

Paper

Design and detailing of structural concrete using strut-and-tie models

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Synopsis

So-called 'details' are as important for a structure's behaviour and safety as the standard problems of design which are covered in the Codes. A unified design concept which covers also the details consistently for all types of concrete structure is described in this paper. It is based on strut-and-tie models, including the truss model for beams as a special case.

After the principles of the method and the modelling process are explained, simplified rules are proposed for dimensioning all the individual members of the model and their nodes. Some examples show the application of the method and demonstrate, also, its use for the improvement of the conceptual design of details.

Introduction

Certain parts of structures are designed with almost exaggerated accuracy while other parts are designed using rules-of-thumb or judgment based on past experience. However, all parts of a structure are of similar importance. A unified design concept, which is consistent for all types of structure and all their parts, must be based on realistic physical models. Strut-and-tie models, a generalisation of the well-known truss analogy for beams, are proposed here as the appropriate approach for designing structural concrete, which includes both reinforced and prestressed concrete structures.

It was actually at the turn of the last century, when Ritter² and Morsch³ introduced the truss analogy. This method was later refined and expanded by Leonhardt⁴, Rüschi⁵, Kupfer⁶, and others, until Thürlimann's Zürich school⁷, with Marti⁸ and Mueller⁹, created its scientific basis for a rational application in tracing the concept back to the theory of plasticity.

Collins and Mitchell¹⁰ further considered the deformations of the truss model and derived a rational design method for shear and torsion.

In various applications, Bay, Franz, Leonhardt, Kupfer and Thürlimann had shown that strut-and-tie models could be usefully applied to deep beams and corbels. From that point, the present authors and other members of the Institute for Concrete Structures at the University of Stuttgart began their efforts systematically to expand such models to entire structures and all structures^{11,12}.

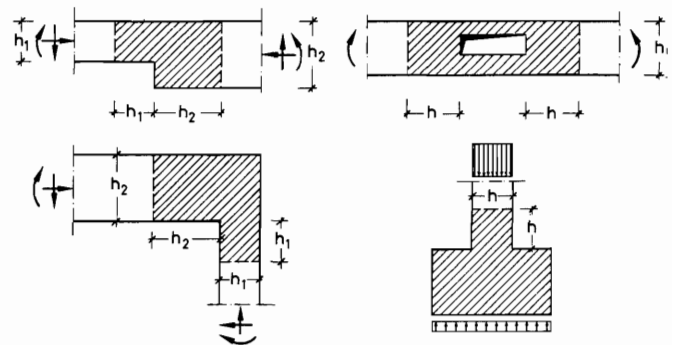
The method had been explained already in some detail in the American PCI Journal¹. The interested reader is referred to this paper as a basis of the present contribution. Here, the development of strut-and-tie models and the dimensioning of their struts, ties and nodes will be repeated only briefly. Concerning the design of nodes, some material which goes beyond ref. 1 is added.

Then, the method is applied to a few new examples, including some comparison with test results. Some of the examples given show that the strut-and-tie method is useful not only in dimensioning given members but also in developing an adequate conceptual design for a critical detail.

The structure's B- and D-regions

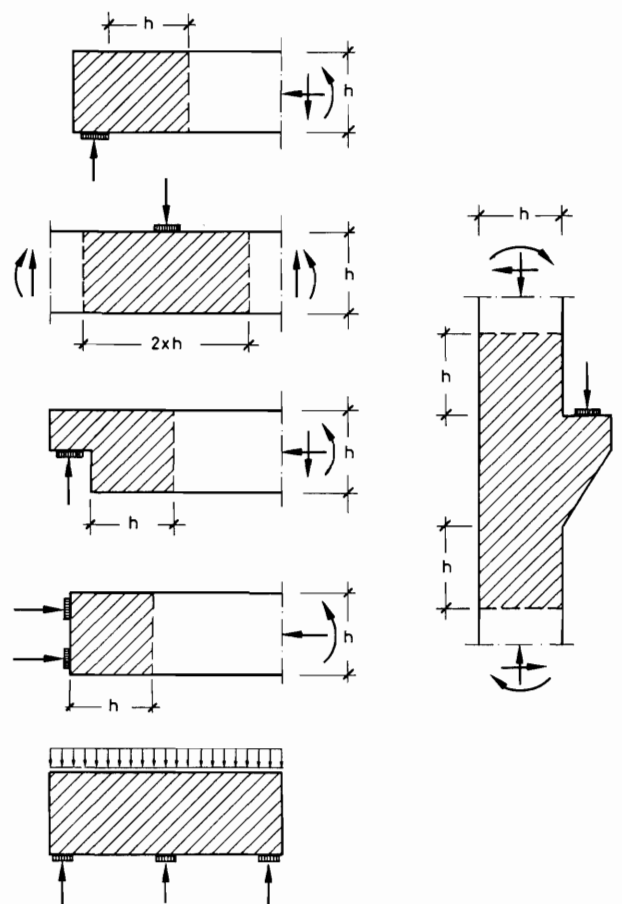
Those regions of a structure, in which the Bernoulli hypothesis of linear strain distribution is assumed valid, will be referred to as B-regions (where B stands for beam or Bernoulli). Their internal forces or stresses can be derived from moments, shear and axial forces analysed by means of the statical system of beams, frames, plates, etc. If uncracked, the stresses are calculated using the bending theory for linear elastic material. For cracked B-regions the truss models or the standard methods of Codes apply.

These standard methods are not applicable to the other regions and details of a structure, where the strain distribution is significantly non-linear, e.g. near concentrated loads, corners, bends, openings and other discontinuities (Fig 1). Such regions will be called D-regions, where D stands for



(a)

Fig 1(a). D-regions (shaded areas) with non-linear strain distribution due to geometrical discontinuities



(b)

Fig 1(b). D-regions (shaded areas) with non-linear strain distribution due to statical and/or geometrical discontinuities

discontinuity, disturbance or detail. The internal flow of forces in D-regions can be reasonably well described by strut-and-tie models.

Not much accuracy is necessary in determining the dividing sections between B- and D-regions. These sections can be assumed to lie approximately in a distance h from the geometrical discontinuity or the concentrated load, where h is equal to the depth of the adjacent B-region (Fig 1). This assumption is justified by Saint-Venant's principle¹.

Principles of strut-and-tie model design

In a strut-and-tie model the struts represent concrete stress fields with prevailing compression in the direction of the strut. Accordingly, the ties normally represent one or several layers of tensile reinforcement. However, model ties can occasionally also stand for concrete tensile stress fields. This is evident from models of practically approved details, the structural safety of which can be explained only if concrete ties are assumed in places where no reinforcement is provided. Typical examples are slabs without stirrups or bar anchorages without spiral or transverse reinforcement.

