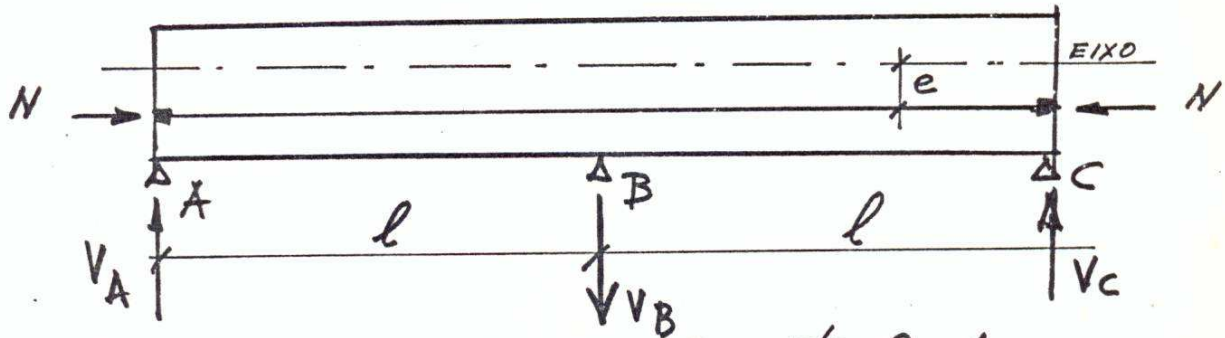
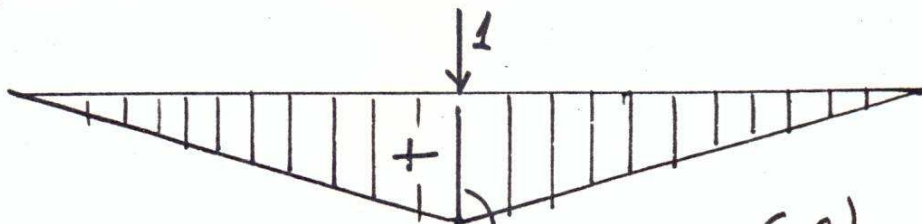
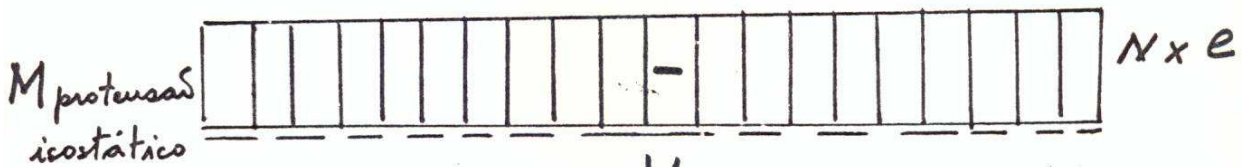
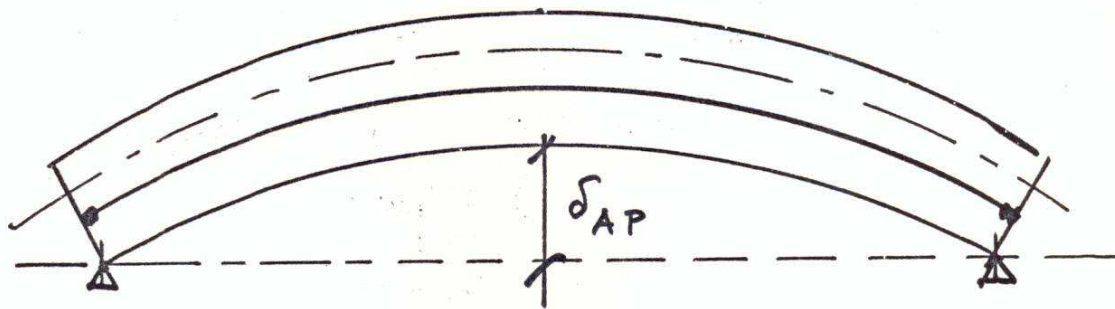




HIPERESTÁTICO DE PROTENSÃO.

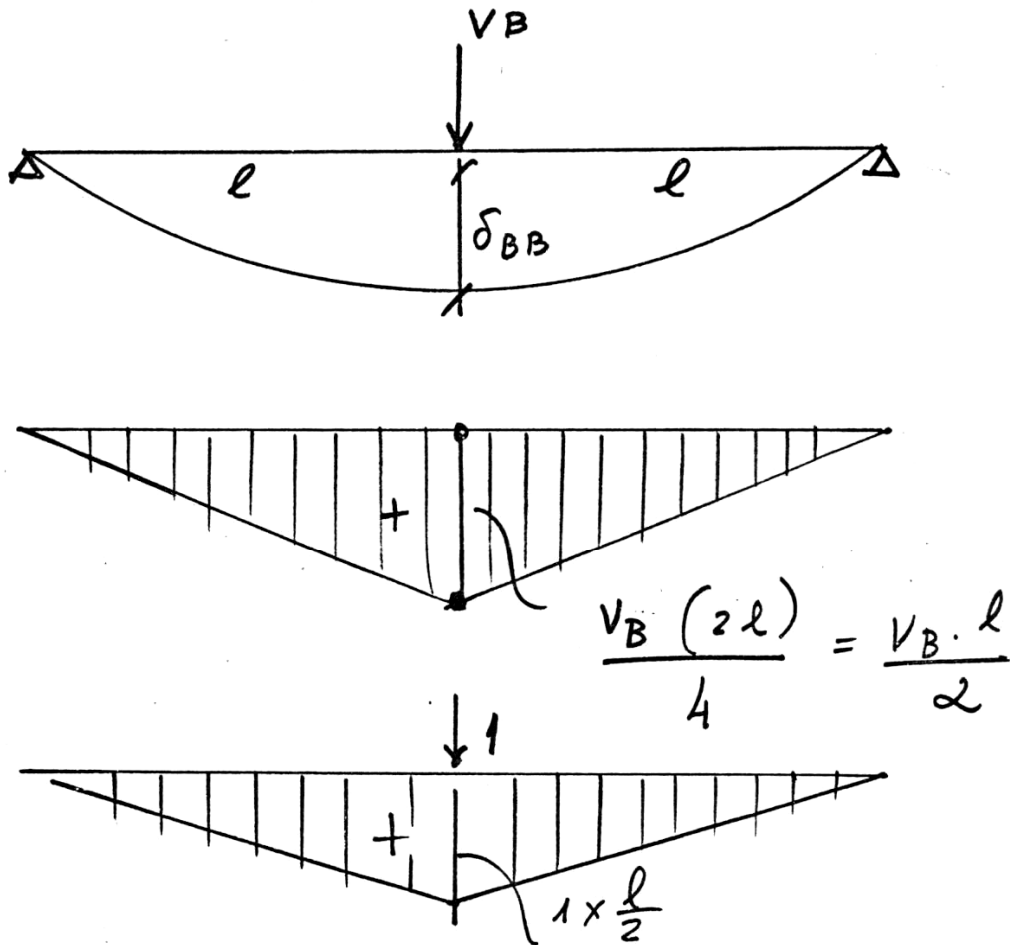


Retirando o apoio central. teremos.



$$M = \frac{1 \times (2l)}{4} = \frac{1 \cdot l}{2}$$

$$\delta_{AP} = -\frac{1}{2} \times (N \cdot e) \times \left(\frac{l}{2}\right) \times \frac{2l}{EI} = -\left(\frac{N \cdot e \cdot l^2}{2EI}\right)$$



$$\delta_{BB} = 2 \times \frac{1}{3} \times \left(\frac{V_B \cdot l}{2} \right) \times \left(\frac{l}{2} \right) \times \frac{l}{EI} = \frac{V_B \cdot l^3}{6 EI}$$

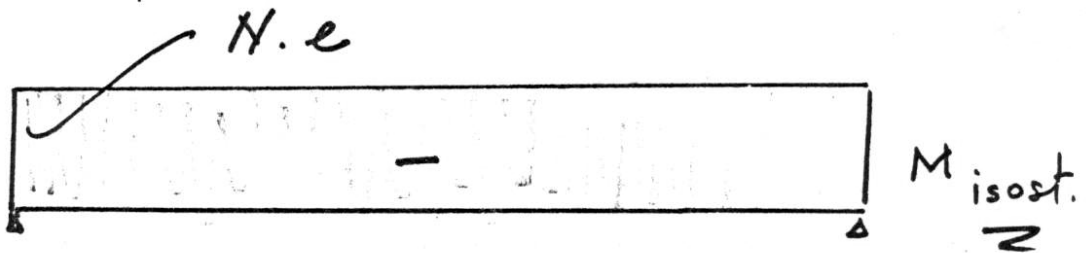
$$\delta_{AP} + \delta_{BB} = 0 \Rightarrow \frac{N \cdot e \cdot l^2}{2EI} = \frac{V_B \cdot l^3}{6EI}$$

$$V_B = 3 \cdot N \cdot \frac{e}{l}$$

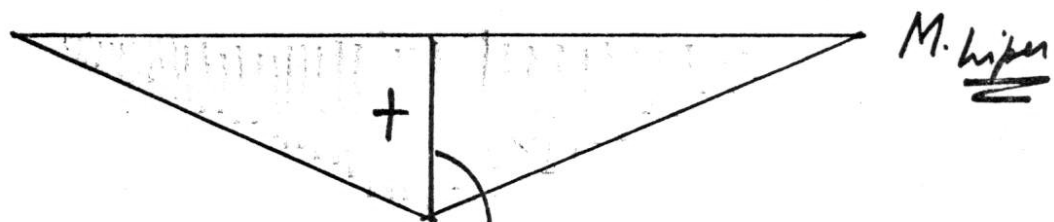
$$V_A = V_C = \frac{1}{2} V_B = \frac{3}{2} N \cdot \frac{e}{l}$$



