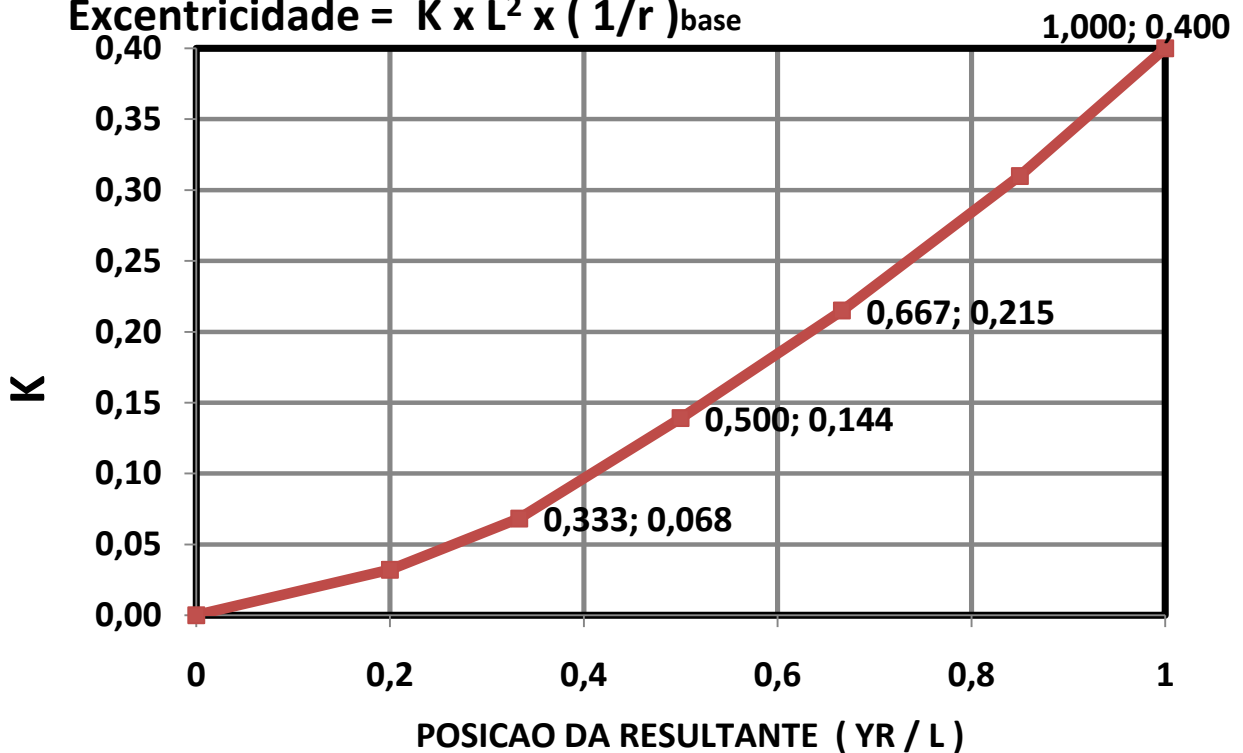
COM CURVA DEFORMADA PARABÓLICA

$$\text{Excentricidade} = K \times L^2 \times (1/r)_{\text{base}}$$



COM CURVA DEFORMADA SENOIDAL

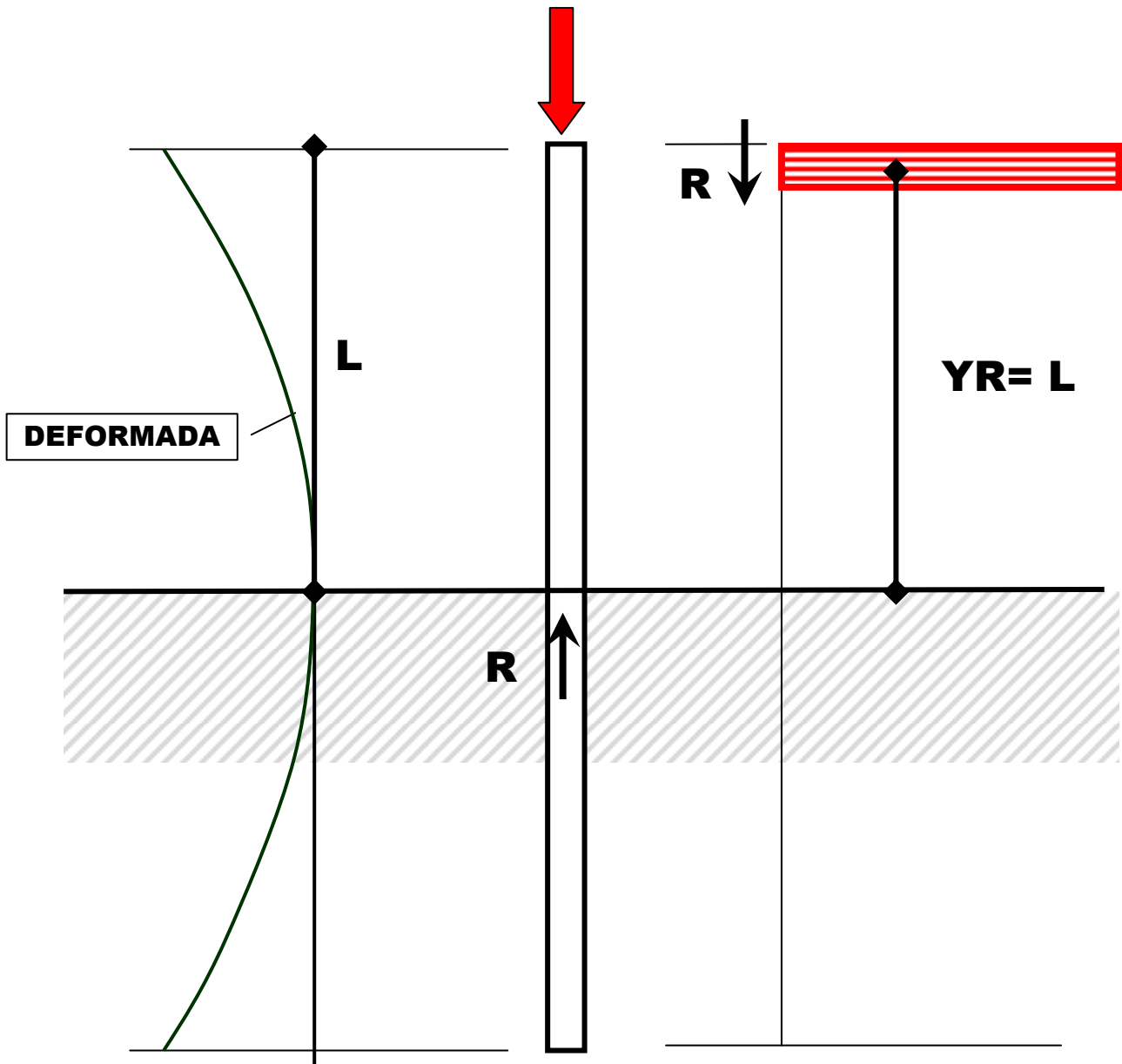
$$\text{Excentricidade} = K \times L^2 \times (1/r)_{\text{base}}$$