If a suitable model of a D-region is known, the forces of the struts and ties will be calculated, thereby satisfying equilibrium between applied loads and inner forces. The struts, ties and their nodes will be dimensioned or checked to carry the inner forces, as described later.

This method implies that the structure is designed according to the lower bound theorem of the theory of plasticity. However, since structural materials, in particular concrete, permit only limited plastic deformations, the internal structural system (the strut-and-tie model) has to be chosen in a way that the deformation capacity is not exceeded at any point, before the assumed state of stress is reached in the rest of the structure.

In highly stressed regions, this ductility requirement is fulfilled by adapting the struts and ties of the model to the direction and size of the internal forces as they would appear from the theory of elasticity.

In normally or lightly stressed regions the direction of the struts and ties in the model may deviate considerably from the elastic pattern without exceeding the structure's ductility. The ties, and hence the reinforcement, may be arranged according to practical considerations. The structure adapts itself to the assumed internal structural system.

This method of orientating the strut-and-tie model along the force paths indicated by the theory of elasticity obviously neglects some ultimate load capacity which could be utilised by a pure application of the theory of plasticity. On the other hand, it has the major advantage that the same model can be used for both the ultimate load and the serviceability check. If, for some reason, the purpose of the analysis is to find the actual ultimate load, the model can easily be adapted to this stage of loading by shifting its struts and ties in order to increase the resistance of the structure. In this case, however, the rotation capacity of the model has to be considered.

Strut-and-tie modelling obviously provides the structural analyst with some freedom of choice which can be used to aim either at the safest or at the cheapest or at an otherwise optimised solution. Modelling therefore requires some design experience as does the choice of a representative overall statical system or of a reasonable finite element net.

The modelling process also covers much of what is normally called detailing and therefore requires considerable knowledge about practicable reinforcement layout; on the other hand, it is just in this field where strut-and-tie models replace experience and guesswork by a more systematic and understandable design.

Modelling of individual D-regions

Before modelling of a D-region begins, all the forces and reactions acting on the D-region must be evaluated (Fig 2(a)). The forces or stresses in sections bounded by B-regions are taken from B-region design.

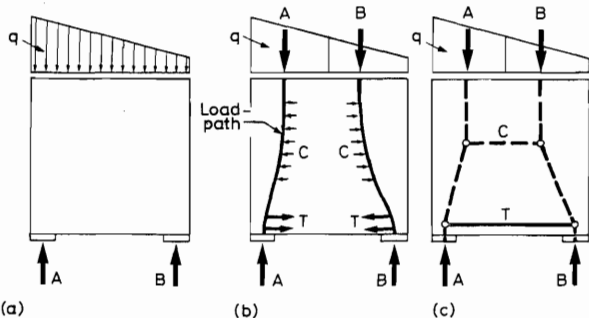


Fig 2. The load path method: (a) the structure and its loads; (b) the load paths through the structure; (c) the corresponding strut-and-tie model

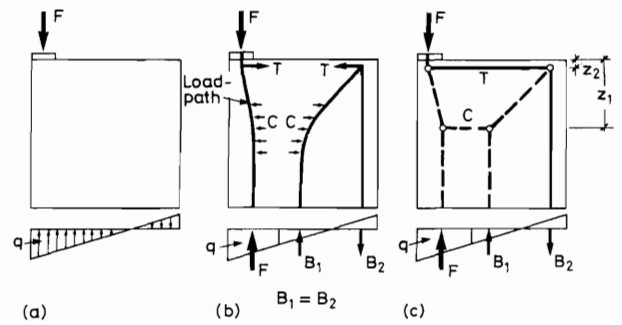


Fig 3. The load path method, including a 'U-turn': (a) the structure and its loads; (b) the load paths through the structure; (c) the corresponding strut-and-tie model

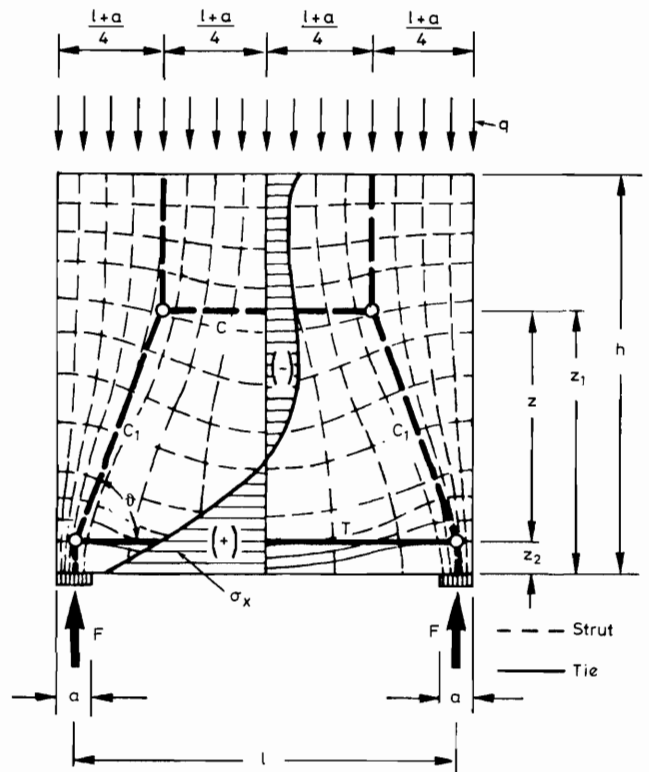


Fig 4. Elastic stress trajectories, distribution of elastic stresses, and corresponding strut-and-tie model

New strut-and-tie models can be systematically developed by tracing the flow of forces through the structure, using the 'load path' method.

The stress diagrams of all the forces applied to the D-region boundaries are subdivided in such a way that the individual stress resultants on opposite sides of the D-region correspond in magnitude and can be connected by streamlined 'load paths' which do not cross each other (Fig 2(b)). After sketching the load paths smoothly curved and replacing them by polygons, further struts and ties must be added for transverse equilibrium (Fig 2(c)). Obviously, in some cases the stress diagrams or the loads are not completely balanced with the load paths described; then the load path for the remaining forces enters the structure and leaves it after a U-turn on the same side (Fig 3).

Developing a model of a D-region is much simplified if the elastic stresses and principal stress directions are available from an elastic FEM analysis. The direction of struts can then be taken in accordance with the mean and main direction of principal compressive stresses, or the more important struts and ties can be located at the centre of gravity of the stress diagram of typical sections (Fig 4).

When modelling, the angles between struts and ties, in particular those with relatively high forces, should be chosen larger than 45° (better 60°) in order to avoid incompatibility problems.