$$M_{\text{isostático}} + M_{\text{hiperestático}} = M_{\text{protensor}}$$

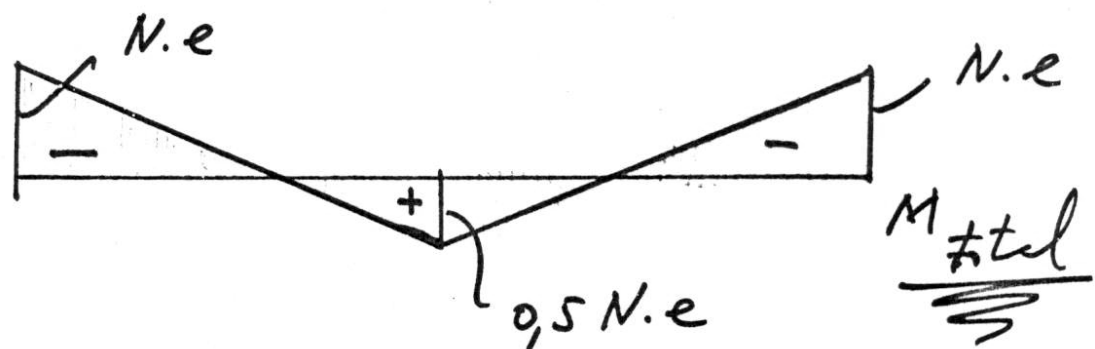


+



$$V_B \cdot \frac{l}{2} = 3N \cdot \frac{e}{l} \times \frac{l}{2} =$$

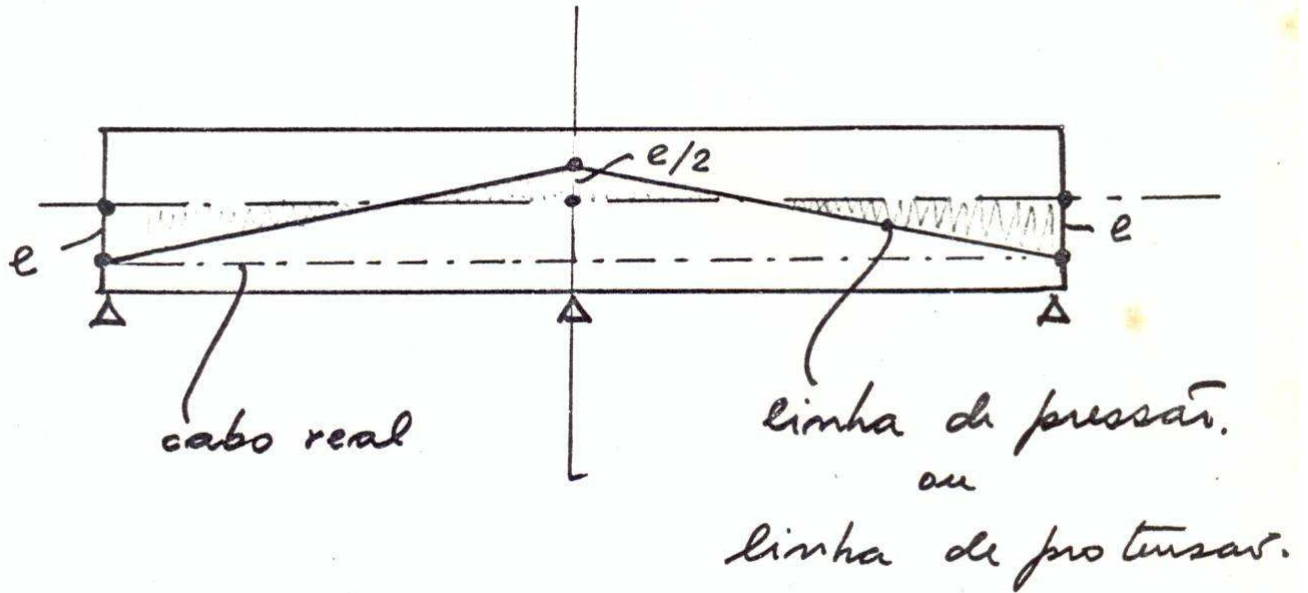
$$= \frac{3}{2} N.e = 1,5 N.e$$



$$M_{\text{Total}} = M_{\text{Isostático}} + M_{\text{Hiperestático}}$$



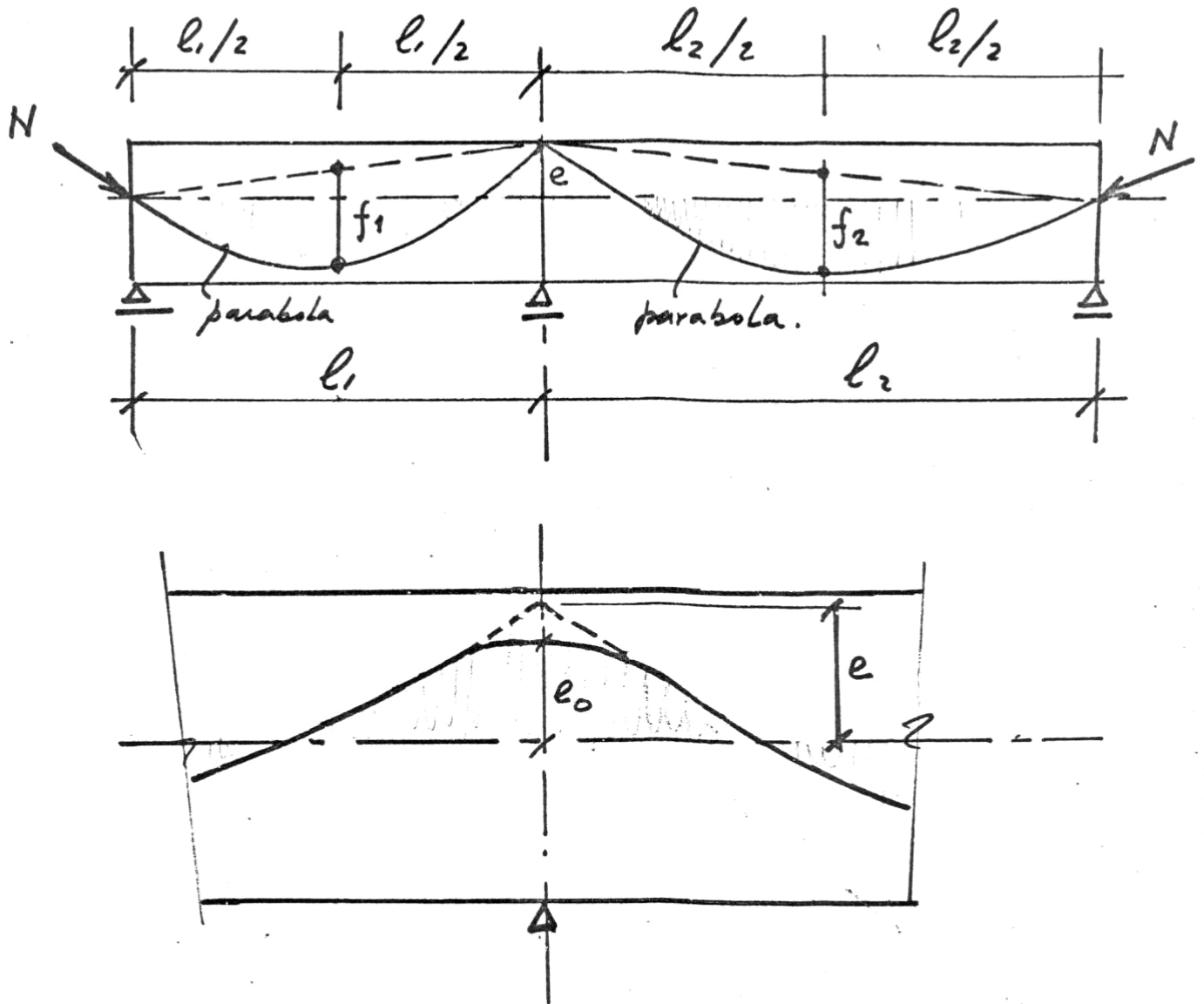
A linha de pressas da protensas seria:



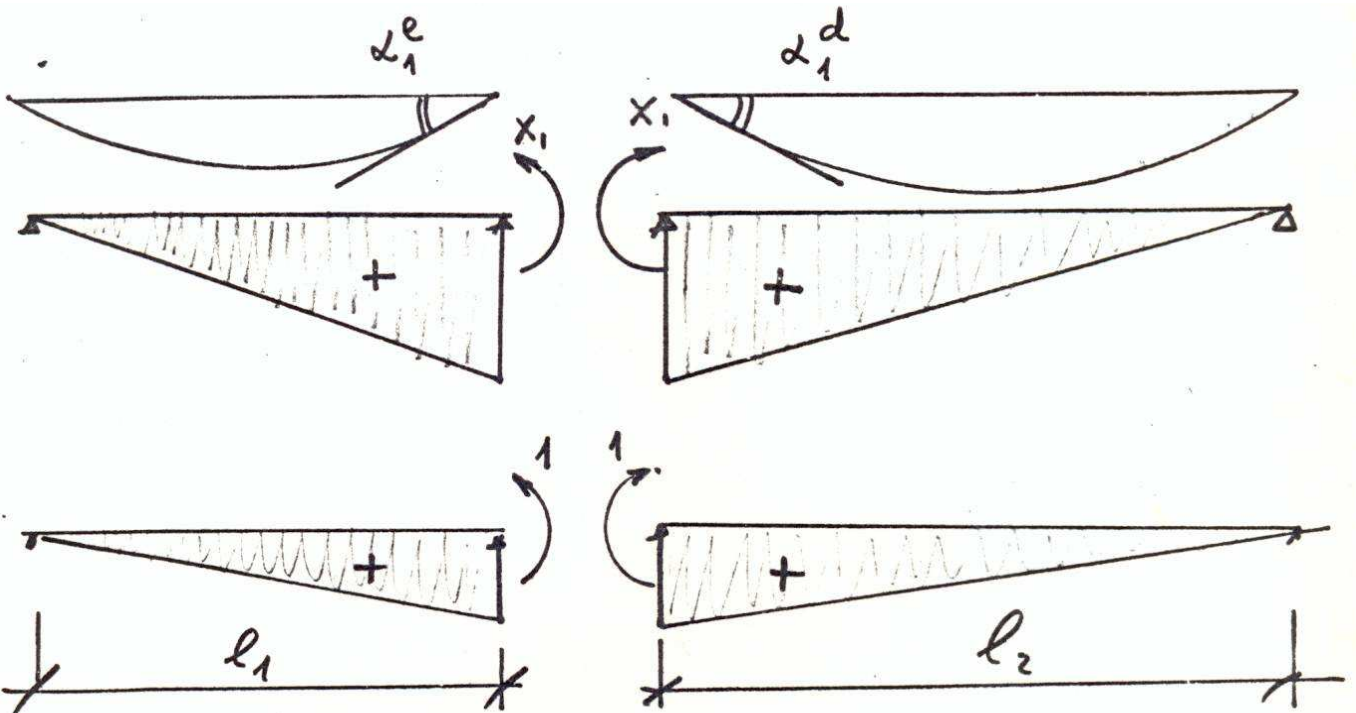
— + —



Outro exemplo:



Para o cálculo do hiperestático usaremos o cabo parabólico, sem a concordância sobre o apoio. Consideraremos portanto o cabo com excentricidade e .



$$\alpha_1^e = \frac{1}{3} \cdot X_1 \cdot 1 \cdot \frac{l_1}{EJ}$$

$$\alpha_1^d = \frac{1}{3} \cdot X_1 \cdot 1 \cdot \frac{l_2}{EJ}$$

— + —

Resumo.

$$\alpha_0 = \alpha_0^e + \alpha_0^d = \frac{N l_1 (f_1 - e)}{3EJ} + \frac{N l_2 (f_2 - e)}{3EJ}$$

$$EJ \alpha_0 = \frac{N}{3} [l_1 f_1 + l_2 f_2 - e(l_1 + l_2)]$$

$$\alpha_1 = \alpha_1^e + \alpha_1^d = \frac{X_1 \cdot l_1}{3EJ} + \frac{X_1 \cdot l_2}{3EJ}$$

$$EJ \alpha_1 = \frac{X_1}{3} (l_1 + l_2)$$



Para que haja continuidade geométrica devemos ter:

$$\alpha_0 = \alpha_1$$

$$\frac{N}{3} [l_1 f_1 + l_2 f_2 - e(l_1 + l_2)] = \frac{X_1}{3} (l_1 + l_2).$$

$X_1 = M_{\text{protensão hiperestática}}$

$$X_1 = N \left(\frac{l_1 f_1 + l_2 f_2}{l_1 + l_2} - e \right)$$

Caso particular viga simétrica:

$$l_1 = l_2 = l$$

$$f_1 = f_2 = f$$

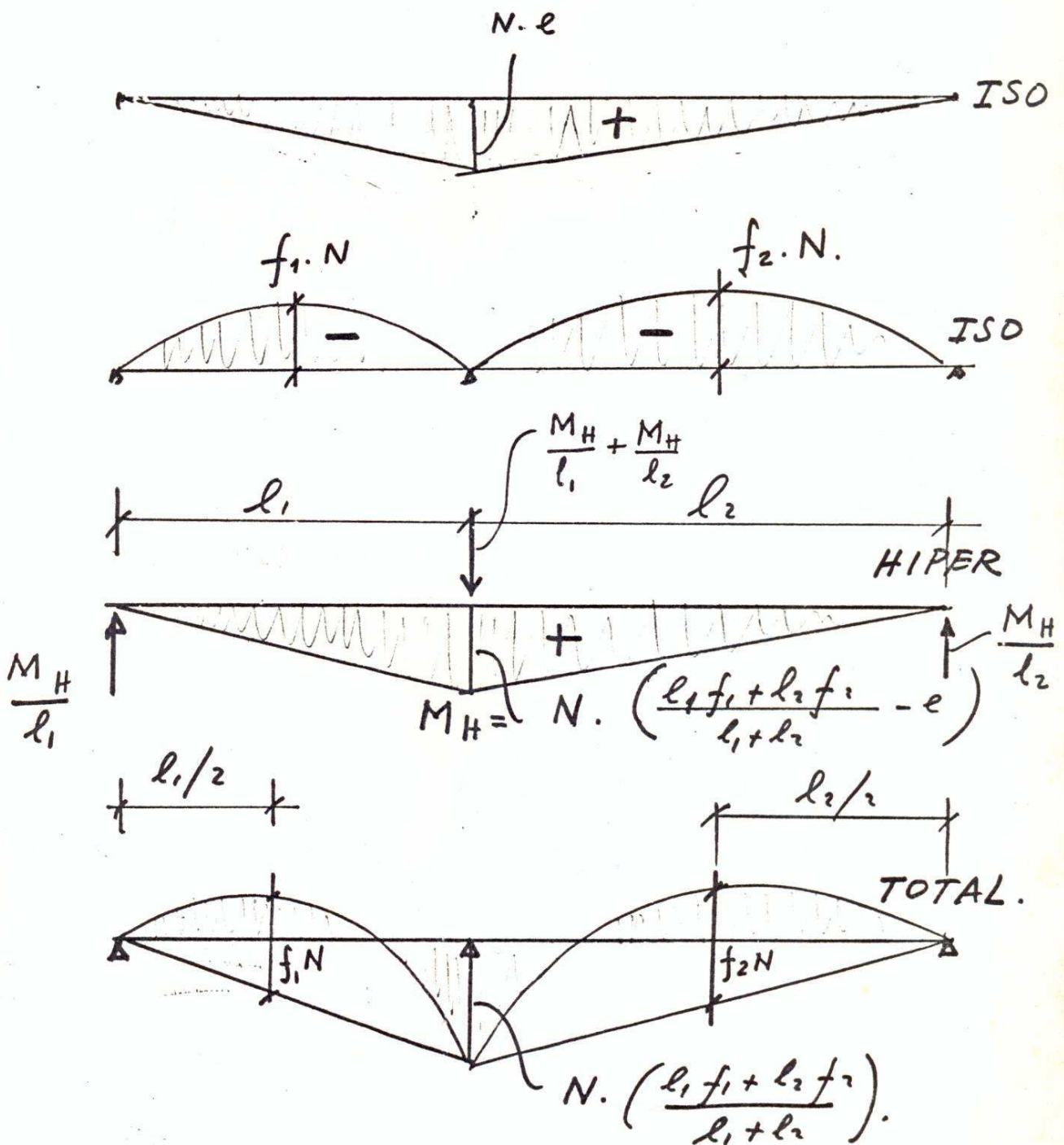
$$X_1 = N (f - e)$$

se $f = e$

$$X_1 = 0.$$



Temos portanto o seguinte dia-
grama final.