Os valores são praticamente iguais

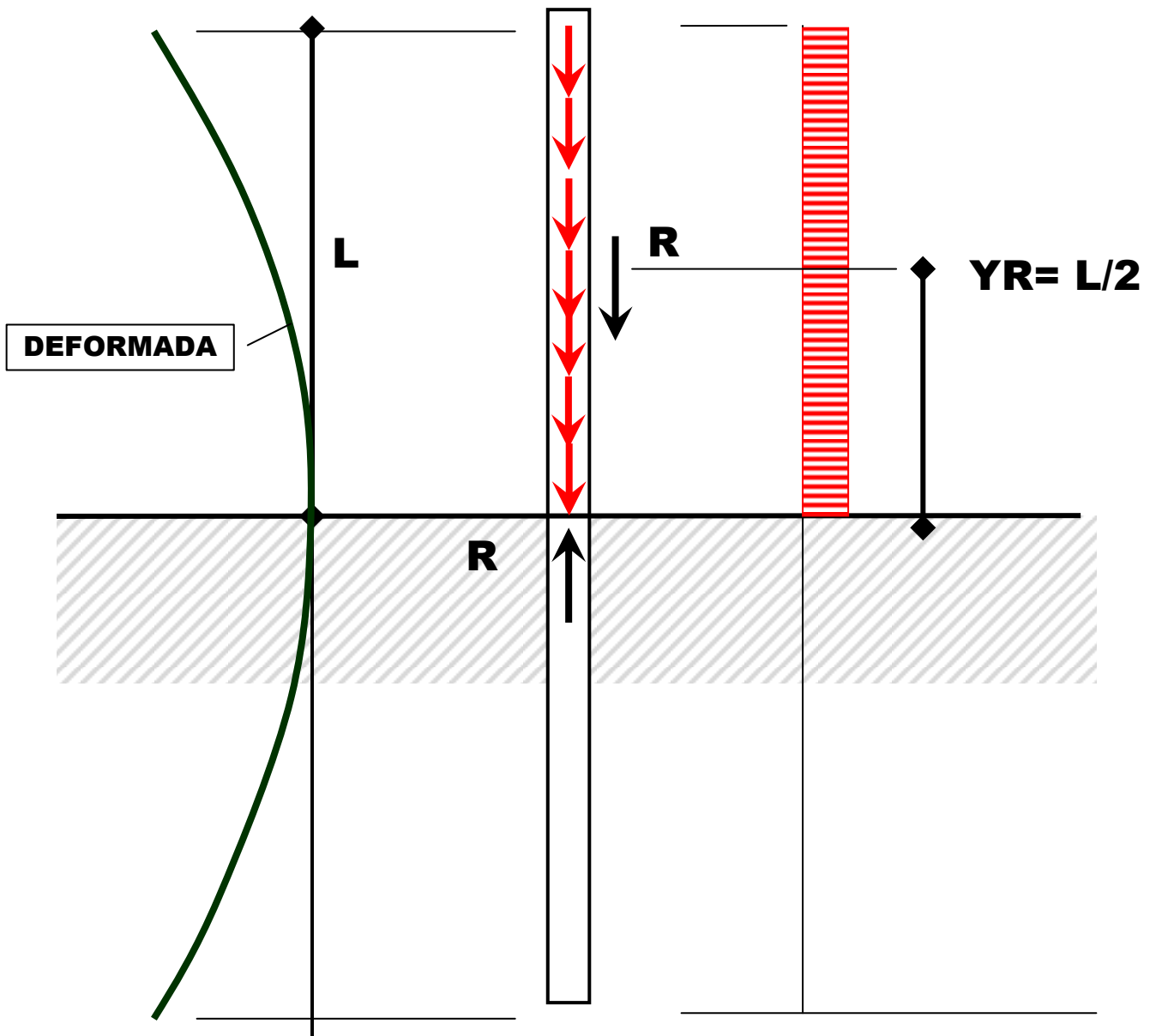
$$\text{Excentricidade} = K \times L^2 \times (1/r)$$

$$K = 0,40$$



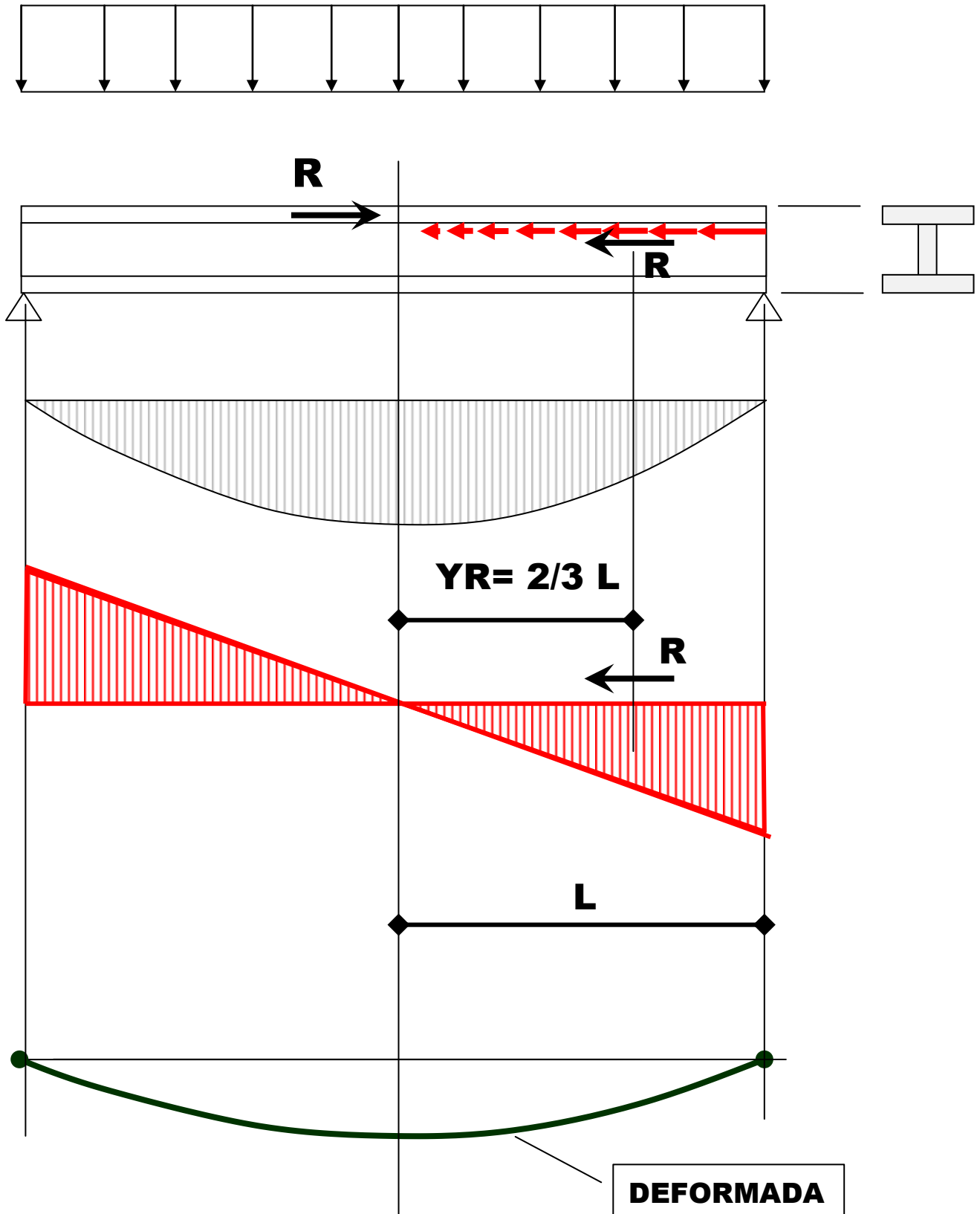
$$\text{Excentricidade} = K \times L^2 \times (1/r)$$