The resulting models are quite often kinematic, which means that the geometry of such a model is strictly related to a particular load configuration

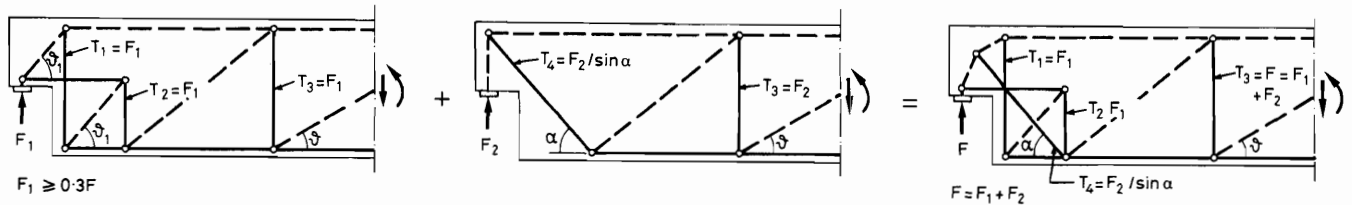


Fig 5. The combination of two models for the dapped beam is better than the individual models

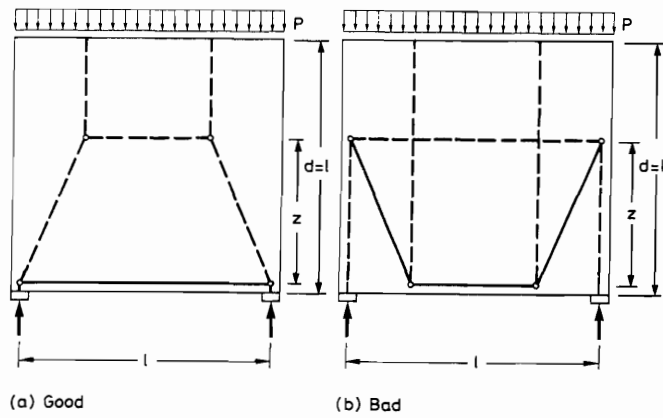


Fig 6. The good model (a) has shorter ties than the bad model (b)

and cannot be used for other loads without modification. Therefore, the governing load combinations have to be investigated. This disadvantage is not a peculiarity of the strut-and-tie method but is inherent to the non-linear material properties of cracked concrete.

Accordingly, superposition of two models is possible only if the combined model satisfies the requirements on reasonable angles between struts and ties. By combining two simple models it is sometimes possible to develop much better but rather complicated models (Fig 5). Instead of investigating such a hyperstatic model with representative stiffnesses of the struts and ties, it is normally more adequate to immediately allot the loads to the two simple models with an eye on the expected stiffness ratio of the individual models.

Doubts could arise as to whether the correct model has been chosen out of several possible ones (Fig 6). In selecting the model, it is helpful to realise that loads try to use the path with the least forces and deformations. Since reinforcement ties are much more deformable than concrete struts, the model with the least and shortest ties is the best. This simple criterion for optimising a model may be formulated as follows:

$$\sum F_i l_i \epsilon_{mi} = \text{minimum}$$

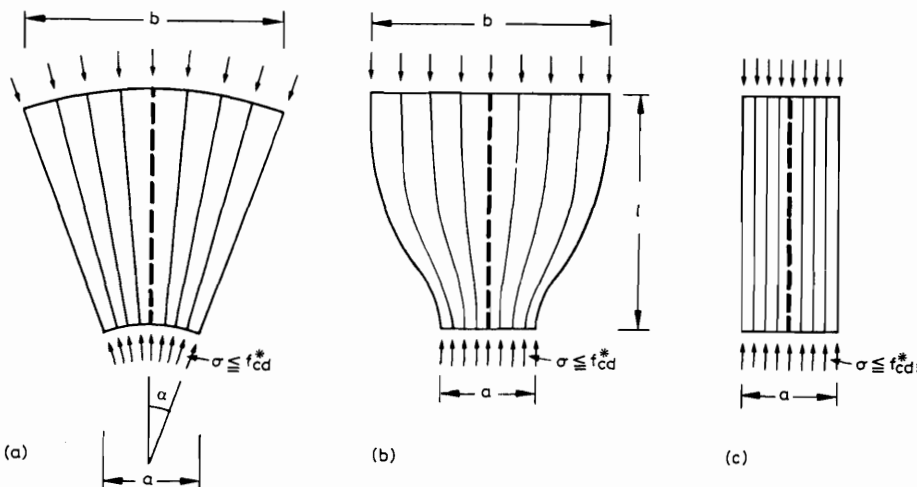


Fig 7. The basic compression fields: (a) the 'fan'; (b) the 'bottle'; (c) the 'prism'

where

F_i is the force in strut or tie i

l_i is the length of member i

ϵ_{mi} is the mean strain of member i

The contribution of the concrete struts can mostly be omitted in the above criterion.

Dimensioning the struts, ties and nodes

Reinforced and unreinforced ties

Normally tie forces are carried by reinforcement. Its cross-section follows from the tie force in the ultimate limit state and the design yield strength of the steel.

For crack distribution the reinforcement shall be distributed over the tensile zone. Crack widths can be analysed if the reinforced tie is considered as a prismatic reinforced bar with an effective concrete area.

The tensile strength of concrete should be utilised for equilibrium forces only if no progressive failure must be expected and if local failure zones are assumed. Thereby restraint forces and microcracks have to be taken into account even in 'uncracked' concrete. Further, some positive experience with similar details and loading should be available.

Concrete struts or compression stress fields

To cover all cases of compression stress fields, three typical configurations are sufficient:

(a) The fan-shaped stress field (Fig 7(a)) is an idealisation of a stress field with negligible curvature. It does not develop transverse stresses.

(b) The bottle-shaped stress field (Fig 7(b)), with its bulging stress trajectories, develops considerable transverse stresses: compression in the bottle neck and tension further away. The transverse tension can cause longitudinal cracks and initiate an early failure. It is therefore necessary to reinforce the stress field in the transverse direction or to consider the transverse tension when determining the failure load of the strut. The transverse tension can be determined from a strut-and-tie model of the stress field. Diagrams simplify its dimensioning (Fig 8).

(c) The prismatic or parallel stress field (Fig 7(c)) is a frequent special case of the preceding two stress fields.

The fan-shaped and the bottle-shaped stress fields are frequently found in D-regions where concentrated loads are introduced into a structure and spread out. The prismatic stress field is typical for B-regions.