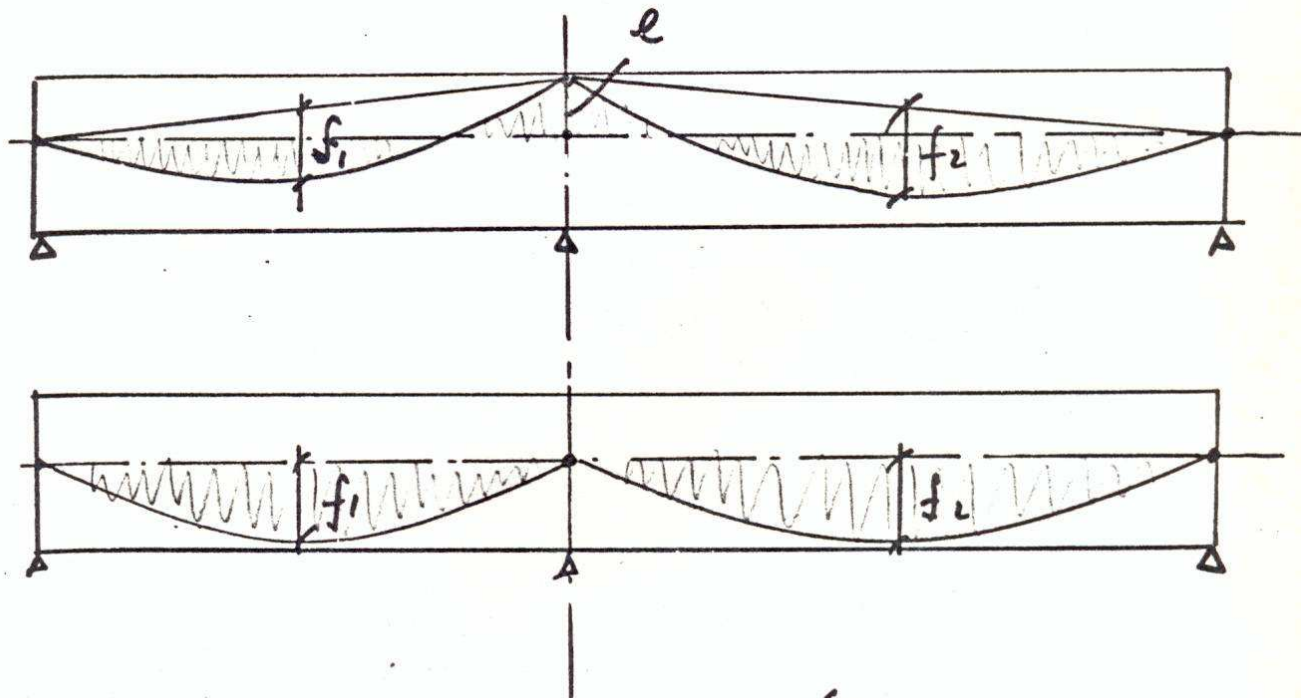
Observamos que o momento total de protensão no apoio central é

$$M_T = M_H + M_I = N \left(\frac{l_1 f_1 + l_2 f_2}{l_1 + l_2} - e \right) + N \cdot e =$$

$$M_T = N \left(\frac{l_1 f_1 + l_2 f_2}{l_1 + l_2} \right)$$

i.e

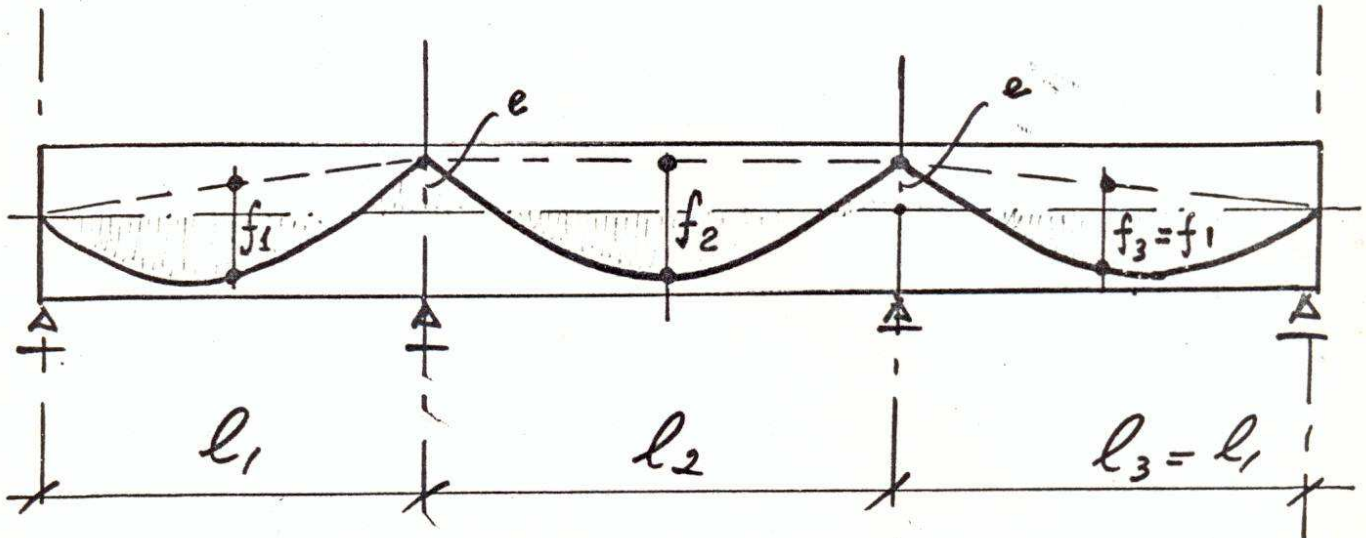
o momento total de protensão M_T no apoio independe de e . (!)



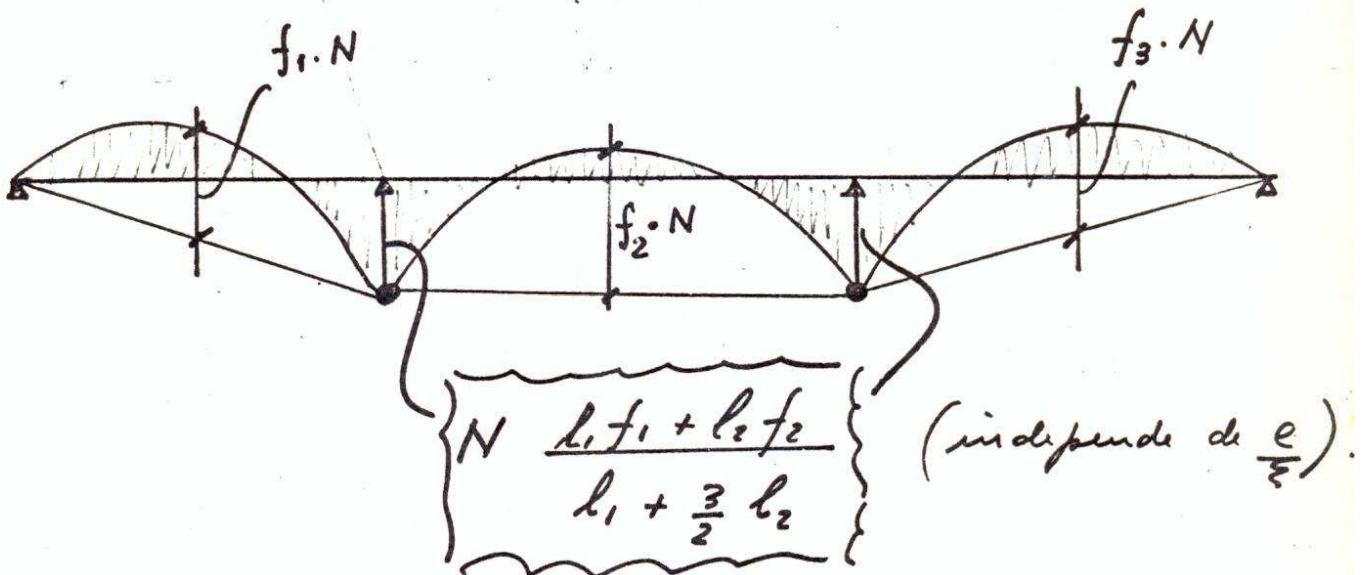
As duas vigas acima têm o mesmo diagrama final de protensão (pg 9).



Outro exemplo:



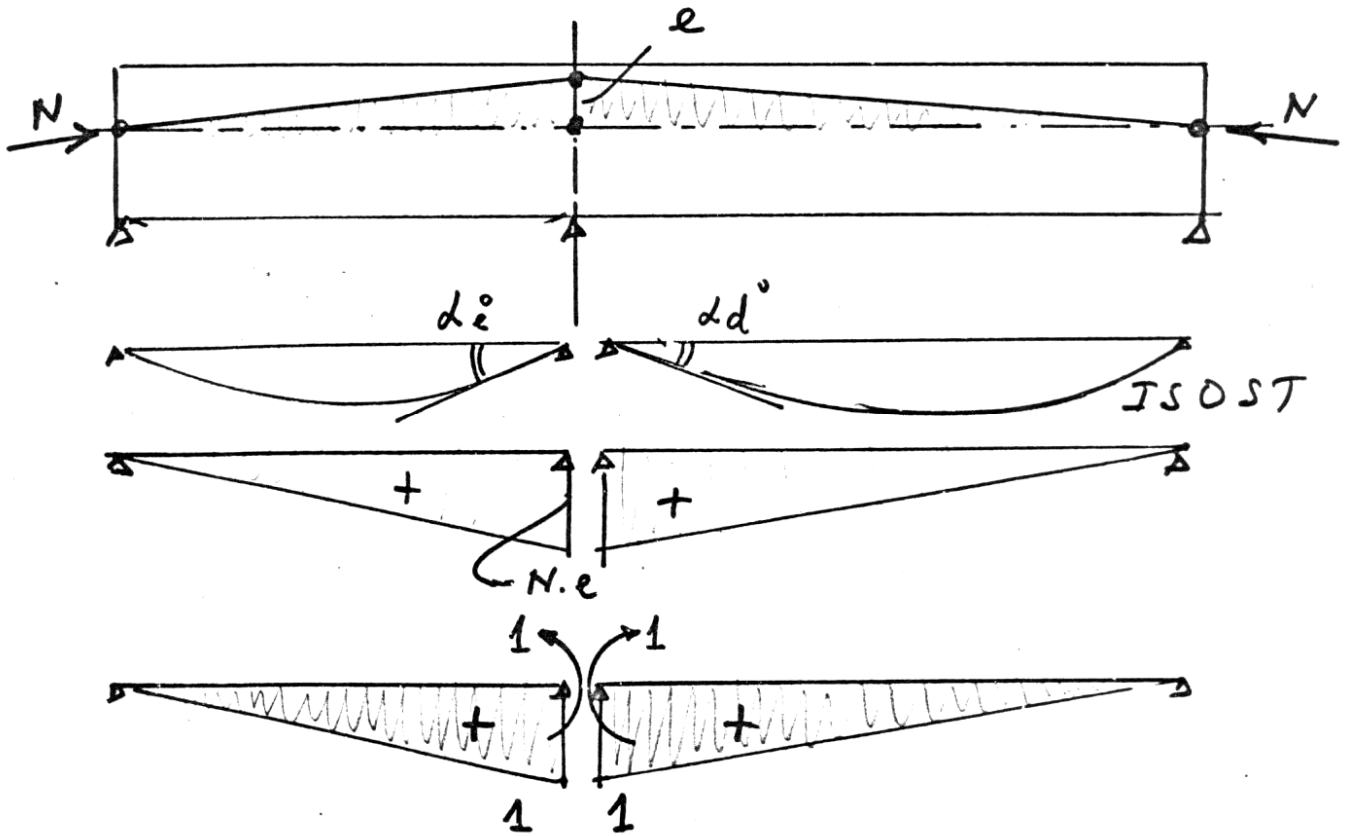
MOMENTO TOTAL DE PROTENSAO.