$$K = 0,133$$



$$\text{Excentricidade} = K \times L^2 \times (1/r)$$

$$K = 0,20$$



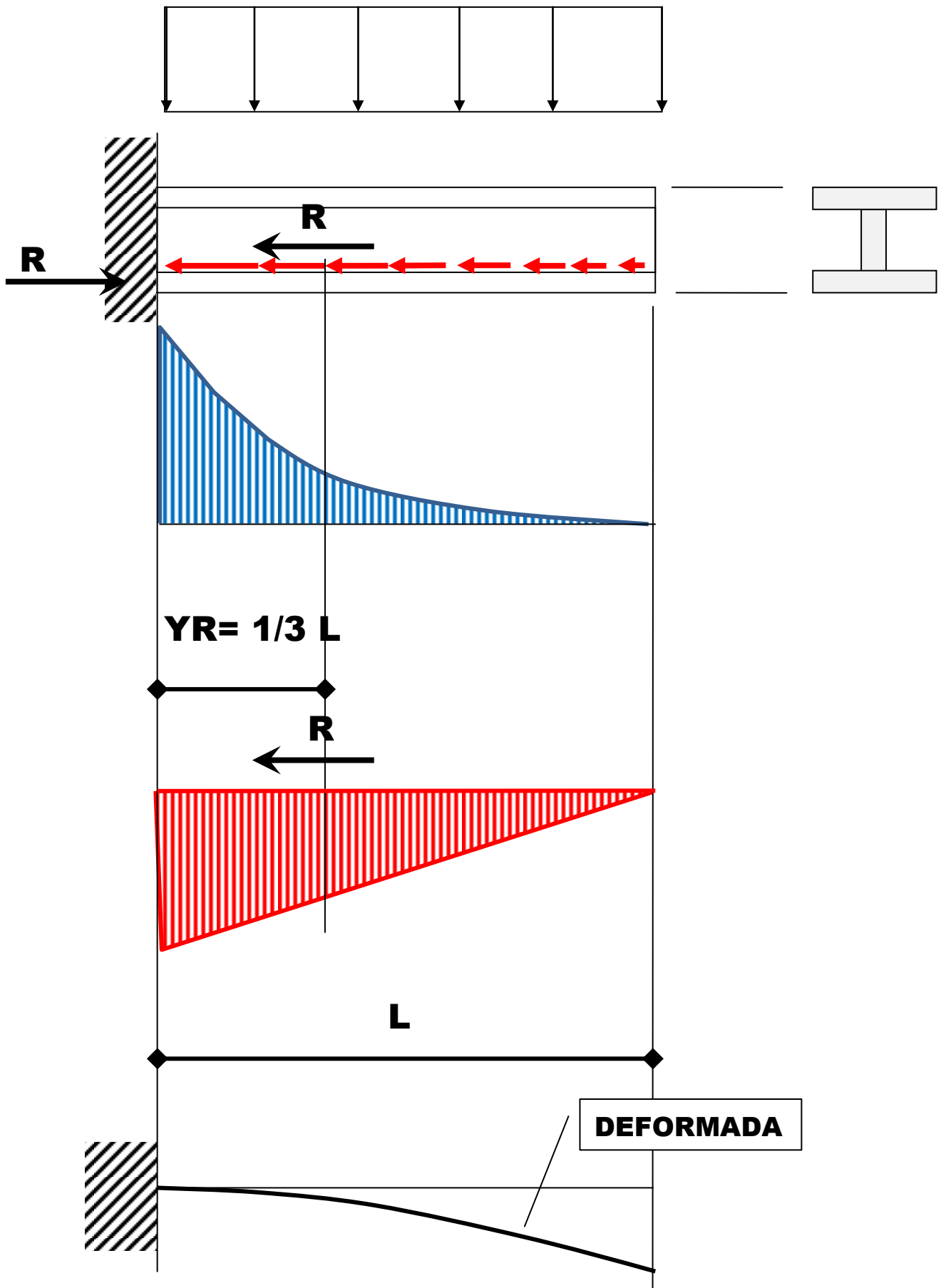
VIGA PRÈ-MOLDADA



<http://www.prefabricatspujol.com/productes/obra-publica/infraestructures-carreteres-i-ferroviaries/bigues-doble-t-serie-gran.html>

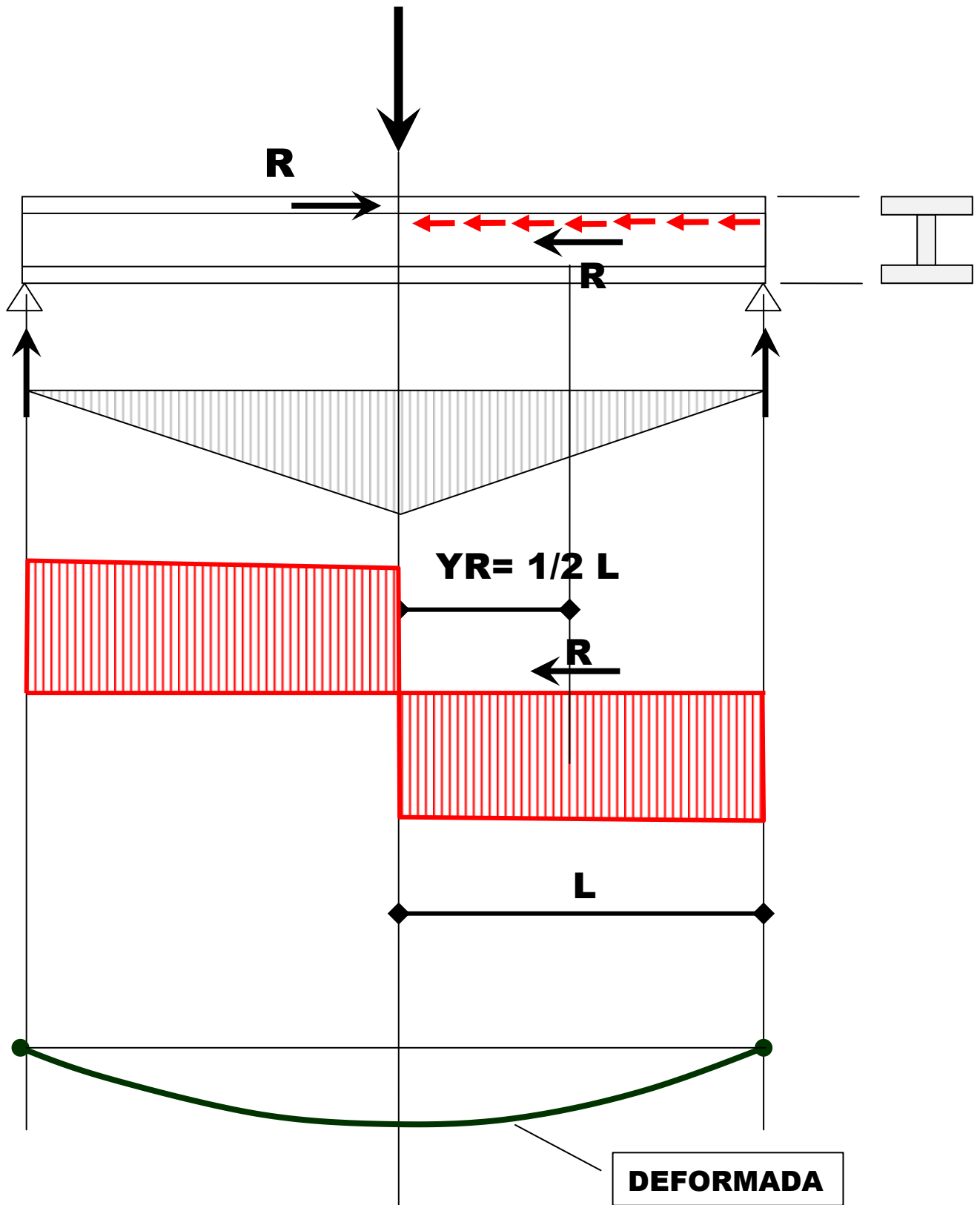
$$\text{Excentricidade} = K \times L^2 \times (1/r)$$

$$K = 0,07$$



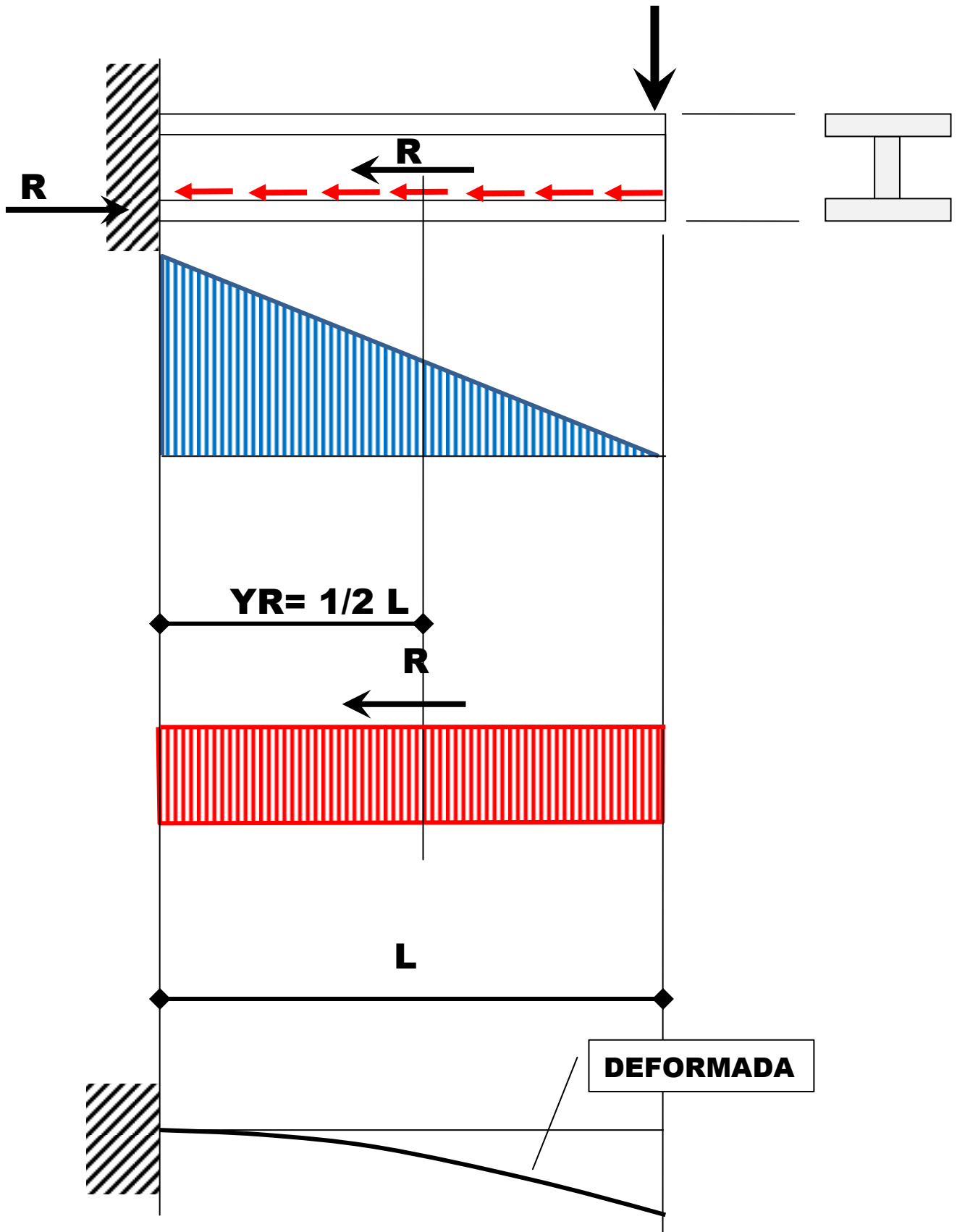
$$\text{Excentricidade} = K \times L^2 \times (1/r)$$