The strength of the concrete in compression stress fields depends to a considerable extent on the multiaxial state of stress and on disturbances

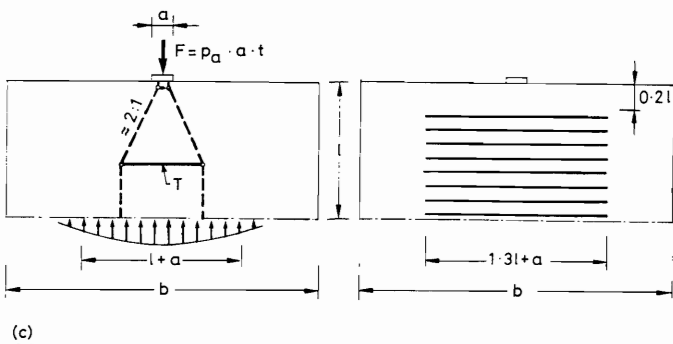
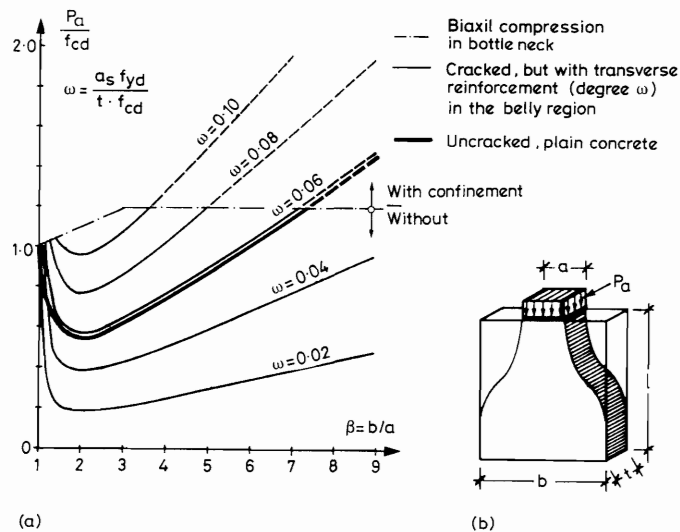


Fig 8. Dimensioning plane bottle-shaped stress fields: (a) diagrams giving safe pressure values p_a with regard to cracking and crushing of plain unreinforced concrete stress fields and yielding of transverse reinforcement; (b) geometry of the stress field; (c) model and reinforcement layout of stress field with transverse reinforcement

from cracks and reinforcement. For practical dimensioning of all kinds of stress-fields, the following simplified design strength values f_{cd}^* are proposed:

- $f_{cd}^* = 1.0 f_{cd}$ for an undisturbed and uniaxial state of stress
- $f_{cd}^* = 0.8 f_{cd}$ for compression fields with cracks parallel to the compression stresses
- $f_{cd}^* = 0.6 f_{cd}$ for compression fields with skew cracks

where f_{cd} denotes the concrete compressive design strength for uniaxial compression according to the Code of Practice.

The design values given above for cracked concrete are meant for structural concrete, whose crack widths are limited in the usual manner. The values for cracked concrete shall also be applied for concrete with transverse tension below the expected tensile strength and if tensile reinforcement is crossing

the stress field. Skew cracks are not expected if the theory of elasticity is closely followed during modelling. However, skew cracks may also be left over from a previous loading case with a different stress situation.

The increase in strength due to 2- or 3-dimensional states of compressive stresses may be taken into account if the simultaneously acting transverse compressive stresses are reliable.

Before deciding on one of the given strength values, both transverse directions must always be considered.

The nodes

The nodes are in reality, regions where forces are deviated over a certain length and widths. The 'smeared' or 'continuous' nodes, where wide concrete stress fields join each other or with closely distributed reinforcing bars, are not critical; it is sufficient to ensure safe anchorage of the reinforcing bars in the smeared node and to catch the outermost fibres of the deviated compressive stress field with reinforcement (Fig 9).

On the other hand, where concentrated forces are applied the deviation of forces is locally concentrated in 'singular' or 'concentrated' nodes. These have to be carefully designed in order to balance the oncoming forces of the struts and ties without excessive deformations resp. cracks.

Though numerous cases of different singular nodes exist, in most cases their forces balance each other in the node region through direct compressive stresses. Also bond is essentially a load transfer via compressive stresses which are supported by the ribs of the steel bar and by radial pressure in bent bars. However, in many cases also concrete tensile stresses develop transverse to the model plane ('third direction').

The stress distribution in singular nodes is mostly so complicated that it cannot be analysed individually with bearable expenditure. But experience shows that some types of node and detail are repeated again and again in quite different structures and can be designed safely by simplified rules: (a) The geometry of the node has to be tuned with the applied forces. Therefore reinforcement anchored in the node should be distributed over a certain height u with due regard to the widths of the oncoming stress fields and the magnitude of their forces; further, it should be adequately distributed in the transverse direction in order to keep transverse tensile stresses low.

(b) The average compressive stresses in the node region boundaries have to be checked to be less than

- $f_{cd}^* = 1.1 f_{cd}$ in nodes where only compression struts meet, thus creating a 2- or 3-dimensional state of compressive stresses in the node region
- $f_{cd}^* = 0.8 f_{cd}$ in nodes where tensile bars are anchored and an allowance in strength must be made for bond action

Suitable node region boundaries and the corresponding compression stresses can easily be determined, as shown in the typical nodes in Figs 10-13. As for all nodes, also the stresses of the oncoming struts have to be checked as described earlier.

(c) Safe anchorage of ties in the node has to be assured: minimum radii of bent bars and anchorage lengths of bars are selected following the Code. The anchorage must be located within and 'behind' the node (Figs 11 and 13). The anchorage begins where the transverse compression stress trajectories meet the bar and are deviated. The bar must extend to the other end of the node region. If this length is less than required by the Code, the bar may be extended beyond the node region and introduce some of its forces from behind.

Node N1 (Fig 10) is typical for a node of compression struts in a corner. Two alternative node region boundaries are shown for the same node, both

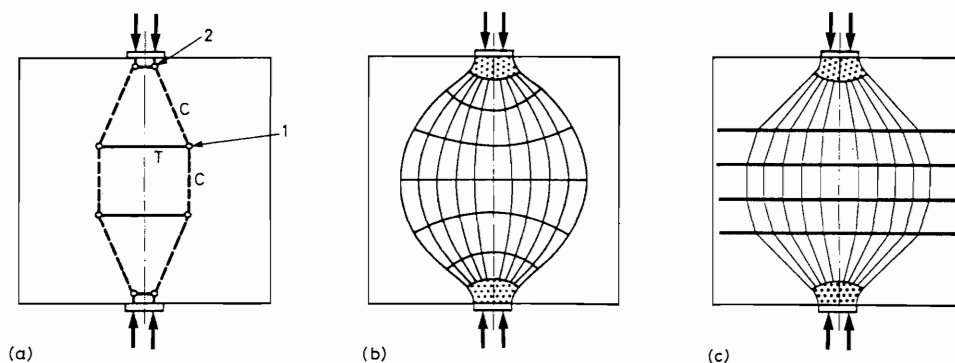


Fig 9. 'Smeared nodes' 1 and 'singular nodes' 2 in a D-region; (a) model, (b) and (c) stress fields and node regions