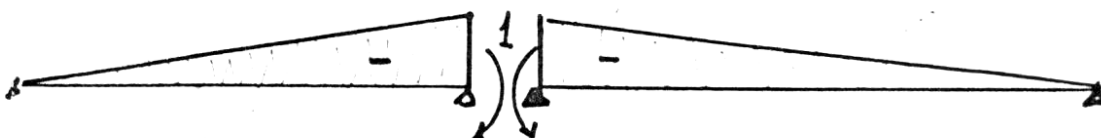
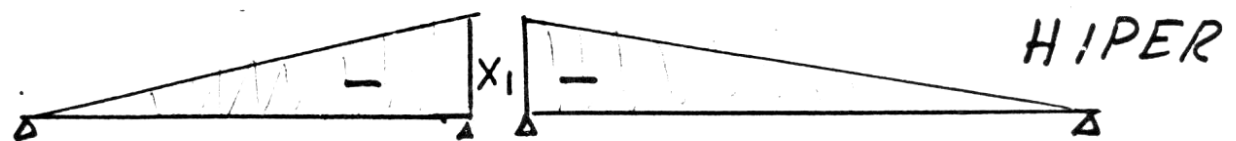
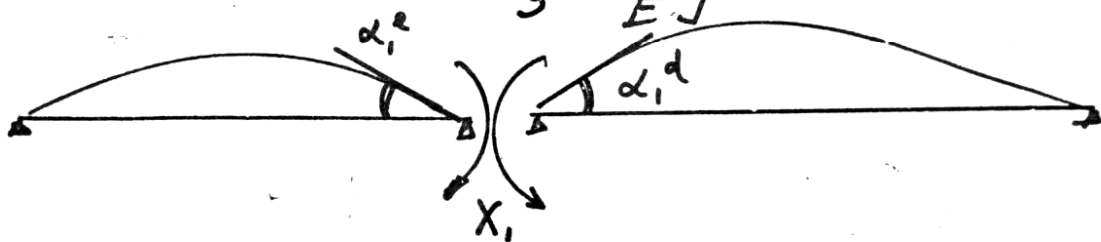
Podemos dizer que a excentricidade nos apoios não tem influência no momento total de protensão.



Curvatura:



$$\alpha^0 = \alpha_e^0 + \alpha_d^0 = \frac{1}{3} \frac{N \cdot e}{EJ} (l_1 + l_2)$$



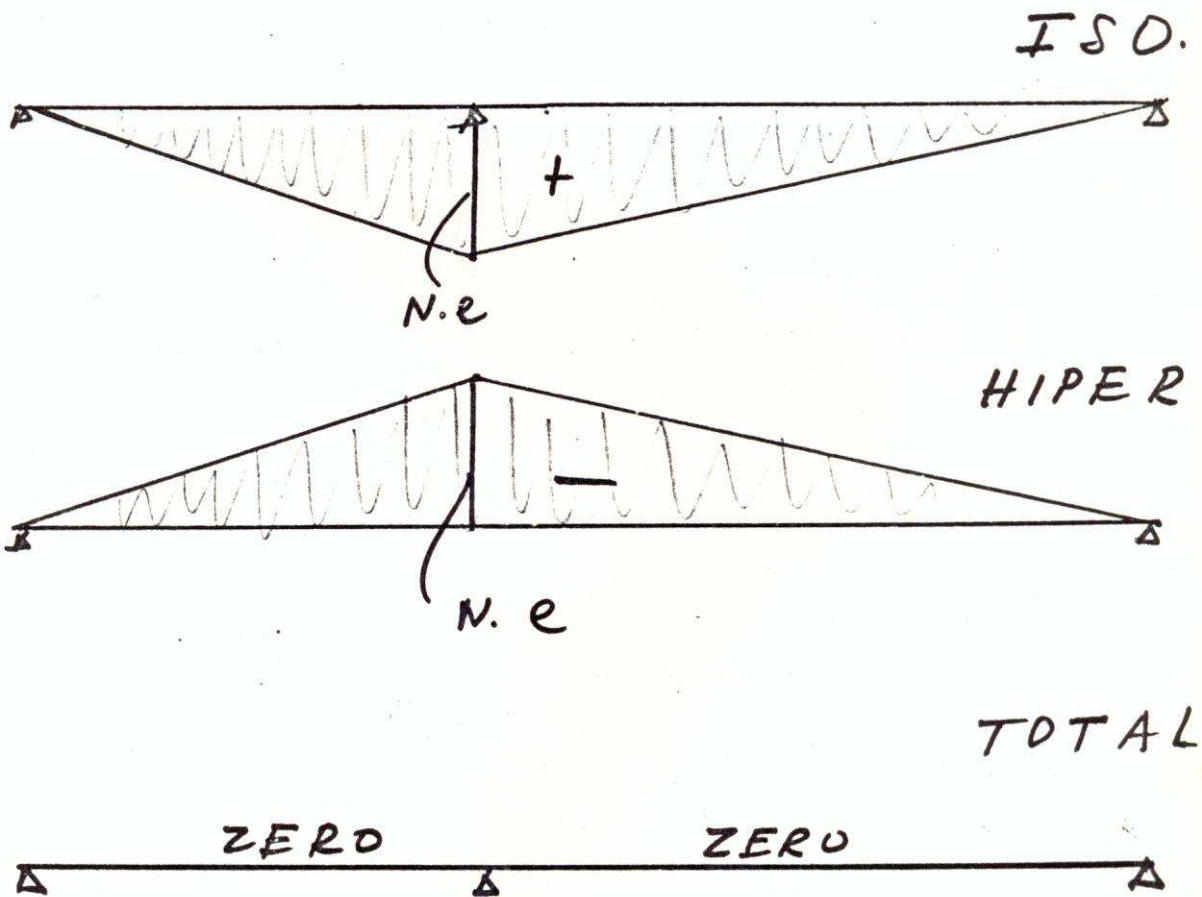
$$\alpha_1 = \alpha_e^1 + \alpha_d^1 = \frac{1}{3} X_1 \cdot \frac{l_1 + l_2}{EJ}$$



$$\alpha_0 = \alpha_1$$

$$\frac{1}{3} \frac{N \cdot e}{EJ} (l_1 + l_2) = \frac{X_1}{3} \frac{(l_1 + l_2)}{EJ}$$

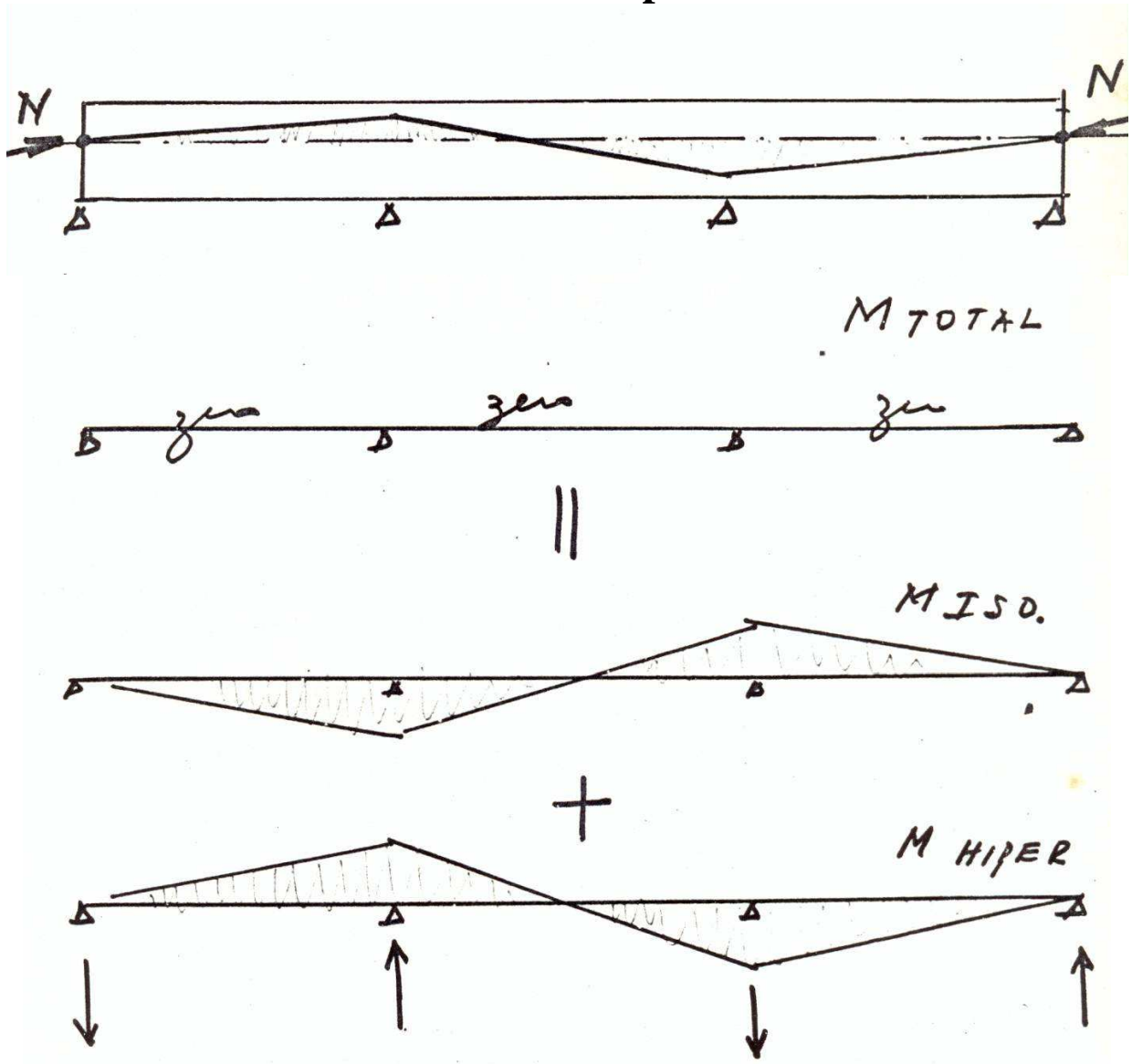
$$X_1 = N \cdot e$$



Em vigas contínuas, cabos retos não produzem momentos fletor e sim força normal. (Se os cabos nas seções extremas atuarem no C.G.).

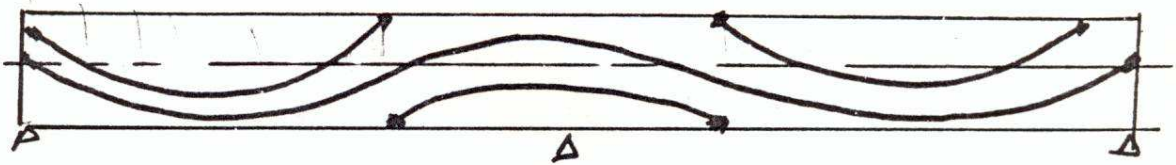


Outro exemplo:

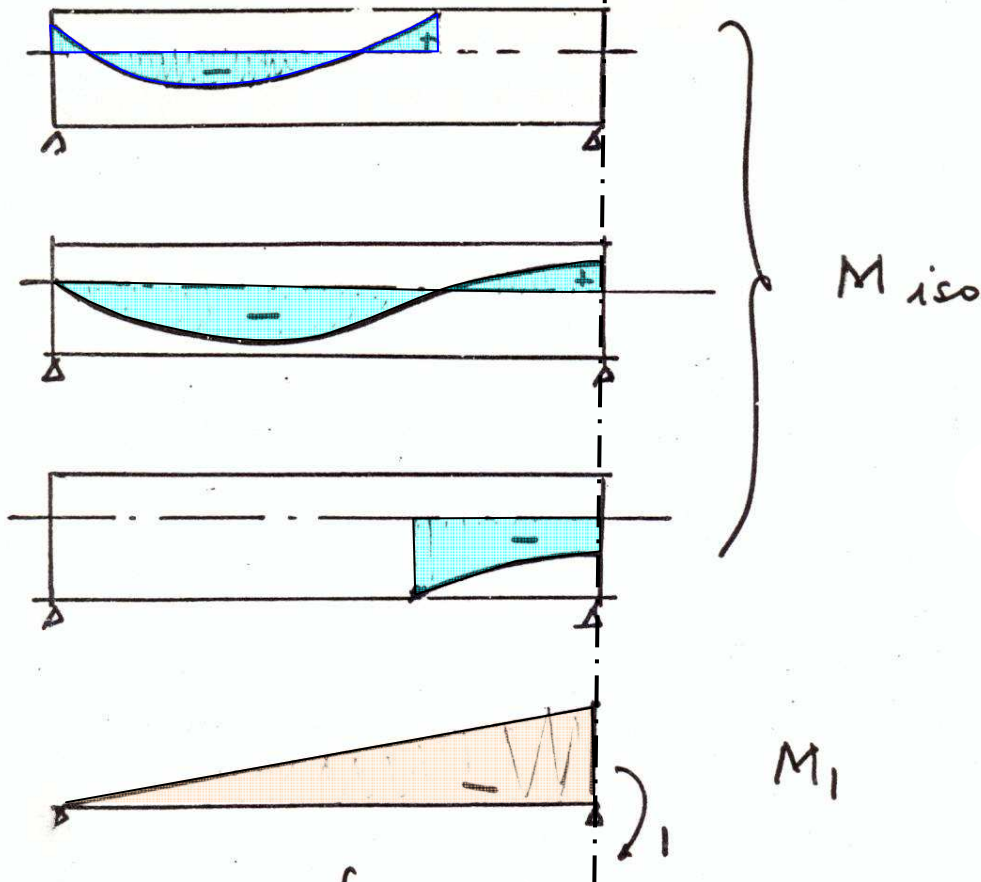




Num caso genérico :



Cálcula-se as rotações do sistema
instático e depois faz-se a
compatibilidade de rotações.