$$K = 0,133$$



$$\text{Excentricidade} = K \times L^2 \times (1/r)$$

$$K = 0,133$$



FOLHA EM BRANCO

ANEXO 03 - EXCENTRICIDADES - DEDUÇÕES

CRITÉRIO DO PROF. P. B. FUSCO - ESTRUTURAS DE CONCRETO - SOLICITAÇÕES NORMAIS - PÁGINA 182

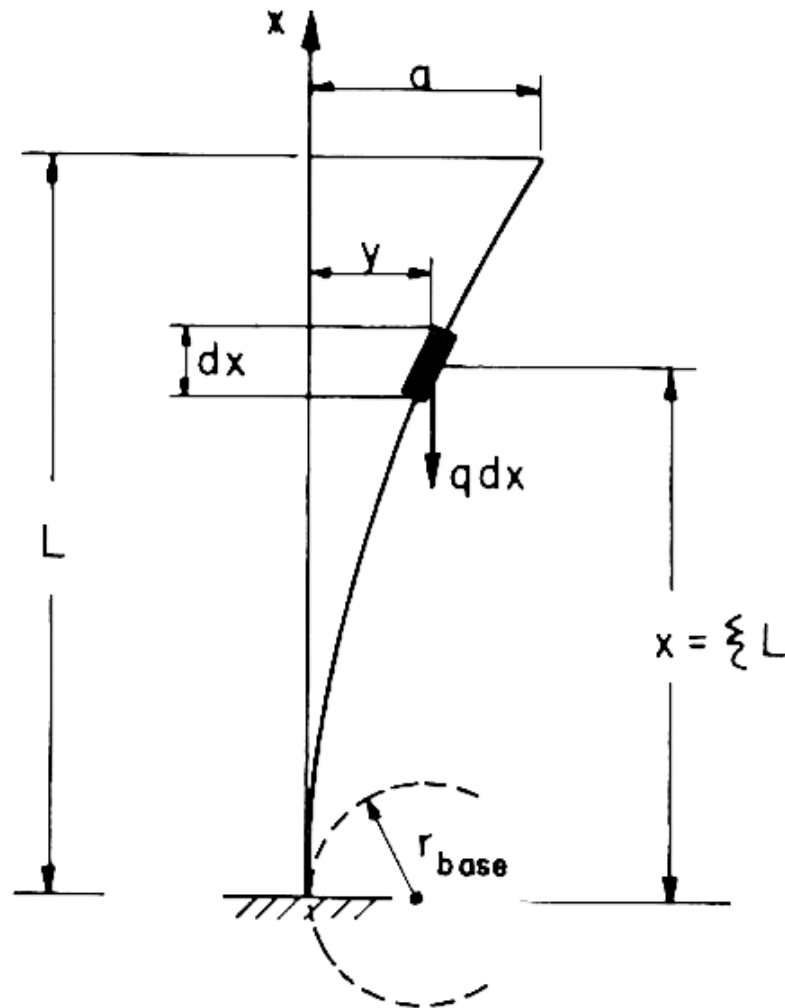


Fig. 9.4.6-1 Cargas axiais uniformemente distribuídas.

Sendo $(1/r)_{base}$ a curvatura da seção da base do pilar, no caso do pilar padrão, carregado na extremidade e de linha elástica senoidal, é obtida a flecha

$$a = \frac{\ell_e^2}{10} \left(\frac{1}{r}\right)_{base}$$

que também pode ser escrita

$$a = 0,4 L^2 \left(\frac{1}{r}\right)_{base}$$

uma vez que $\ell_e = 2 L$.

No caso presente, sendo o carregamento distribuído ao longo do comprimento do pilar, admite-se que a linha elástica seja *parabólica*, com a mesma flecha a que o pilar padrão e com a mesma curvatura da seção da base, resultando então a equação

$$y = 0,4 x^2 \left(\frac{1}{r}\right)_{base} \quad (9.4.6-1)$$

Com essa hipótese, obtém-se o seguinte valor para o momento de 2.^a ordem na seção da base:

$$M_{2, base} = \int_0^L q_d y dx$$

logo

$$M_{2, base} = 0,4 \left(\frac{1}{r}\right)_{base} q_d \int_0^L x^2 dx = 0,4 \left(\frac{1}{r}\right)_{base} q_d \frac{L^3}{3}$$

Chamando-se de R_{vd} a carga total que produz efeitos de 2.^a ordem, tem-se

$$q_d = \frac{R_{vd}}{L}$$

resultando

$$M_{2, base} = R_{vd} \frac{L^2}{7,5} \left(\frac{1}{r}\right)_{base}$$

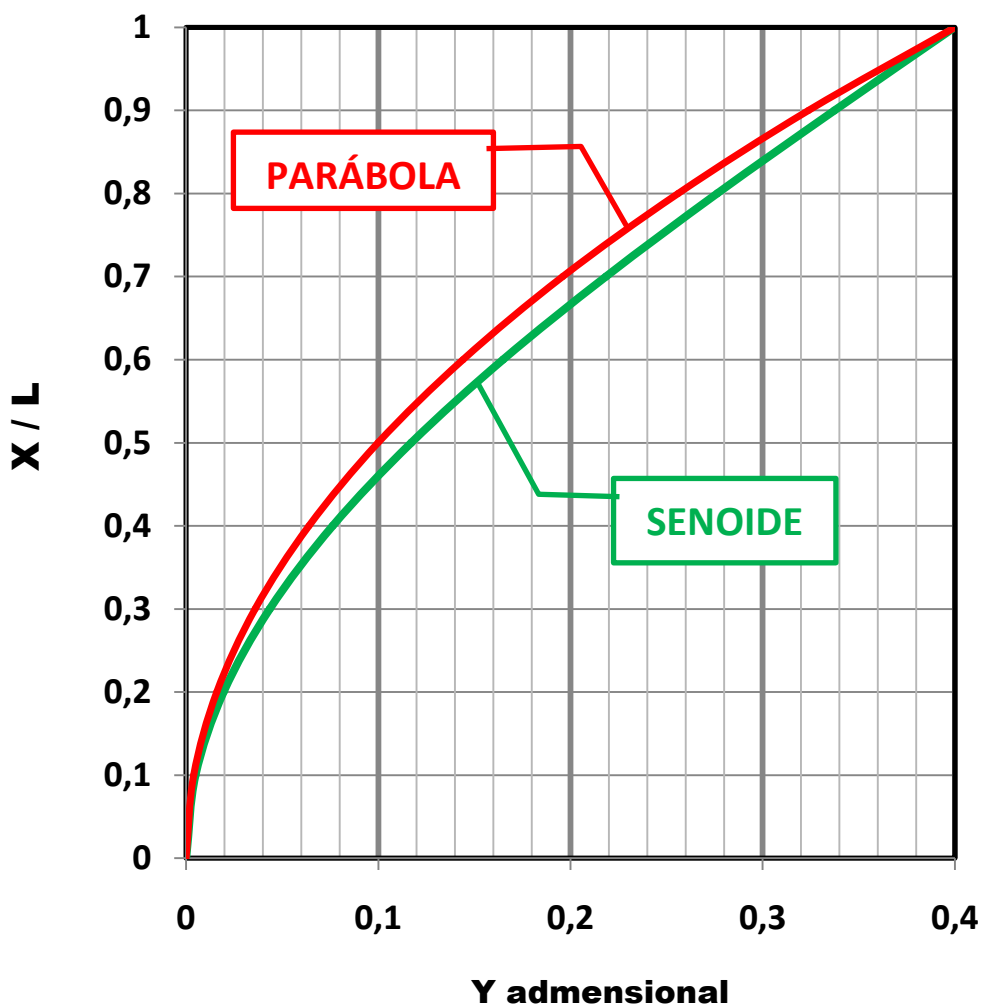
$$\text{PARÁBOLA... } M = R_{vd} \times \left(\frac{1}{7,5} = 0,1333\right) \times \left[L^2 \times \left(\frac{1}{r}\right)_{base}\right]$$