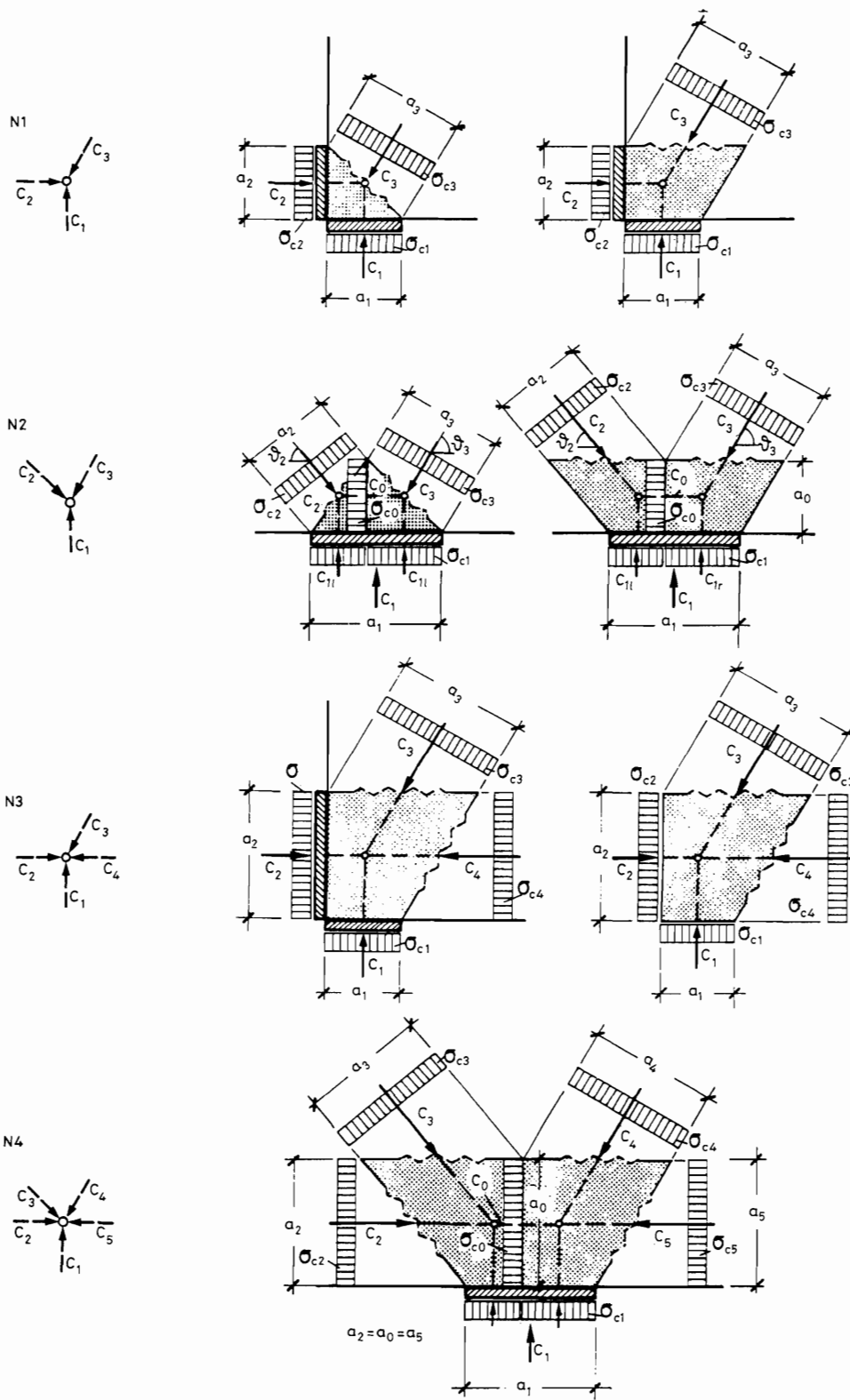


Fig 10. Nodes without anchorage of reinforcement

leading to the same results. The node is safe, if

$$\sigma_{c1}, \sigma_{c2} \text{ and } \sigma_{c3} \leq 1.1 f_{cd}$$

Node N2 (Fig 10) is a combination of two nodes N1. It is realistic and convenient to choose a_0 large enough,

$$a_0 \geq a_1 \cos \theta_2 \sin \theta_2 = a_1 \cos \theta_3 \sin \theta_3,$$

in order that the bearing pressure $\sigma_{c1} \leq 1.1 f_{cd}$ governs the node's design.

Nodes N3 and N4 (Fig 10) are typical for loads or support forces applied to the edge of a structure with a chord force running parallel to this edge through the node. Normally, the concrete compressive stresses

σ_{c0} and $\sigma_{c1} \leq 1.1 f_{cd}$ govern the design.

Node N5 (Fig 11) applies to the anchorage of ties far from the edges, i.e. inside the structure in the plane of the model. As for all nodes with ties, the anchorage length must be checked.

Node N6 (Fig 11) is typical for end supports. The height u in deep beams should be chosen

$$u \approx 0.15 h \leq 0.2 h \leq 0.2 l$$

where h is the height of D-region and l is the span of deep beam.