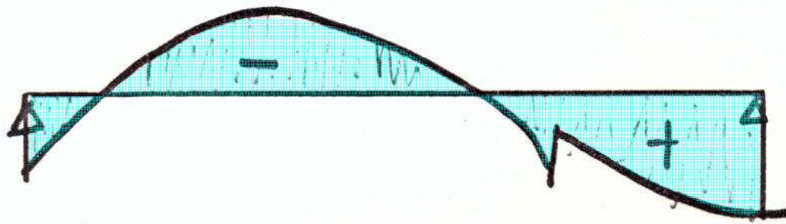


$$\alpha_e^0 = \int \left(\frac{M_{iso} \cdot M_1}{EJ} \right) dx.$$

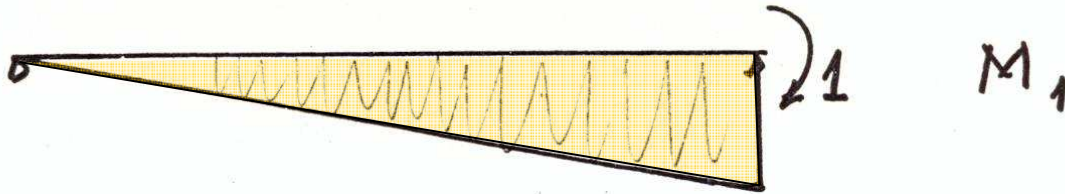
...



Podemos usar o diagrama total de protensão.



M_{iso}
(todos os cabos).



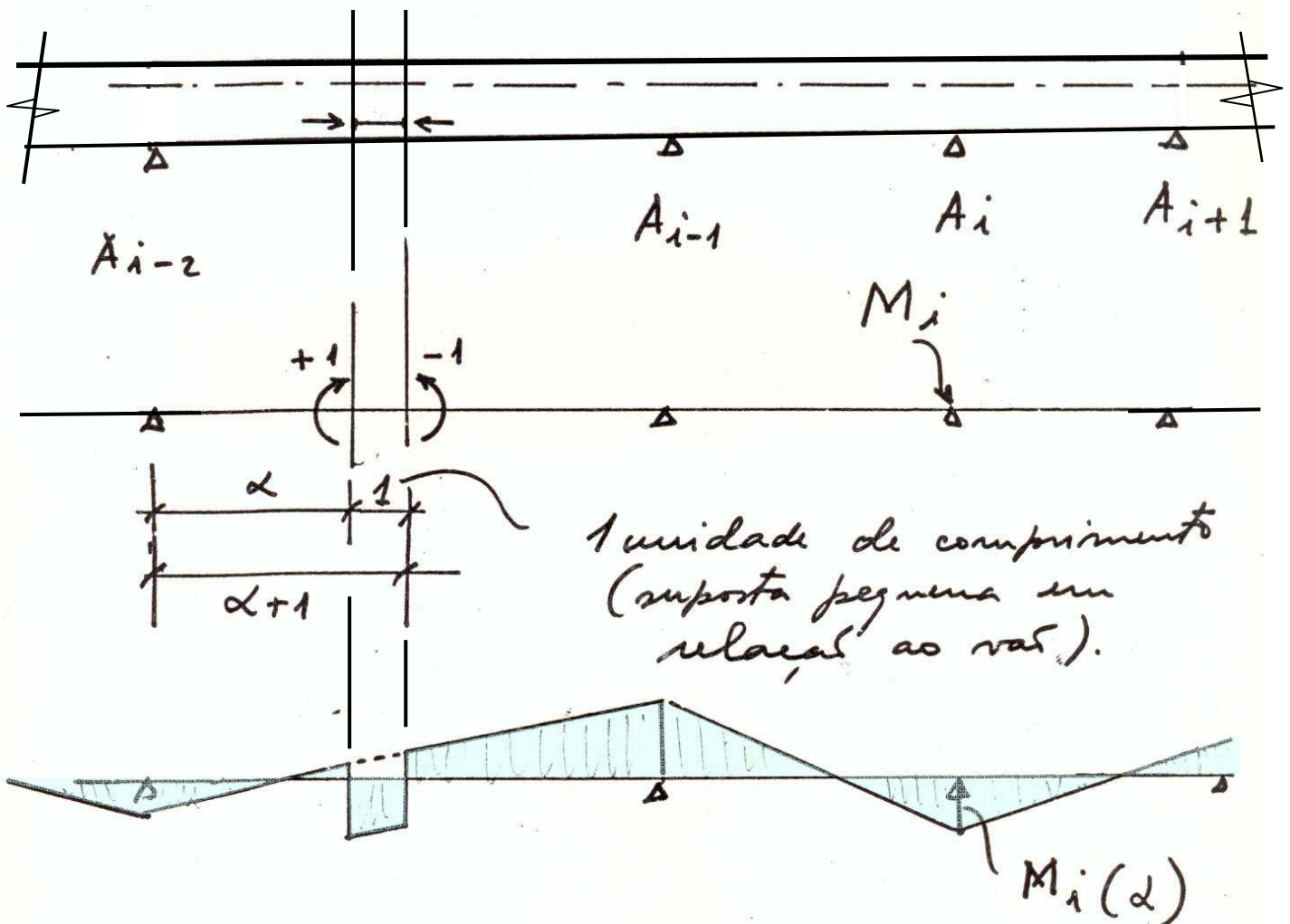
$$\Delta_e = \int \frac{M_{iso} \cdot M_1}{EI} \cdot dx$$

(Integradas pela fórmula de Simpson ou Trapezoidal ...).



Linha de influência de Momento hiperestático de protensão.

(Nomenclatura de M. Albiges.)

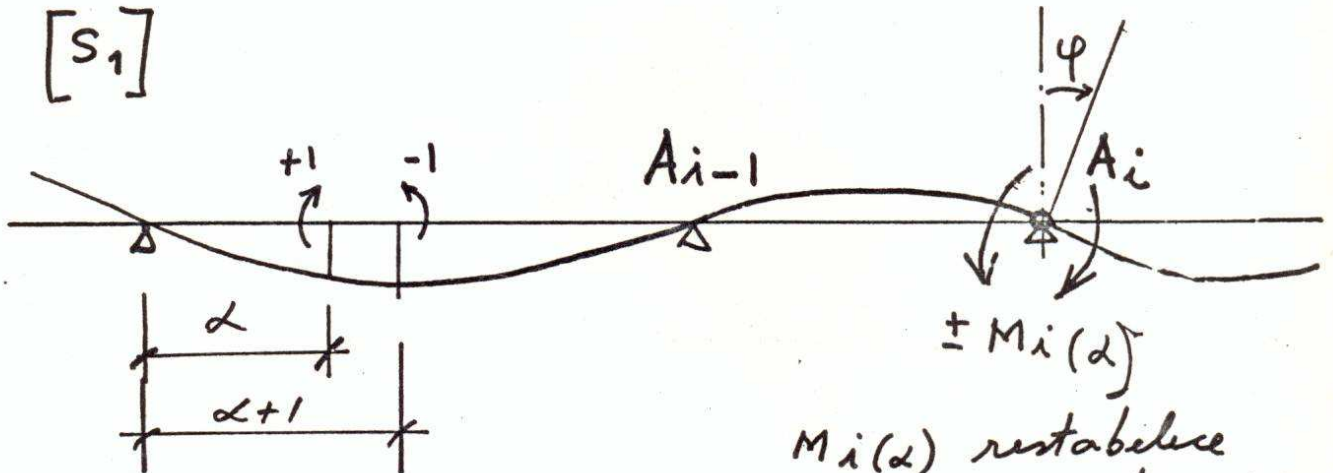


Como varia $M_i(\alpha)$ quando
o par de momentos 1 caminha
ao longo da viga?



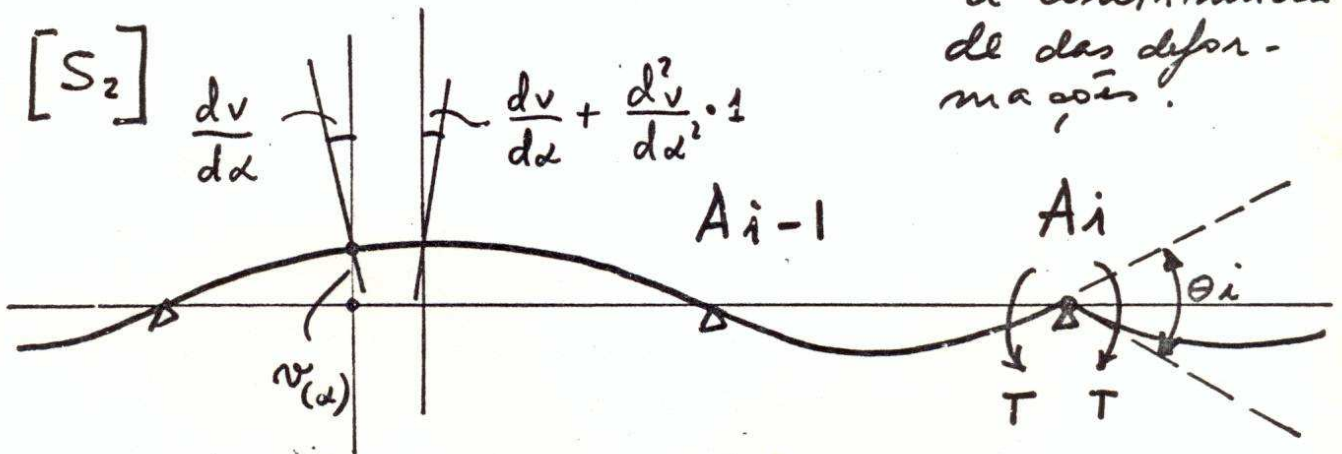
Introduzamos no apoio i uma rótula e consideremos 2 sistemas.

$[S_1]$



$M_i(\alpha)$ restabelece a continuidade das deformações.

$[S_2]$



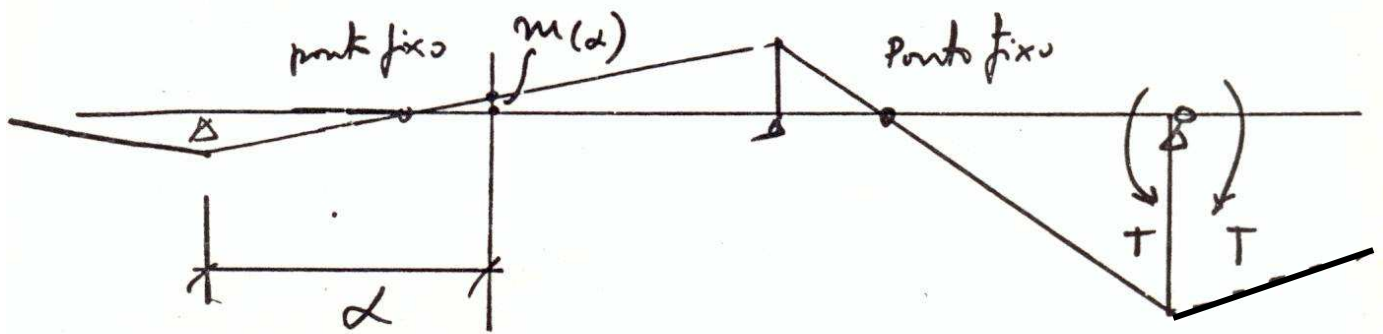
Aplicando o teorema de reciprocidade:

$$+ M_i(\alpha) \cdot \theta_i + 1 \times \frac{dv}{dx} - 1 \left(\frac{dv}{dx} + \frac{d^2v}{dx^2} \right) = + T \cdot \varphi - T \cdot \varphi$$

$$M_i(\alpha) = \frac{1}{\theta_i} \frac{d^2v}{dx^2}$$



Se $m(\alpha)$ é o momento fletor no sistema 2 $[S_2]$



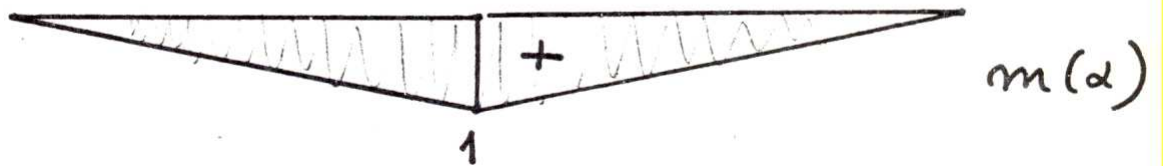
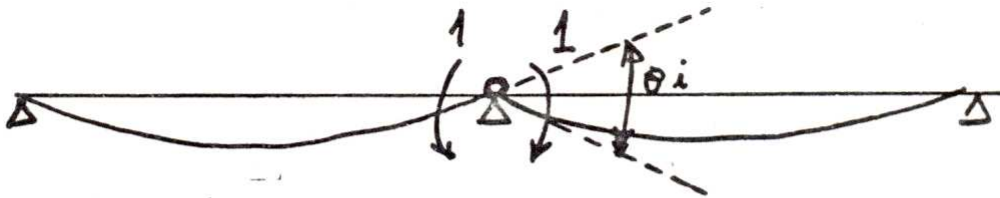
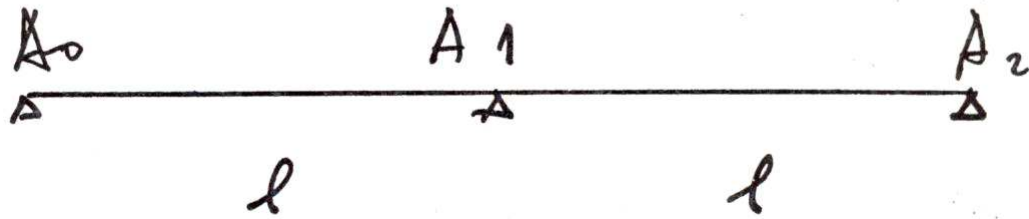
$$\frac{d^2 v}{d\alpha^2} = \frac{m(\alpha)}{EI(\alpha)}$$

$$M_i(\alpha) = \frac{1}{\theta_i} \cdot \frac{m(\alpha)}{EI(\alpha)}$$

A linha de influência de $M_i(\alpha)$ é portanto afim à linha $\frac{m(\alpha)}{EI(\alpha)}$