$$\text{SENOIDE... } M = R_{vd} \times (0,1454) \times \left[L^2 \times \left(\frac{1}{r}\right)_{base}\right]$$

APROXIMAÇÃO DA BARRA DEFORMADA PELA FLAMBAGEM

PROF. P. B. FUSCO - ESTRUTURAS DE CONCRETO - SOLICITAÇÕES NORMAIS - PÁGINA 182

SENOIDE X PARÁBOLA



$$\text{Parábola} = y = 0,4 \times \left[L^2 \times \left(\frac{1}{r} \right)_{base} \right] \times \left(\frac{x}{L} \right)^2$$

$$\text{Senóide} = y = 0,4 \times \left[L^2 \times \left(\frac{1}{r} \right)_{base} \right] \times \left(1 - \cos \left(\frac{\pi}{2} \times \left(\frac{x}{L} \right) \right) \right)$$

$$Y \text{ adimensional} = \frac{y}{\left[L^2 \times \left(\frac{1}{r} \right)_{base} \right]}$$

CARREGAMENTO TRIANGULAR SUPERIOR

$$\text{Linha elástica parábola} = \left[y = 0,4 \times x^2 \times \left(\frac{1}{r} \right)_{base} \right]$$

$$\text{carregamento} = p = (q/L) \cdot x$$

$$\begin{aligned} M_{base} &= \int_0^L \left(\frac{q}{L} \times x \right) \times dx \times y = \\ &= \int_0^L \left(q \times \frac{x}{L} \right) \times dx \times \left[y = 0,4 \times x^2 \times \left(\frac{1}{r} \right)_{base} \right] = \\ &= 0,4 \times \frac{q}{L} \times \left(\frac{1}{r} \right)_{base} \times \int_0^L x^3 \times dx = 0,4 \times \frac{q}{L} \times \left(\frac{1}{r} \right)_{base} \times \frac{L^4}{4} = \\ &= 0,2 \times \left(\frac{q \times L}{2} = Rvd \right) \times L^2 \times \left(\frac{1}{r} \right)_{base} = \\ &= Rvd \times \left[\text{excentricidade} = \mathbf{0,20} \times L^2 \times \left(\frac{1}{r} \right)_{base} \right] \end{aligned}$$

CARREGAMENTO TRIANGULAR INFERIOR

$$\text{Linha elástica parábola} = \left[y = 0,4 \times x^2 \times \left(\frac{1}{r} \right)_{base} \right]$$

$$\text{carregamento} = p = (q/L) \cdot (L - x)$$

$$\begin{aligned} M_{base} &= \int_0^L \left(\frac{q}{L} \times (L - x) \right) \times dx \times y = \\ &= \int_0^L \left[q \times \left(1 - \frac{x}{L} \right) \right] \times dx \times \left[y = 0,4 \times x^2 \times \left(\frac{1}{r} \right)_{base} \right] = \\ &= 0,4 \times q \times \left(\frac{1}{r} \right)_{base} \times \int_0^L \left(x^2 - \frac{x^3}{L} \right) \times dx = 0,4 \times q \times \left(\frac{1}{r} \right)_{base} \times \left(\frac{L^3}{3} - \frac{L^3}{4} = \frac{L^3}{12} \right) = \\ &= 0,4 \times \left(\frac{q \times L}{2} = Rvd \right) \times \frac{L^2}{6} \times \left(\frac{1}{r} \right)_{base} = \\ &= Rvd \times \left[\text{excentricidade} = (0,0666 \cong \mathbf{0,07}) \times L^2 \times \left(\frac{1}{r} \right)_{base} \right] \end{aligned}$$

CARREGAMENTO TRIANGULAR SUPERIOR

$$\text{Linha elástica senoide} = y = 0,4 \times \left[L^2 \times \left(\frac{1}{r} \right)_{base} \right] \times \left(1 - \cos \left(\frac{\pi}{2} \times \left(\frac{x}{L} \right) \right) \right)$$

$$\text{carregamento} = p = q \cdot x / L$$

$$M_{base} = \int_0^L p \times dx \times y =$$

$$M_{base} = \int_0^L \left(q \times \frac{x}{L} \right) \times dx \times \left[y = 0,4 \times \left[L^2 \times \left(\frac{1}{r} \right)_{base} \right] \times \left(1,0 - \cos \left[\left(\frac{\pi}{2} \right) \times \left(\frac{x}{L} \right) \right] \right) \right] =$$

PARTE 1

$$\begin{aligned} M_{base} &= \int_0^L \left(q \times \frac{x}{L} \right) \times dx \times \left[0,4 \times \left[L^2 \times \left(\frac{1}{r} \right)_{base} \right] \times (1,0) \right] = \\ &= \left[0,4 \times \left[L^2 \times \left(\frac{1}{r} \right)_{base} \right] \right] \times \left(\frac{q}{L} \right) \times \left[\frac{x^2}{2} \right]_0^L = \left(Rvd = \frac{q \times L}{2} \right) \times \left[0,4 \times \left[L^2 \times \left(\frac{1}{r} \right)_{base} \right] \right] \end{aligned}$$

PARTE 2

$$\begin{aligned} M_{base} &= \int_0^L \left(q \times \frac{x}{L} \right) \times dx \times \left[y = 0,4 \times \left[L^2 \times \left(\frac{1}{r} \right)_{base} \right] \times \left(-\cos \left[\left(\frac{\pi}{2} \right) \times \left(\frac{x}{L} \right) \right] \right) \right] = \\ &= \left\langle \frac{q \times 0,4}{L} \times \left[L^2 \times \left(\frac{1}{r} \right)_{base} \right] \right\rangle \times (-) \int_0^L \left[(x) \times \cos \left[\left(a = \frac{\pi}{2L} \right) \times (x) \right] \right] \times dx = \\ &= \langle \rangle \times (-) \times \left[\left(\frac{1}{a^2} = \frac{4L^2}{\pi^2} \right) \times \left[\frac{d}{dx} (x) = 1,00 \right] \times \cos \left[\left(\frac{\pi}{2L} \right) \times (x) \right] + \left(\frac{1}{a} = \frac{2L}{\pi} \right) \times [(x)] \times \text{sen} \left[\left(\frac{\pi}{2L} \right) \times (x) \right] \right]_0^L + \\ &= -0,1850 \times Rvd \times \left[L^2 \times \left(\frac{1}{r} \right)_{base} \right] \end{aligned}$$

TOTAL PARTE 1 + PARTE 2

$$\text{PARTE 1} = Rvd \times 0,4 \times \left[L^2 \times \left(\frac{1}{r} \right)_{base} \right]$$

$$\text{PARTE 2} = -Rvd \times 0,1850 \times \left[L^2 \times \left(\frac{1}{r} \right)_{base} \right]$$

$$\text{TOTAL senoide} = Rvd \times \mathbf{0,2150} \times \left[L^2 \times \left(\frac{1}{r} \right)_{base} \right]$$

FOLHA EM BRANCO

ANEXO 04 - FOTOS DA TESE DE MESTRADO

https://upcommons.upc.edu/bitstream/handle/2099.1/25533/Victor_Herrando_Lateral_stability_analysis_of_long_precast_prestressed_concrete_beams.pdf



TESI DE MÀSTER

Màster

ENGINYERIA DE CAMINS, CANALS I PORTS

Títol

LATERAL STABILITY ANALYSIS OF LONG PRECAST
PRESTRESSED CONCRETE BEAMS

Autor

VÍCTOR HERRANDO GERMÁN

Tutor

ALBERT DE LA FUENTE ANTEQUERA
JESÚS MIGUEL BAIRÁN GARCÍA

Intensificació

ENGINYERIA D'ESTRUCTURES I CONSTRUCCIÓ

Data

Febrer de 2015

CHAPTER 6.