Single-layer reinforcement shall be placed near the lower edge, where the

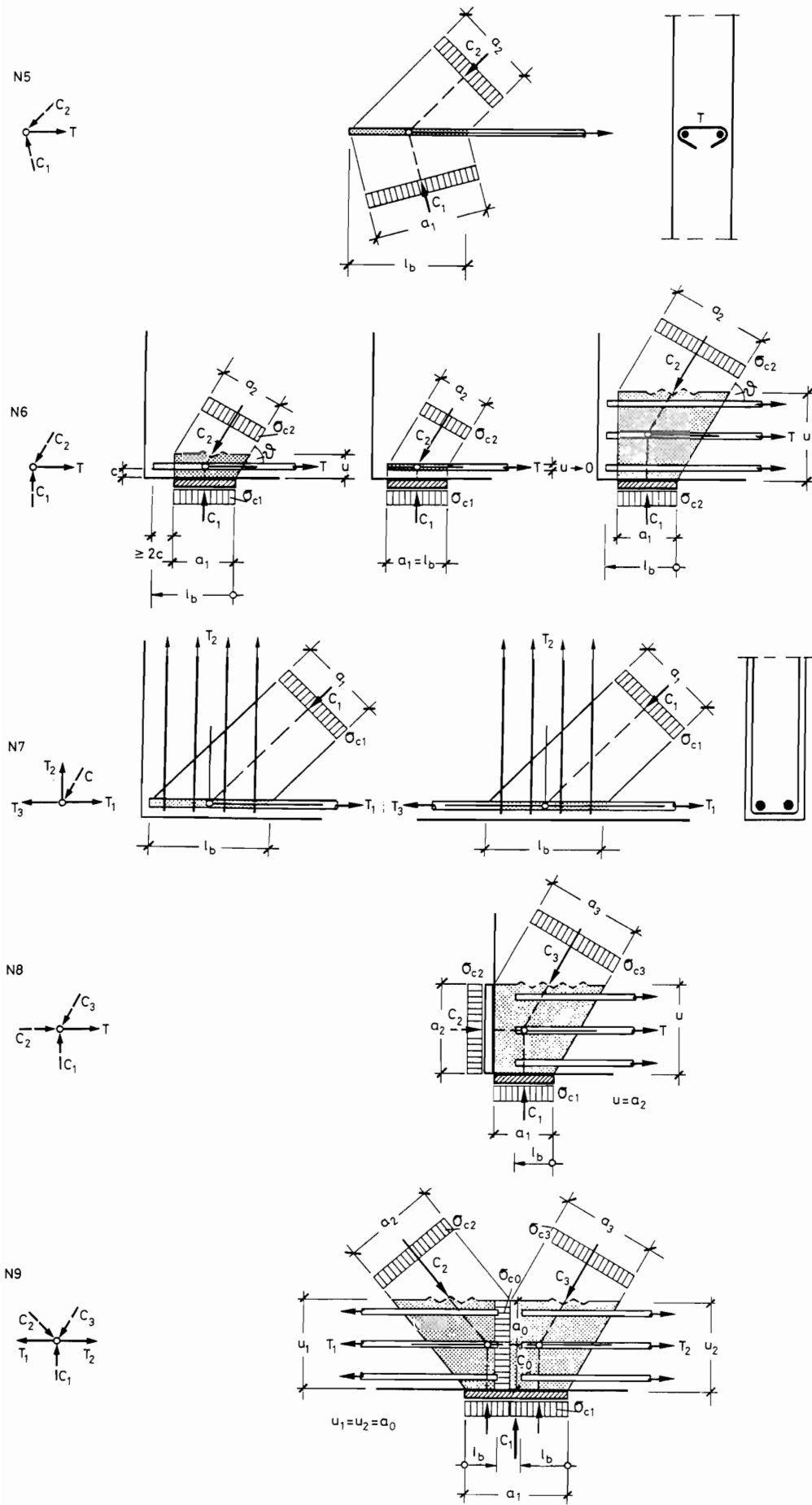


Fig 11. Nodes with anchorage of reinforcement

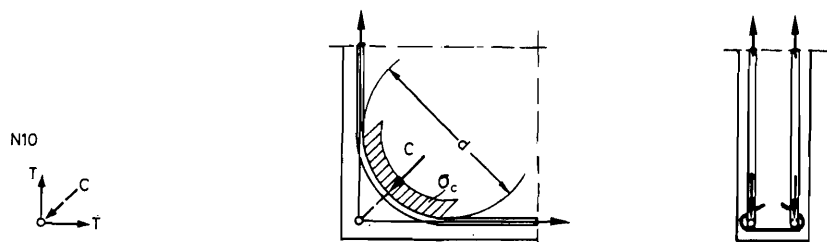


Fig 12. Node with deviation of reinforcement

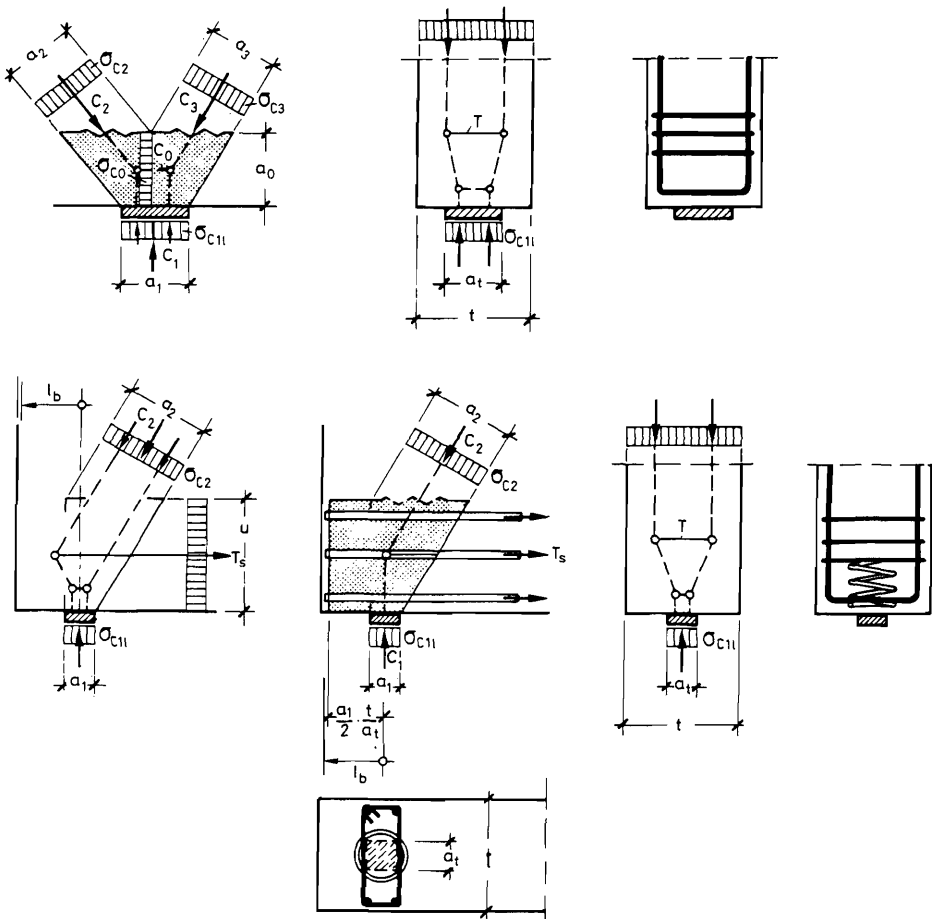


Fig 13. Treatment of local pressure in typical nodes N2 and N6, applicable also to other nodes

deviation forces are largest. Checks include

$$\sigma_{c1} \text{ and } \sigma_{c2} \leq 0.8 f_{cd}$$

Node N7 (Fig 11) is typical in the tension chord of beams or deep beams. Thin, well-distributed bars shall be chosen as reinforcement for tie T_2 and they shall embrace tie T_1 . Concrete stresses $\sigma_c \leq 0.8 f_{cd}$ will rarely be decisive.