Exemplo



$$\theta_i = ?$$



$$\theta_i = 2 \times \left(\frac{1}{3} \cdot 1 \cdot 1 \right) \frac{l}{EI} = \frac{2}{3} \frac{l}{EI}$$

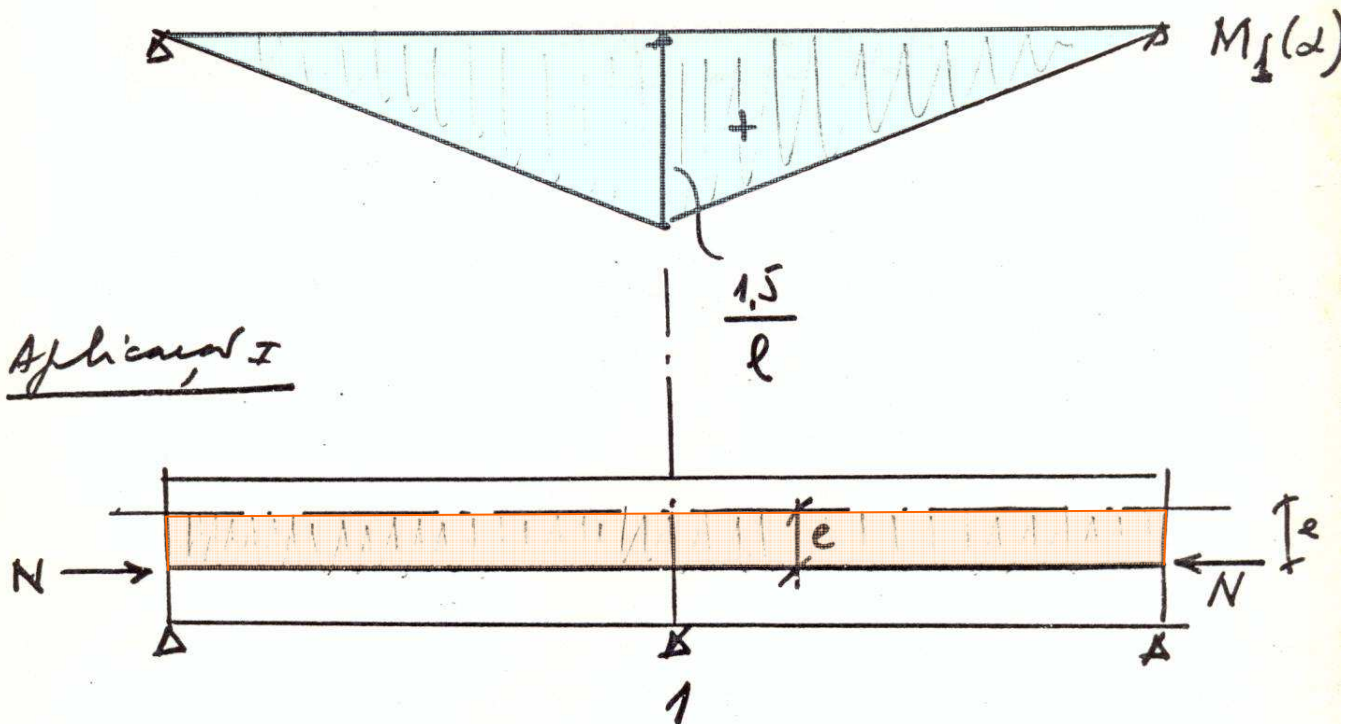


Linha de influência de M .

$$M_i(\alpha) = \frac{1}{\theta_i} \frac{m(\alpha)}{EI(\alpha)}$$

$$\theta_i = \frac{2}{3} \frac{l}{EI}$$

$$M_I(\alpha) = \frac{1}{\frac{2}{3} \frac{l}{EI}} \cdot \frac{m(\alpha)}{EI} = \frac{1,5 \cdot m(\alpha)}{l}$$



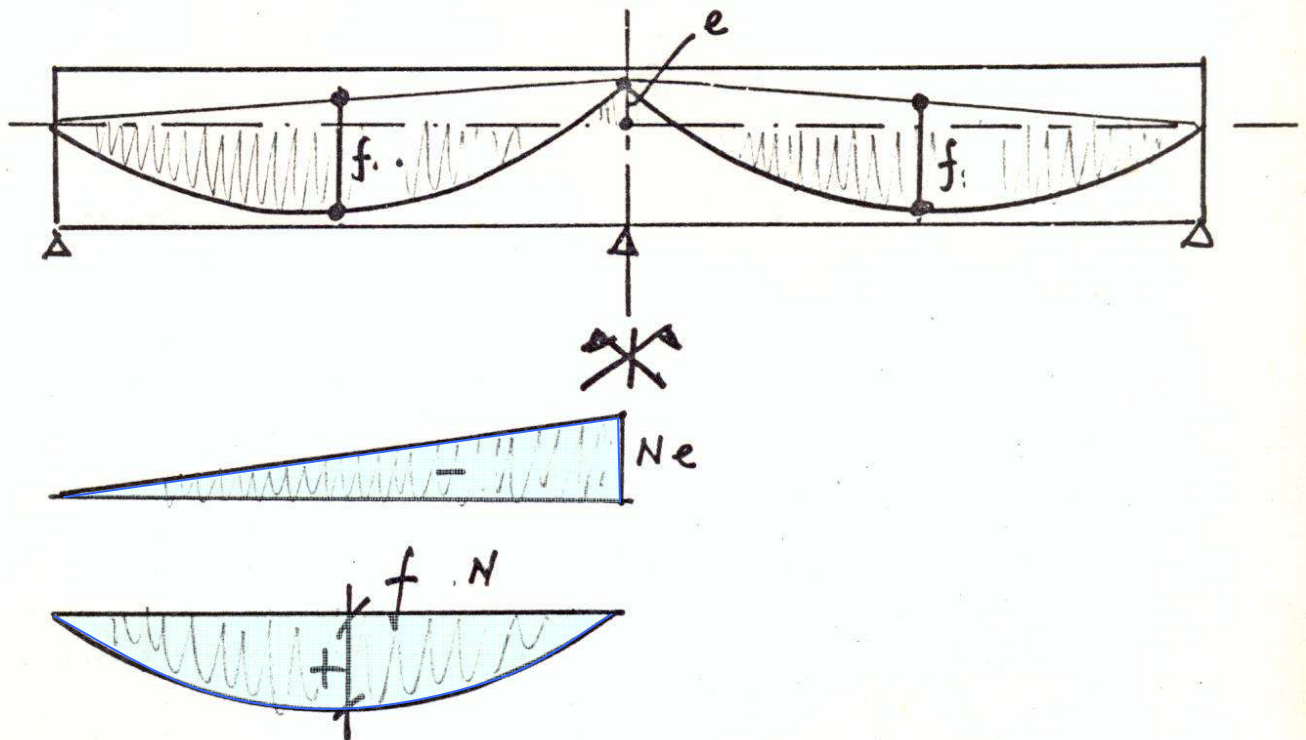
$$M_{HIP.} = 2 \times \frac{1}{2} \times \left(\frac{1,5}{l}\right) \times (N \cdot e) \times l = 1,5 N \cdot e$$

(ver 1º exem.
plo pag 3).

$$\frac{1}{l} \times \frac{1}{l} = \frac{1}{2} i \cdot j \cdot l$$

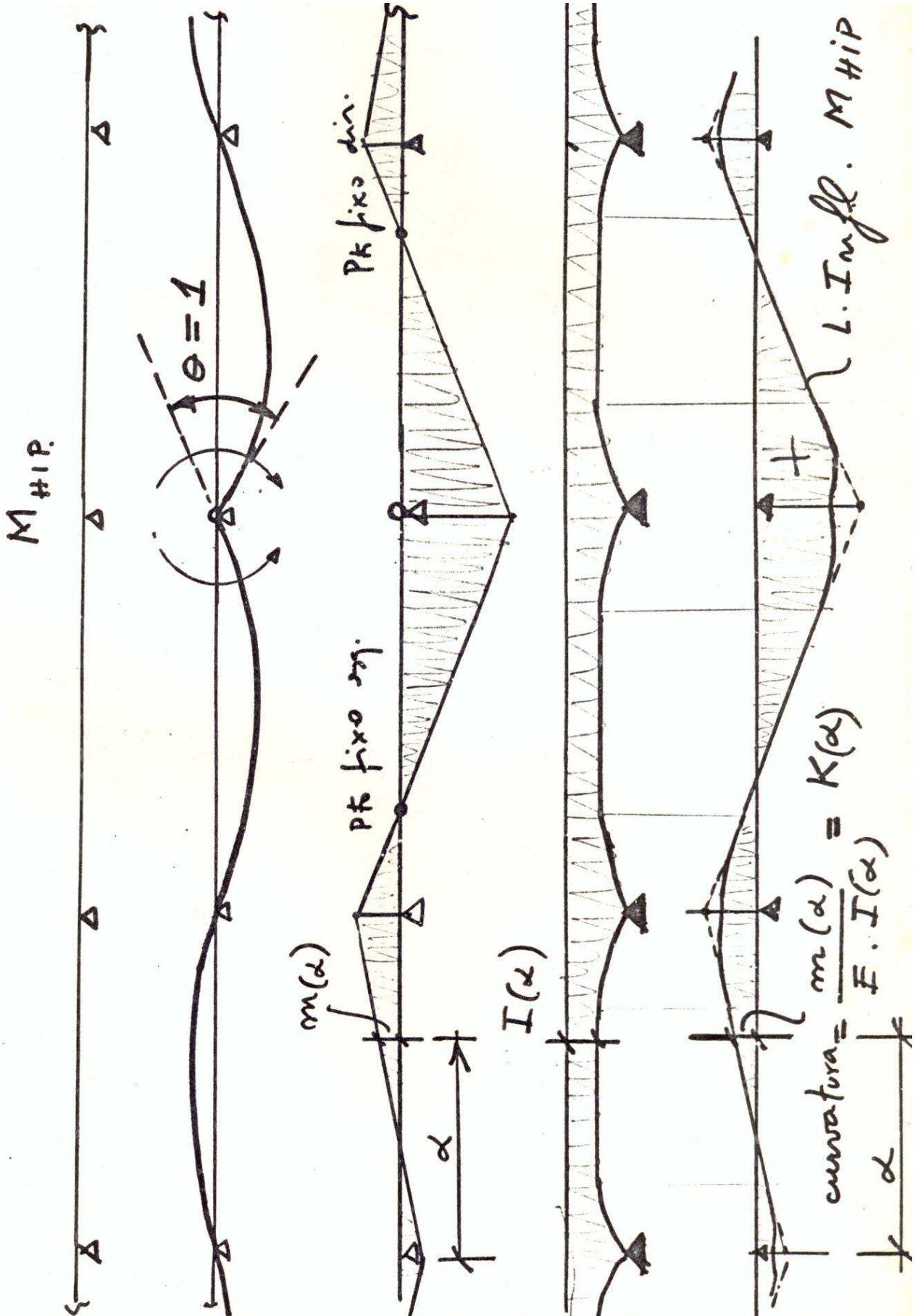


Aplicação 2



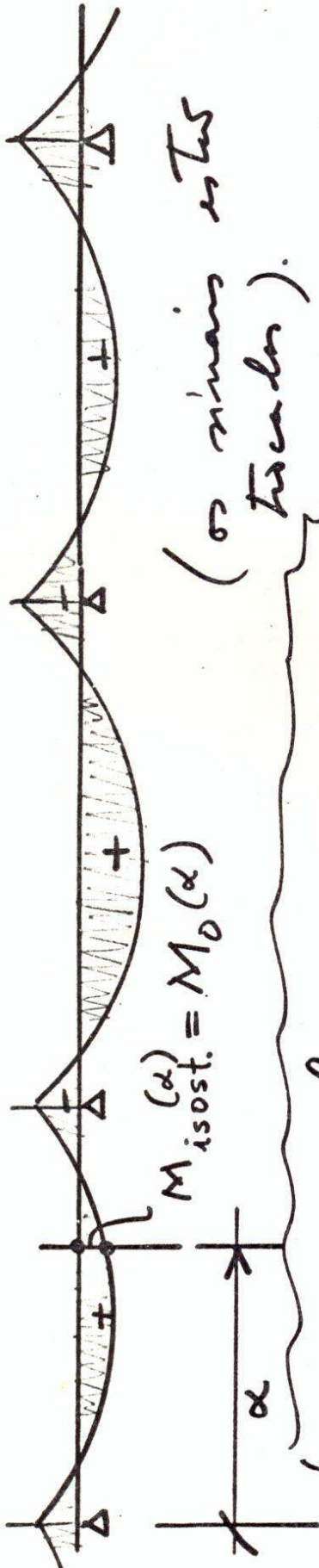
$$M_{HIP} = \left\{ \begin{array}{l} - 2 \times \frac{1}{3} \times (N \cdot e) \times \left(\frac{1,5}{l}\right) \times l = -N \cdot e \\ + 2 \times \frac{1}{3} \times (N \cdot f) \times \left(\frac{1,5}{e}\right) \times l = +N \cdot f \end{array} \right.$$