REAL CASE STUDY

https://upcommons.upc.edu/bitstream/handle/2099.1/25533/Victor_Herrando_Lateral_stability_analysis_of_long_preca_st_prestressed_concrete_beams.pdf



Figure 6.1. Perspective of bridge girder I200B120 during lifting.

https://upcommons.upc.edu/bitstream/handle/2099.1/25533/Victor_Herrando_Lateral_stability_analysis_of_long_preca_st_prestressed_concrete_beams.pdf



Figure 6.2. Bridge girder I200B120 during lifting.

https://upcommons.upc.edu/bitstream/handle/2099.1/25533/Victor_Herrando_Lateral_stability_analysis_of_long_preca_st_prestressed_concrete_beams.pdf



Figure 6.3. Bridge girder I200B120 with modified top flange during lifting.

FOLHA EM BRANCO

ANEXO 05 - EXEMPLO DE FLAMBAGEM LATERAL DE UMA VIGA I DE AÇO

Flambagem lateral com torção em vigas com perfis de alma senoidal

Gílson Queiroz^I; Lourdiane Gontijo das Mercês Gonzaga^{II}; Roberval José Pimenta^{III}; Ricardo Hallal Fakury^{IV}; Francisco Carlos Rodrigues^V

http://www.scielo.br/scielo.php?script=sci_arttext&pid=S0370-44672007000200016



FOLHA EM BRANCO

ANEXO 06 - CONTRAVENTAMENTO : EM CASOS EXTREMOS USAR CONTRAVENTAMENTO HORIZONTAL NA MESA SUPERIOR COM TRELIÇAS DE AÇO.

Ver : PCI Advisory on I - Girder Stability during Handling and Construction - 2009

by Chris D. Hill, John S. Dick and Maher K. Tadros

http://www.aspirebridge.com/magazine/2009Winter/safety_win09.pdf

CONTRAVENTAMENTO

A 9-ft 3-in.-deep, 213-ft-long girder being shipped in Calgary, Alberta, Canada, using a reinforcing steel truss attached to the top flange for additional stiffness.
Photo: Con-Force Structures.



Vão da viga $L = 64\text{m}$; Altura da viga $H = 2,8\text{m}$; $H/L = 1/23$

ANEXO 07 - University of Florida

Civil and Coastal Engineering
Final Report July 2007

Lateral Bracing of Long-Span Florida Bulb-Tee Girders

Principal investigators:

Gary R. Consolazio, Ph.D.

H. R. (Trey) Hamilton, III, Ph.D., P.E.

<https://www.google.com.br/urlbvm=bv.147448319,d.Y2I&cad=rja>

$$w_{cr} = \frac{\sqrt{EI_y GJ}}{L^3} m k_1 k_2$$

In this equation, m is a variable coefficient dependent principally upon the end section conditions, w_{cr} is the critical buckling uniform load (also referred to as the “buckling capacity” in this report), I_y is the moment of inertia about the weak vertical axis of the girder, J is the torsional constant, E is the elastic (Young’s) modulus of concrete, G is shear modulus of concrete, and L is the span length of the beam. If the ends of the beam are restrained, the influence of the shape of the cross section on buckling loads may be considered by introducing the coefficient k_2 which takes into account additional restraint to lateral deflections of the top and bottom flanges. For loads applied at an eccentricity relative to the centroid of the section, the buckling load may be corrected by using a factor k_1 .

= = =

FÓRMULA FINAL

$$C = \frac{\sqrt{E I G J}}{L^3} (24(1.0 - 0.0017L)(1.0 - 0.006\theta))$$

In the equation above, C is the maximum uniform distributed load that a girder can carry (in units of kip/ft), E is the elastic modulus of concrete (in units of kip/ft²), I is the weak axis moment of inertia of the girder (in units of ft⁴), G is the shear modulus of concrete (in units of kip/ft²), J is the torsional constant for the girder (in units of ft⁴), L is the span length (in units of feet), and θ is the skew angle between the bearing pads and the longitudinal axis of the girder (in units of degrees). Capacities predicted by the above equation are based on limit states related to lateral buckling (girder stability) and tensile flange stresses (cracking).

APPENDIX A APPROXIMATE CALCULATION OF TORSIONAL CONSTANT

Accurate determination of torsional constants requires the use of relatively sophisticated numerical approaches that generally employ the “pressurized membrane analogy” of torsion (Young 1989 article 4.4 and 9.2, Timoshenko and Goodier 1970 article 107). Such methods require that the shape of the cross-section be carefully discretized (meshed) and analyzed using computer software.

In lieu of conducting this type of analysis, a variation on a procedure outlined in ACI 318-05 (ACI 2005) may be used to estimate torsional constants for girder cross-sections. In Chapter 13 of ACI 318-05, the following expression is given for estimating the torsional constant of a section consisting of rectangular shapes:

$$J = \sum \left(1 - 0.63 \left(\frac{x}{y} \right) \right) \frac{x^3 y}{3} \quad (\text{A.1})$$

In this equation, x is the shorter overall dimension of each rectangle, and y is the longer overall dimension. To apply Eqn. A.1 to an I-shaped girder, it is recommended that the cross-section be divided into three distinct parts: top flange, web, and bottom flange. Note that subdivision of the cross-section into a larger number of smaller parts will generally worsen, not improve, the estimation of torsional constant when applying Eqn. A.1. The procedure recommended here for subdividing a section is illustrated in Figure A.1 for an FBT78 section. The widths of the rectangles are set equal to the overall dimensions of the corresponding parts in the actual cross-section. The vertical heights of the rectangles are then computed to make the part-areas match between the true cross-sectional shape, and the piecewise rectangular shape.

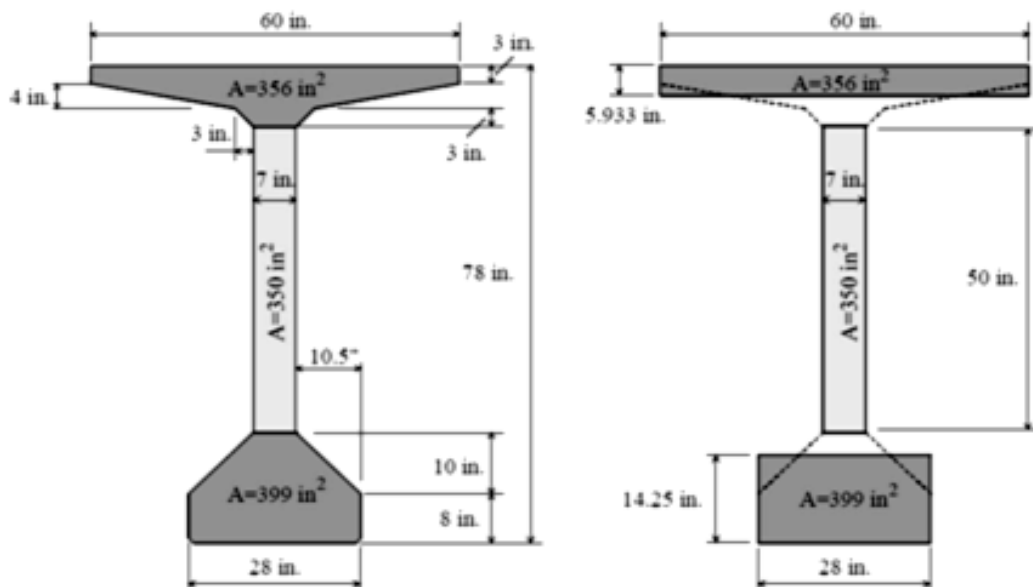


Figure A.1 Actual FBT78 cross-section (left) and recommended equivalent piecewise rectangular cross-section (right)

APPENDIX B EXAMPLE CALCULATIONS - BUCKLING CAPACITY

In this appendix, calculations are presented for computing the buckling capacity of a bridge girder in accordance with equations proposed earlier in this report. The type of bridge girder under consideration in this example is a Florida bulb-tee section. An overview of the geometry of the bridge girder and support conditions is given in Figure B.1. Specific values for the girder considered in this example are summarized in Table B.1.