Node N8 (Fig 11) is a mixture of the nodes N1 and N6, and therefore maximum compression stresses between those of both node types are proposed:

$$\sigma_{c1}, \sigma_{c2} \leq f_{cd}$$

Besides, the rules for typical node N6 apply.

Node N9 (Fig 11) is composed of two nodes N8; checks are accordingly. This node is typically over the support of continuous beams and normally also covered by the Code rules (check the beam's cross-section for M , N and V , bearing pressure, anchorage of chord reinforcement).

Node N10 (Fig 12) is checked via the admissible radius of the bent bar. In nodes with local pressure ($a_t < t$, Fig 13), the transverse tension in the third direction must be covered by transverse reinforcement designed for

$$T = \frac{1}{4} \times \frac{t - a_t}{t} \times C_1$$

Local pressures σ_{c11} may be tolerated up to

$$\sigma_{c11} = \frac{C_1}{a_t a_1} \leq \frac{t}{a_t} f_{cd}^* \leq 3 \cdot 3 f_{cd}$$

General rule

Since singular nodes are bottlenecks of the stresses, it can be assumed that an entire D-region is safe, if the pressure under the most heavily loaded bearing plate or anchor plate is less than $0.6 f_{cd}$ and if all significant tensile forces are resisted by reinforcement and further if sufficient development lengths are provided for the reinforcement.

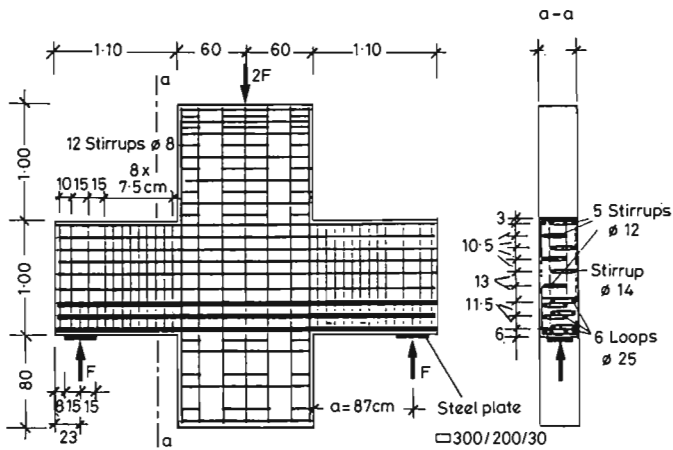
Only if this rule does not lead to a satisfactory result, more sophisticated analysis, as described earlier, is required.

Applications

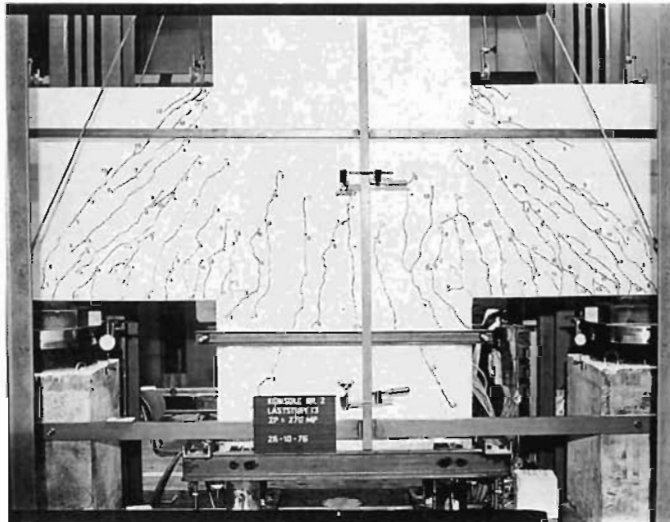
Only a few applications of the strut-and-tie method can be shown here; many more can be found in refs. 1 and 12.

Corbel

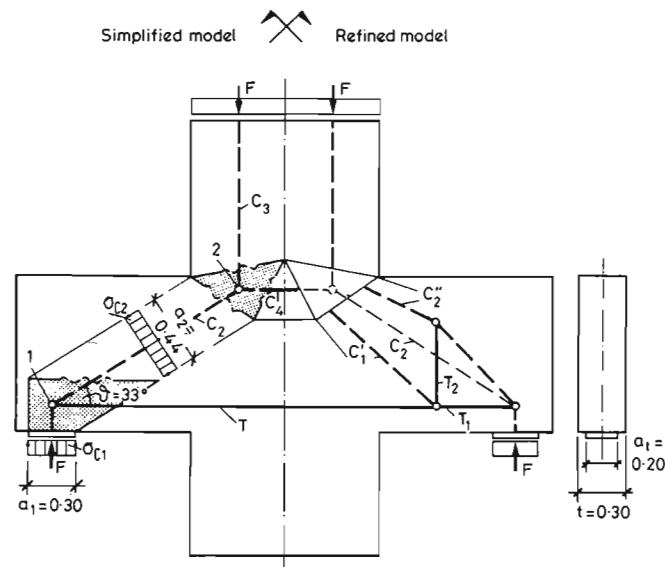
Corbels are D-regions for which strut-and-tie models are applied successfully for a long time. For a check of the method and the design rules given above, a test specimen will be analysed and the results compared with the test results. In order to include also the potential concrete failure in the checks, one of those rare test specimens¹³ is selected for which yielding of the main tie is not the obvious failure criterion.



(a)



(b)



(c)

Fig 14. Corbel: (a) tested specimen no.2¹³; (b) crack pattern at $F = 1350\text{ kN}$ (near failure); (c) simplified and refined strut-and-tie model of internal forces at failure load $F_u = 1.425\text{ MN}$

The test specimen representing two symmetrical corbels was tested upside down (Fig 14(a)). The crack pattern depicts quite well the internal flow of forces (Fig 14(b)), condensed in the simplified model (Fig 14(c) left side). This model can easily be derived by the load path method. However, the precise model geometry is known only after the nodes are dimensioned.

Taking a strut angle $\theta = 33^\circ$ from a first sketch of the model, the following internal forces are derived for the recorded failure load $F_u = 1.425\text{ MN}$:

$$T = C_4 = F_u / \tan \theta = 2.19\text{ MN}$$

$$C_2 = F_u / \sin \theta = 2.62\text{ MN}$$

Tie T :

$$\sigma_s = T / A_s = 359\text{ N/mm}^2 < f_y = 452\text{ N/mm}^2$$

Node 1:

$$\sigma_{c1} = \frac{F_u}{a_1 \times a_t} = \frac{1.425}{0.30 \times 0.20} = 23.8\text{ N/mm}^2$$

$$< \frac{t}{a_1} \times f_c^* = \frac{0.30}{0.20} \times 0.8 \times 26.3 = 31.6\text{ N/mm}^2 \text{ (local pressure)}$$

Transverse tension from local pressure is covered by loops and stirrups. Anchorage and distribution of reinforcement in the node region is adequate.

$$\sigma_{c2} = \frac{C_2}{a_2 \times t} = \frac{2.62}{0.44 \times 0.30} = 19.8\text{ N/mm}^2$$

$$< 0.8 f_c = 21.0\text{ N/mm}^2$$

Node 2. The concrete stresses in this pure compression node (similar to typical node N2) cannot be critical,

$$\sigma_c < 1.1 f_c,$$

if the stresses in the adjacent stress fields are satisfactory.