$$\boxed{M_{HIP} = +N(f - e)} \quad (\text{ver pag. 8}).$$





Carregar a l. Influença com $K(\alpha)$



$$M_{HIP} = \int_0^l K(\alpha) \cdot M_0(\alpha) \cdot d\alpha$$

$$[K] = m^{-1}$$

$$[M_0] = m \times t$$

$$[d\alpha] = m$$

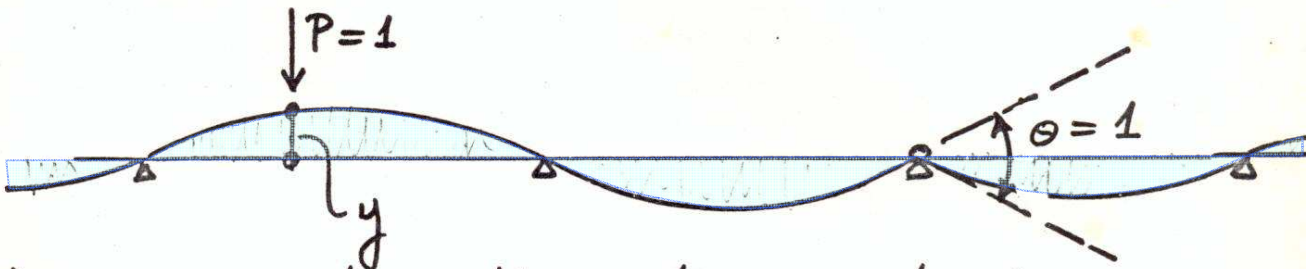
Para integrar o produto podemos usar as tabelas de integrais tipo



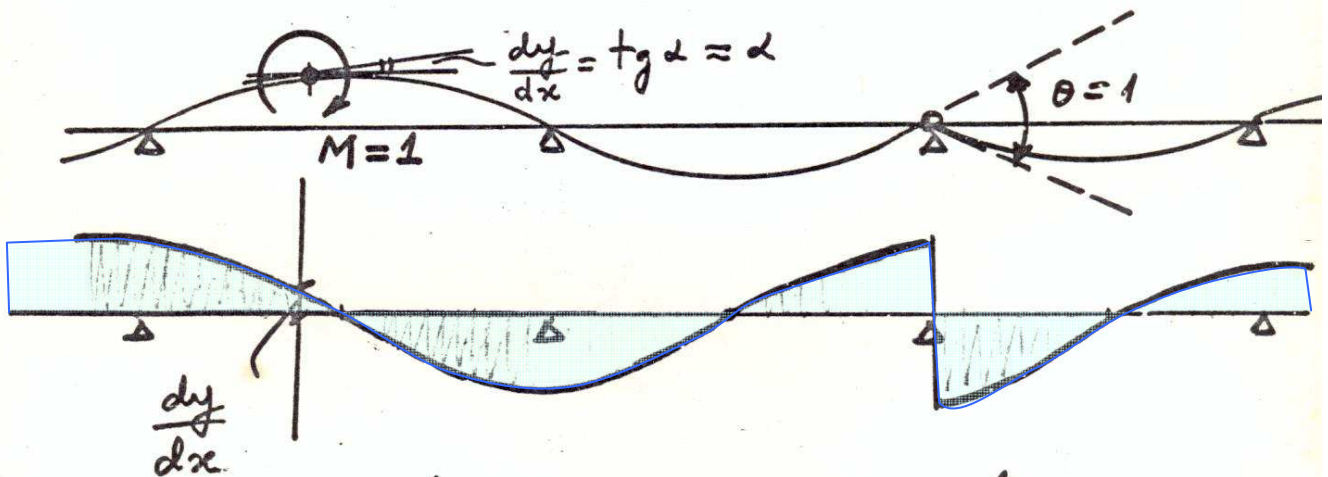


Conceitos de Linha de Influência.

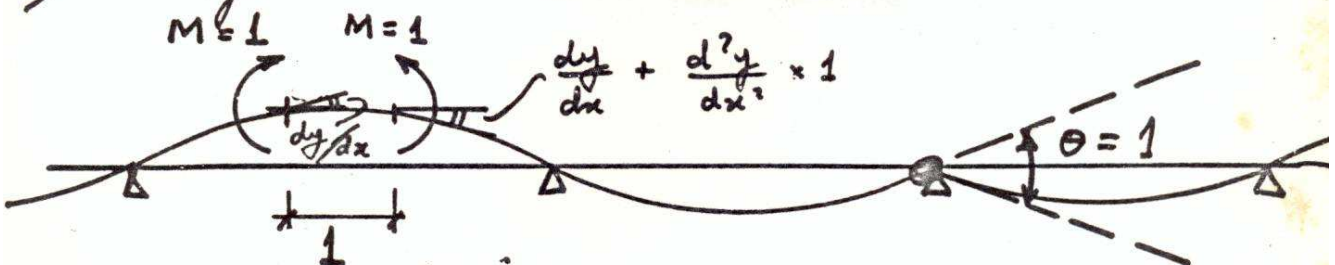
1) Carregamento : Força concentrada.



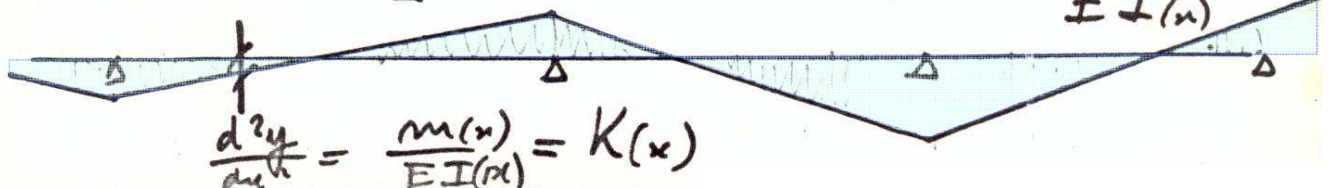
2) Carregamento : Momento concentrado.



3) Carregamento : Par de momentos:



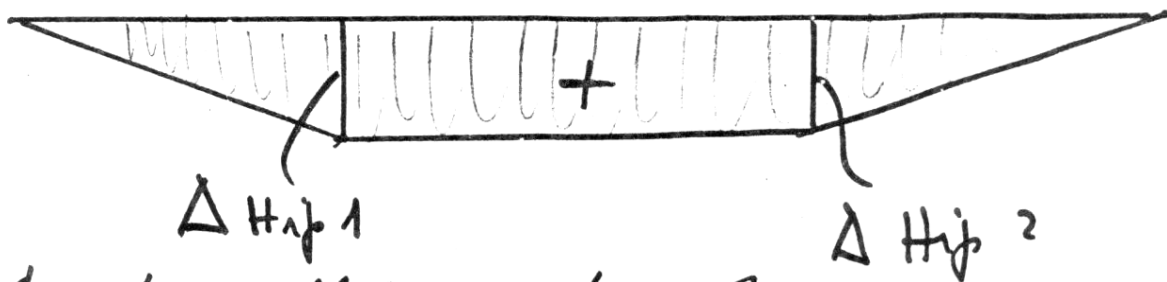
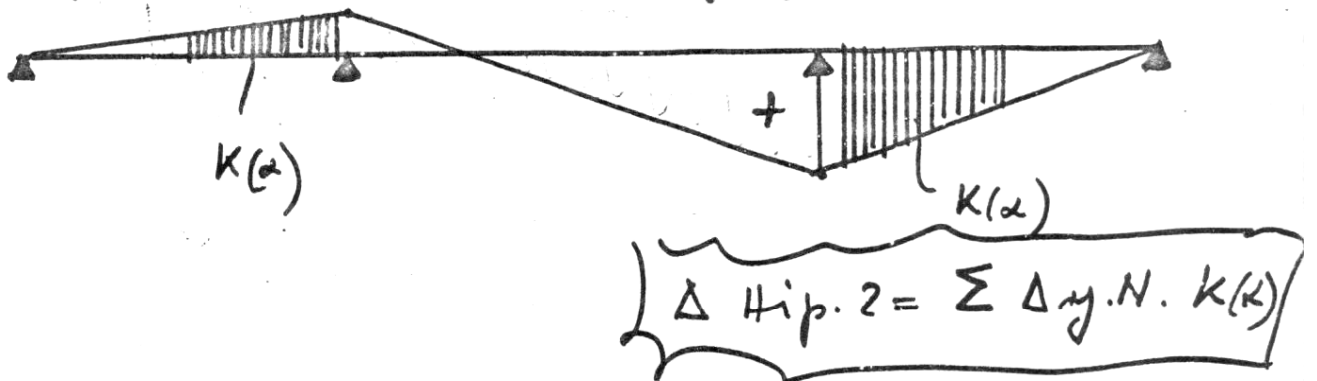
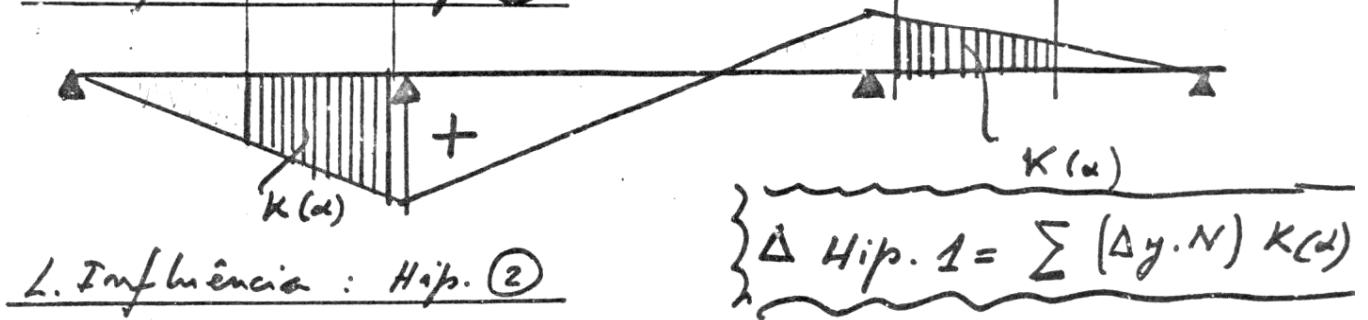
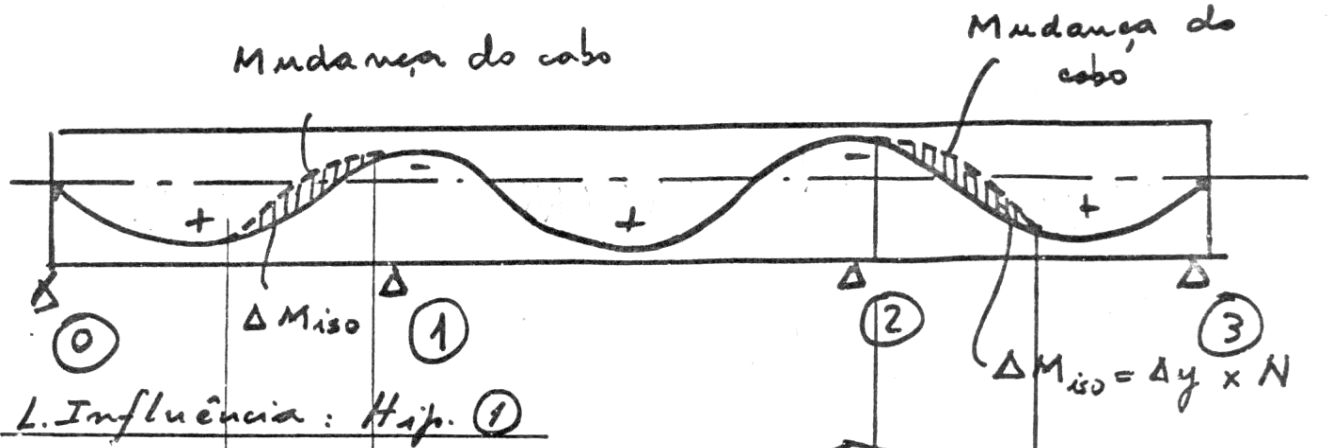
$$1 \times \frac{dy}{dx} - 1 \times \left[\frac{dy}{dx} + \frac{d^2y}{dx^2} \cdot 1 \right] = -\frac{d^2y}{dx^2} = \frac{m(x)}{EI(x)}$$