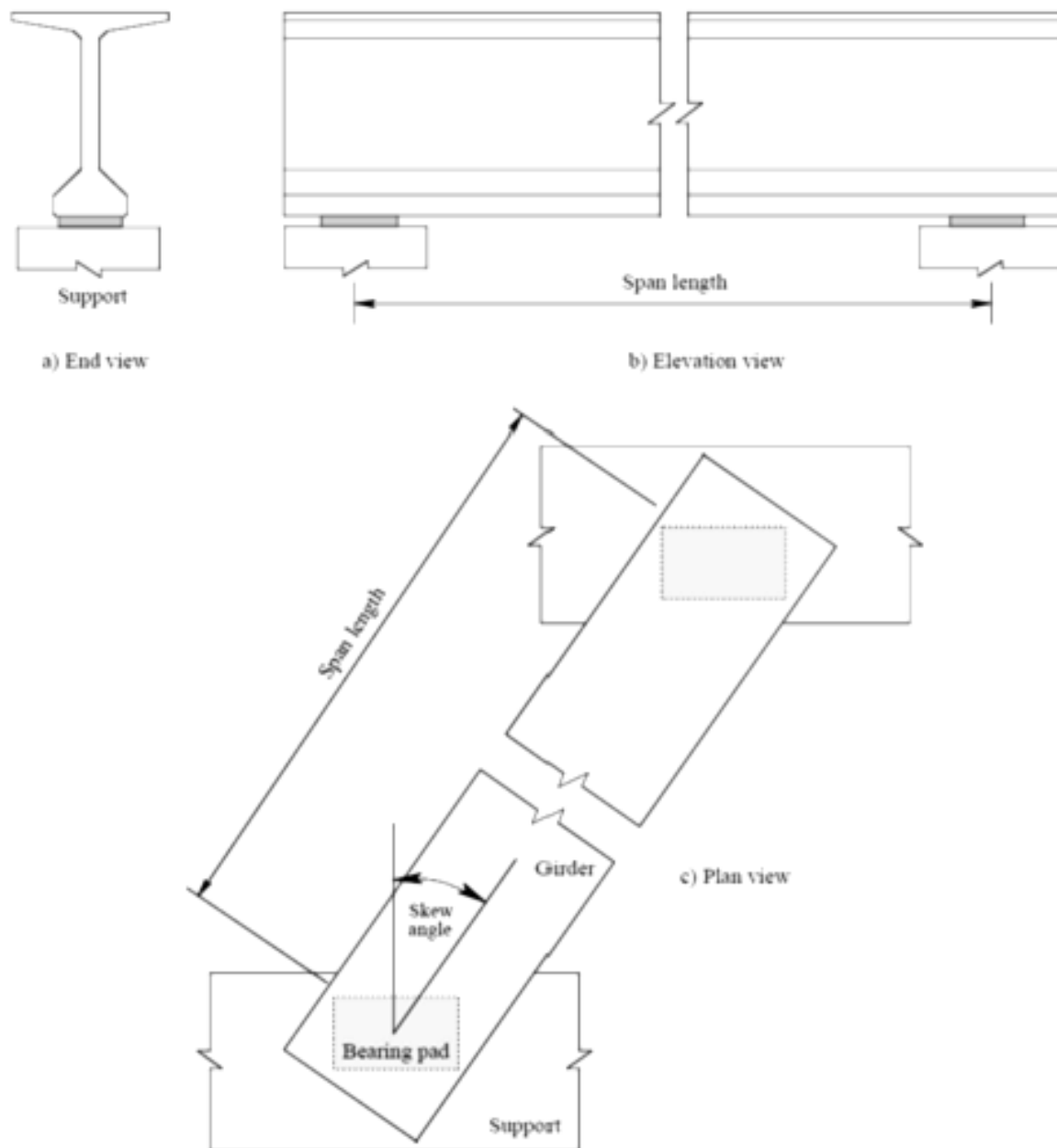


Figure B.1 Geometric configuration of bridge girder

FLORIDA BULB-TEE GIRDER 78IN.DEEP

Table B.1 Girder Parameters

Parameter	Value	Comments
Girder type	FBT78	Florida bulb-tee girder 78 in. deep
Span length	$L = 160$ ft	See figure B.1
Skew angle	$\theta = 30$ deg.	See figure B.1
Section properties	$I = 99295$ in ⁴	Weak axis moment of inertia (Table 3.2, Chapter 3)
Section properties	$J = 35270$ in ⁴	Torsional constant (Table 3.2, Chapter 3)
Section properties	$A = 1105.5$ in ²	Cross-sectional area (Table 3.2, Chapter 3)
Concrete	$f'_c = 6$ ksi	Compressive strength
Concrete	$\gamma = 0.155$ kip/ft ³	Specific weight
Concrete	$\nu = 0.2$	Poisson's ratio

As noted in Sections 8.5 and 11.2 of this report, the following equation may be used to calculate the capacity of a girder in terms of resisting buckling due to uniform vertical loading :

$$C = \frac{\sqrt{EIGJ}}{L^3} (1.0 - 0.0017L)(1.0 - 0.006\theta)$$

In this expression, C is the capacity of the girder (the maximum uniformly distributed vertical load that the girder can carry, in units of kip/ft, see Figure B.2), E is the elastic modulus of concrete (in units of kip/ft²), I is the weak axis moment of inertia of the girder (in units of ft⁴), G is the shear modulus of concrete (in units of kip/ft²), J is the torsional constant for the girder (in units of ft⁴), L is the span length (in units of feet), and θ is the skew angle (in units of degrees). As noted in Sections 8.5 and 11.2, it is recommended that use of this equation be limited to span lengths of 180 ft (or less) for FBT78 girders. Since the span length in this example is $L = 160$ ft., the equation given above may be used.

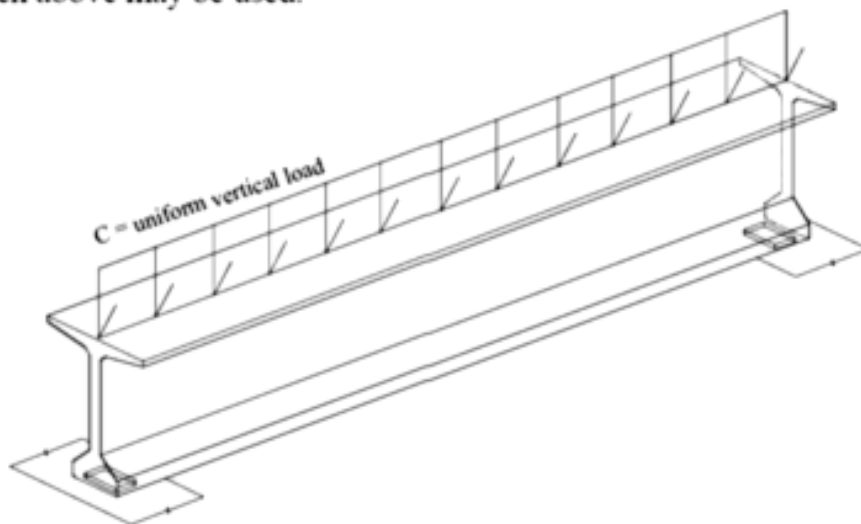


Figure B.2 Capacity defined as vertical load that initiates girder buckling

In order to use the capacity equation, cross-sectional properties I and J must be converted from units of in⁴ to ft⁴ :

$$I = 99295 \text{ in}^4, \text{ therefore } I = \frac{99295}{12^4} = 4.7885 \text{ ft}^4$$

and

$$J = 35270 \text{ in}^4, \text{ therefore } J = \frac{35270}{12^4} = 1.7009 \text{ ft}^4$$