$$\frac{d^2y}{dx^2} = \frac{m(x)}{EI(x)} = K(x)$$



Algumas observações



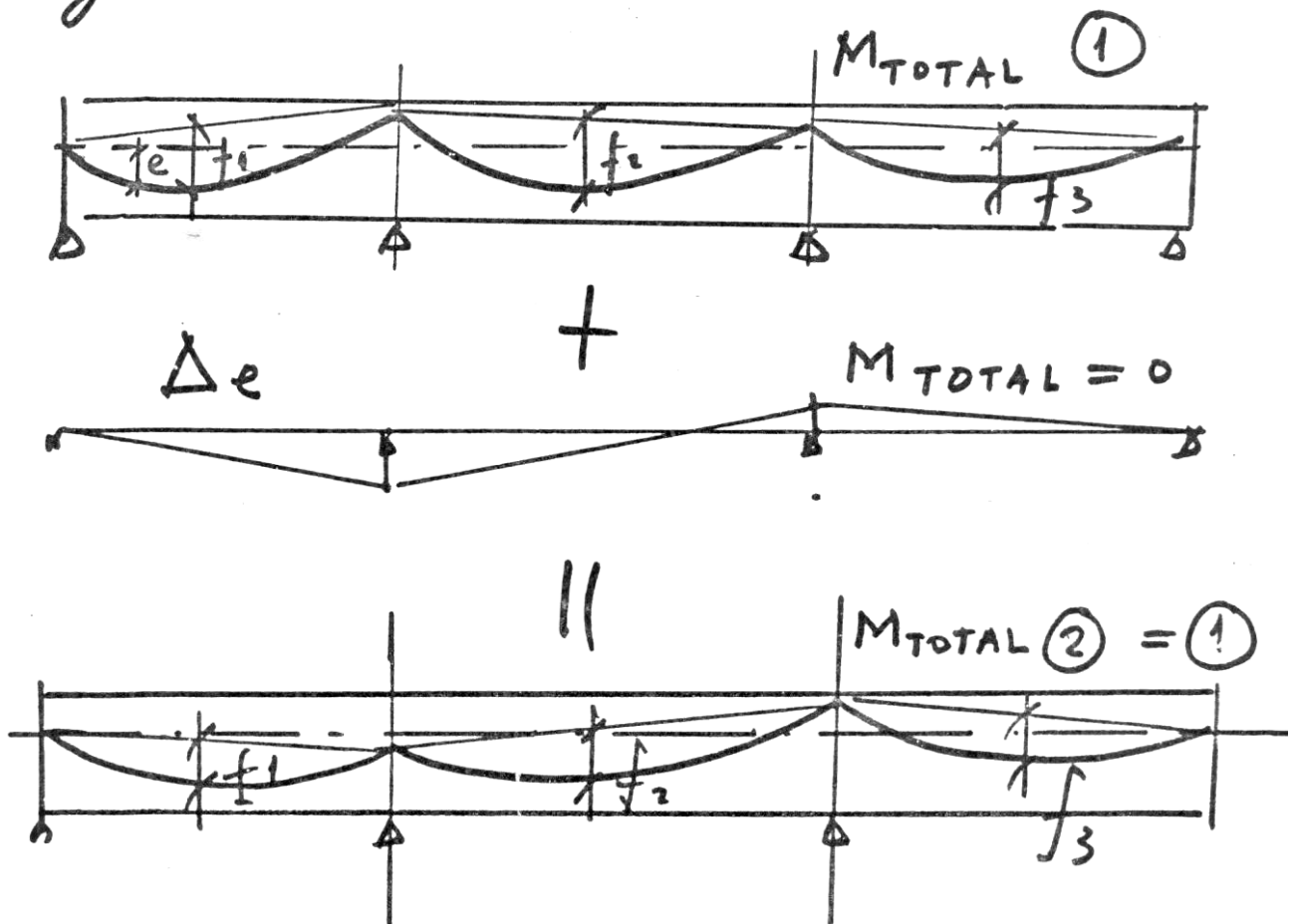
A L. Influência permite avaliar onde deve ser modificada o cabo para mudar convenientemente o hiperestático.



Teorema:

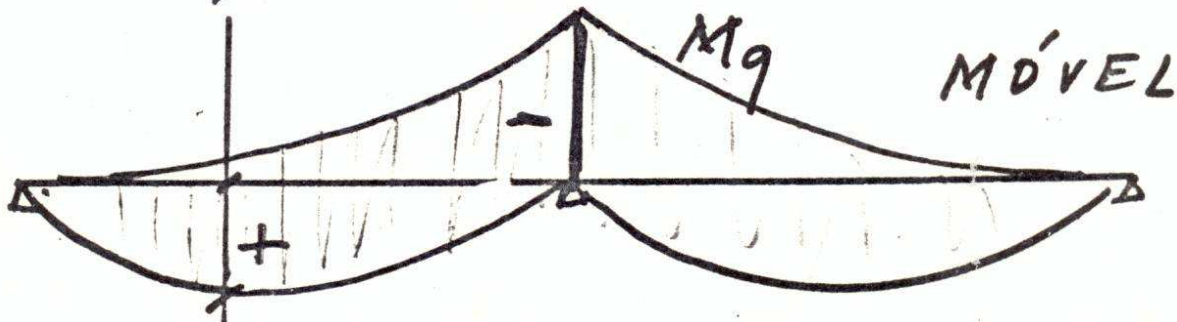
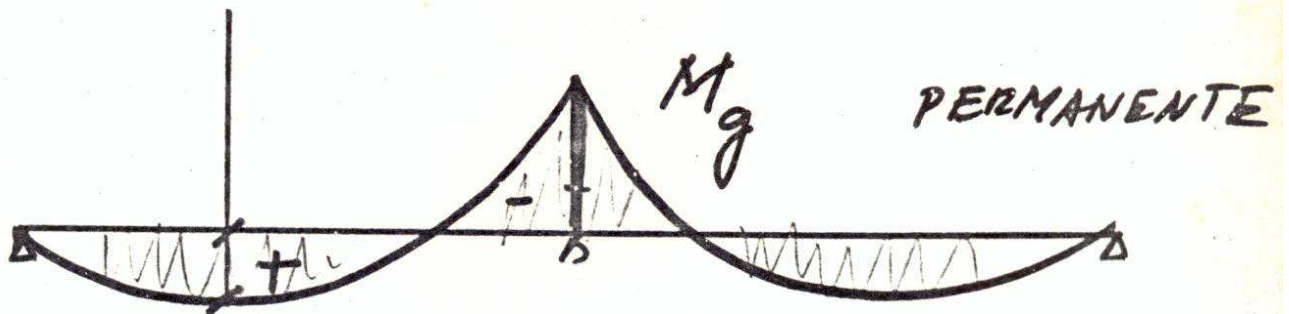
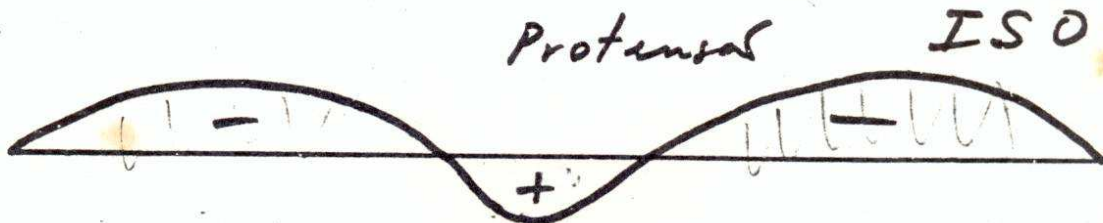
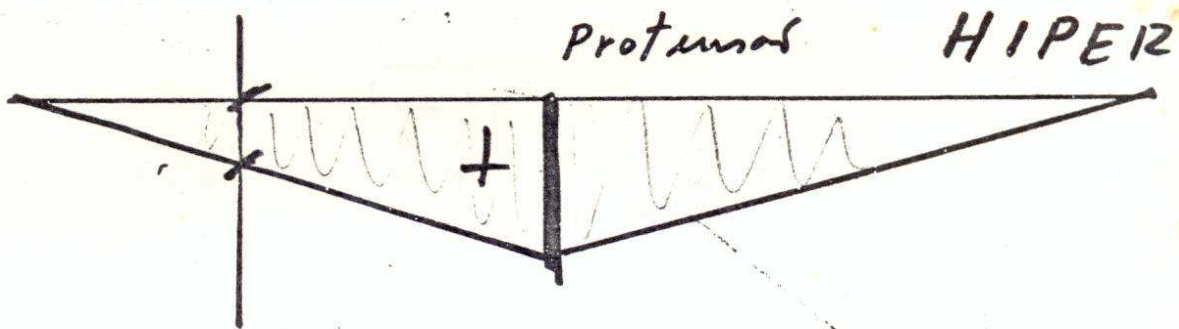
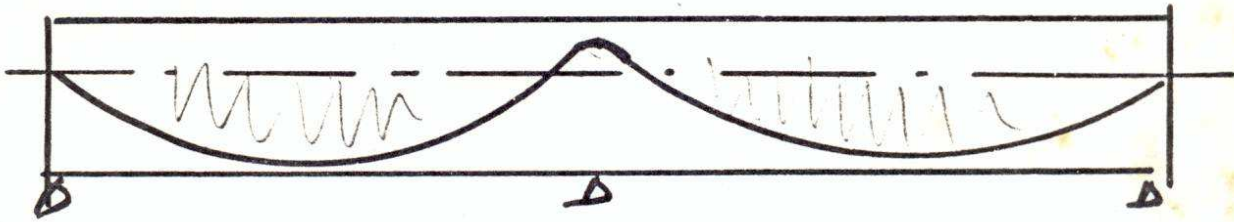
Se deslocarmos os cabos nos apoios mantendo as curvaturas dos membros obtém-se o mesmo diagrama de momentos f_t e de protensões.

Isk é obtido deslocando o cabo em trechos retos entre os apoios.





Verificação à Ruptura.



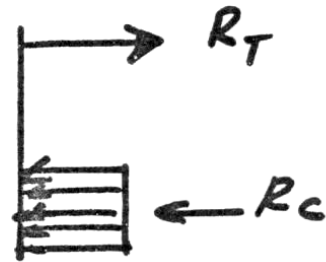
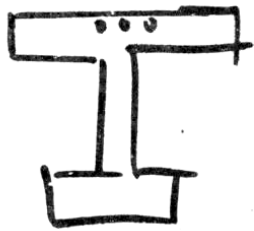


SEÇÃO APÓIO CENTRAL

$$M_d = 1,4 M_g \overset{(-)}{+} 1,4 M_g \overset{(-)}{+} 0,9 M_{HIP} \overset{(+)}{+}$$

M_{HIP} . favorável

$$M_u = \dots$$



Depende só { da seção e
do pré-alongamento
do aço.

Se $M_u \geq M_d$ OK

Se $M_u < M_d$ (NÃO).

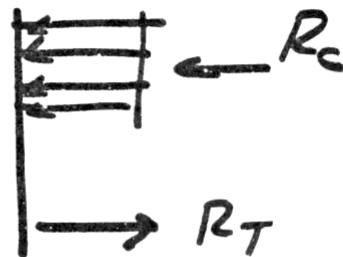
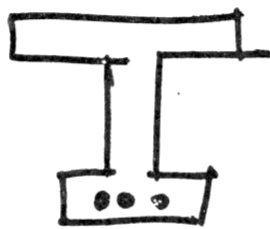


SEÇAS DO VAS M_d (máximo).

$$M_d = 1,4 M_g^{(+)} + 1,4 M_q^{(+)} + 1,0 M_{HIP}^{(+)}$$

Protensas aumenta
a solicitações

$M_u = \dots$



Depende só } de seções e
de pré alongamento
do aço.

Se $M_u \geq M_d$ OK

Se $M_u < M_d$ NAF.

— + —