In addition, the elastic and shear moduli must be computed and converted into appropriate units. The elastic modulus E may be computed as:

$$E = 33000 \gamma^{1.5} \sqrt{f'c}$$

where E is in units of kip/in², γ is in units of kip/ft³, and $f'c$ is in units of kip/in². Then E is given by :

$$E = 33000(0.155)^{1.5} \sqrt{6} = 4933 \text{ kip/in}^2$$

and in units of kip/ft²:

$$E = \frac{4933 \text{ kip/in}^2}{12^2} = 710313 \text{ kip/ft}^2$$

The shear modulus G of concrete can then be computed according to the mechanics relationship:

$$G = \frac{E}{2(1+\nu)} = \frac{710313 \text{ kip/ft}^2}{2(1+0.2)} = \frac{710313 \text{ kip/ft}^2}{2.4} = 295964 \text{ kip/ft}^2$$

Girder buckling capacity C, expressed in units of kip/ft, is then given by :

$$C = \frac{\sqrt{E I G J}}{L^3} 24(1.0-0.0017L)(1.0-0.006\theta)$$

$$C = \frac{\sqrt{(710313)(4.7885)(295964)(1.7009)}}{(160)^3} 24(1.0-0.0017(160))(1.0-0.006(30))$$

$$C = 4.577 \text{ kip/ft}$$

Therefore, a maximum uniform vertical load of approximately 4.6 kip/ft may be applied to the girder before buckling will occur. If the only load carried by the girder in the unbraced condition

is its own self-weight, then the factor of safety against buckling may be computed as follows. The cross-sectional area is given by:

$$A = 1105.5 \text{ in}^2, \text{ therefore } A = \frac{1105.5}{12^2} = 7.677 \text{ ft}^2$$

Volume per unit length of girder is :

$$V = A (1 \text{ ft.}) / \text{ft} = (7.677 \text{ ft}^2)(1 \text{ ft.}) / \text{ft} = 7.677 \text{ ft}^3 / \text{ft}$$

The structural demand, D, on the girder is given by the self-weight of the girder per unit length, expressed in units of kip/ft. The weight of a 1 ft. length of girder is:

$$W = \gamma V = (0.155)(7.677) = 1.190 \text{ kip/ft} \approx 1.2 \text{ kip/ft}$$

The factor of safety against buckling is then equal to the capacity to demand (C/D) ratio :

$$\text{FS} = C/D = (4.6 \text{ kip/ft}) / (1.2 \text{ kip/ft}) = 3.8$$

In Section 11.2, it is recommended that the factor of safety not be less than 3.0 for buckling, therefore in this example, the girder has adequate capacity against buckling. As noted in Sections 8.5 and 11.2, capacities computed using the equation :

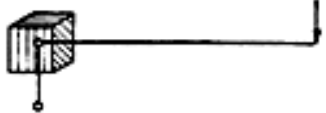
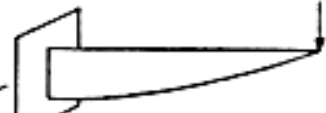

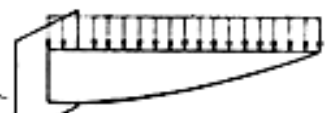
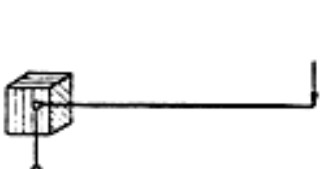


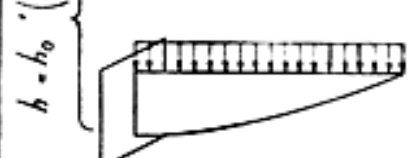

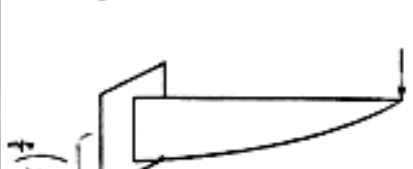
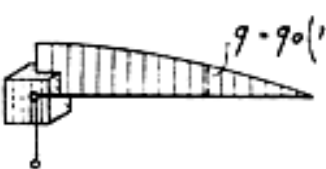
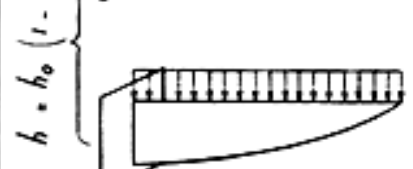

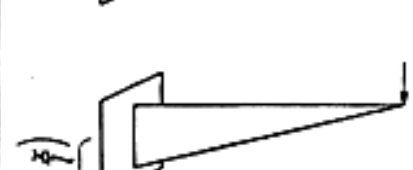
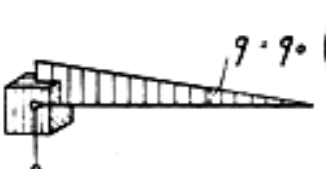
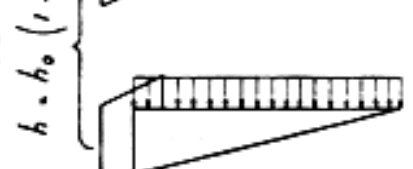
$$C = \frac{\sqrt{E I G J}}{L^3} 24(1.0 - 0.0017L)(1.0 - 0.006\theta)$$

are based on limit states related to lateral buckling (girder stability) and tensile flange stresses (cracking). Additional section capacity calculations based on alternative failure modes (e.g., compression failure in the flange, etc.) must also be conducted. Such calculations are not presented here because specific quantities and locations of prestressing and/or mild reinforcing steel have not been included in the example.

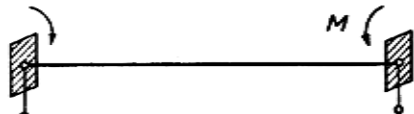
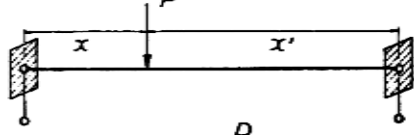
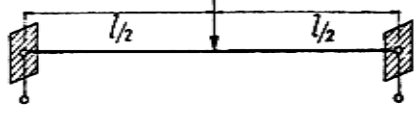
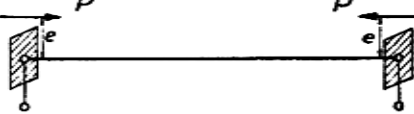
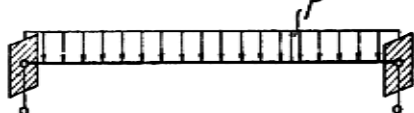
It must also be noted that the detrimental effects of horizontal wind loading are not included in the buckling capacity calculations presented in this example. The presence of horizontal wind pressure on a girder will reduce the magnitude of vertical gravity loading required to initiate buckling and instability. Hence, calculations in addition to those shown here must be performed in order to assess girder capacity under conditions of combined horizontal (wind) and vertical (gravity) loading.

ANEXO 08 - SIDNEY M. G. DOS SANTOS - TESE DE CÁTEDRA - UFRJ -1950

Quadro V - P_k ou $pl = m \cdot \frac{\sqrt{B' R_l}}{l^2}$

Seccão constante		Seccão variavel	
Carregamentos	m	Carregamentos	m
	4,013		3,81
	12,85		12,06
	$4,013 \left(1 - \frac{h}{2l} \sqrt{\frac{B'}{R_l}}\right)$		3,21
	$12,85 \left(1 - 0,975 \frac{h}{2l} \sqrt{\frac{B'}{R_l}}\right)$		11,24
	15,87		7,31
	19,08		10,43
	22,64		2,41
	26,61		9,62

QUADRO VI

Carregamentos	m	Formulas
	π	$M_k = m \cdot \frac{\sqrt{B R l}}{l}$
	(V. Ab. IV)	} $P_k = m \cdot \frac{\sqrt{B' R l}}{l^2}$
	16,93	
	π	$M_k = m \cdot \frac{\sqrt{B' R l}}{l}$
	$\sqrt{1 + \frac{l}{\pi e} \sqrt{\frac{R l}{B_2}}}$	$(pl)_k = m \cdot \frac{\sqrt{B' R l}}{l^2}$
	28,32	

Observação : Os valores obtidos pelos diversos autores são idênticos.

Referências básicas sobre Kippe :

- 1 - Prandtl Ludwig - Kipperscheinungen - Dissertação - Munique 1900
- 2 - Timoshenko P. Stephen - Theory of Elastic Stability - 1931 / 1961 (2ª edição) ,
- 3 - Sidney M.G. dos Santos - KIPPE ou Flambagem transversal - Tese de Cátedra - UFRJ - 1950
- 4 - Kollbrunner F. Curt - Knicken, Biegedrillknicken, Kippen - 1961 (2ª edição)

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