Strut C_2 . The diagram for bottle-shaped stress fields (Fig 8) will be used.

Reinforcement ratio: vertical $\omega_y = 0.08$, horizontal $\omega_x = 0.13$.

For $\omega = 0.08$ the diagram predicts a minimum capacity

$$p_a \approx 0.75 f_c = 19.7\text{ N/mm}^2$$

which almost exactly coincides with the pressure $\sigma_{c2} = 19.8\text{ N/mm}^2$ determined for node 1 in the ultimate condition. Indeed, the strut C_2 failed in the test after yielding of the vertical reinforcement. The same width a_2 is necessary in the other bottle neck of the stress field where it joins node 2. This determines the geometry of node 2 and finally that of the simple model.

However, it must be pointed out that the strut angle $\theta = 33^\circ < 45^\circ$ indicates a rather poor orientation of the simple model at the elastic behaviour. A refined model is given in Fig 14(c), right side. This model immediately explains the forces in the yielding vertical stirrups (tie T_2) and leads to reduced stresses and anchor forces in node 1, which therefore cannot be critical. The geometry and the checks for node 2 are unchanged if the resultant C_2 of struts C_2' and C_2'' is considered. Stresses in the diagonal struts are not higher than in the simple model.

Deep beam

The deep beam tested by Leonhardt & Walther¹⁴ shall be evaluated using the strut-and-tie method. Dimensions and reinforcement layout are given in Fig 15(a).

$$f_c = 30.2\text{ N/mm}^2 \text{ concrete prism strength}$$

$$f_{sy} = 428\text{ N/mm}^2 \text{ yield strength of main reinforcement}$$

$$f_{su} = 547\text{ N/mm}^2 \text{ rupture strength of main reinforcement}$$

The test specimen failed at a total load $F_u = 1195\text{ kN}$ after rupture of the principal reinforcement.

For a first approximation the model from Fig 4 will be used (Fig 15(b)), neglecting the deviation of bars near the support and the mesh reinforcement. The lever arm of the chord is assumed to be not much larger than expected from the theory of elasticity:

$$z = 0.72 l = 1.04\text{ m}$$

When the tension chord begins to yield,

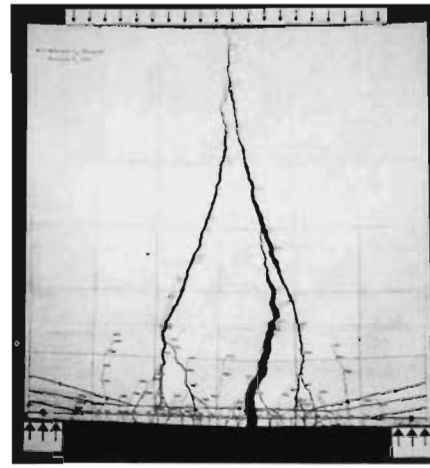
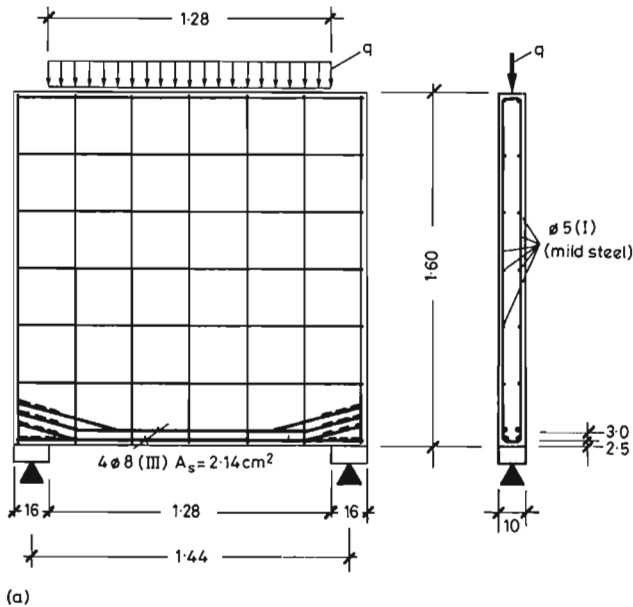
$$T_{1y} = A_s \times f_y = 2.14 \times 42.8 = 91.6\text{ kN},$$

the corresponding load amounts to

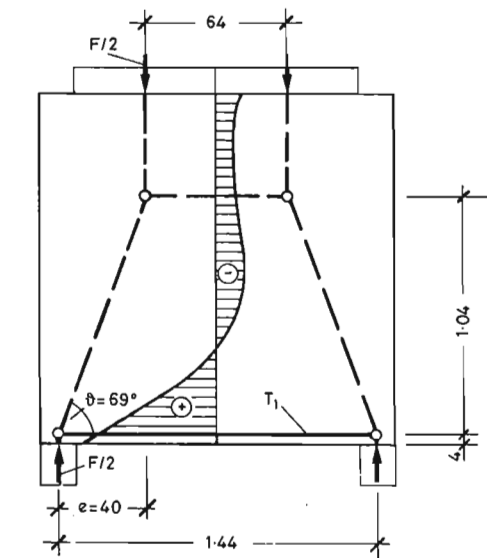
$$F_y = \frac{2 \times T_{1y} \times z}{e} = \frac{2 \times 91.6 \times 1.04}{0.40} = 476\text{ kN}.$$

This is already more than the usual design would predict, but only 40% of the measured failure load.

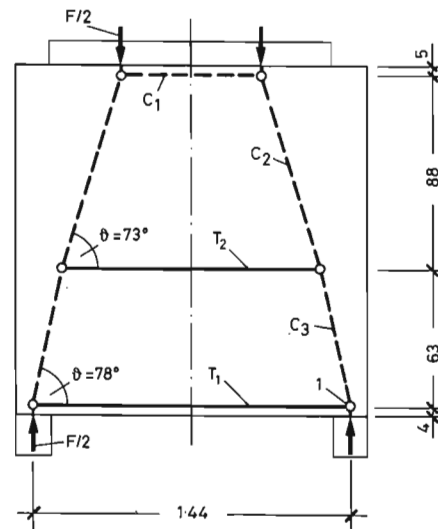
However, for an explanation of the recorded ultimate load, the model



(c)



(b)



(d)

Fig 15. Deep beam: (a) tested specimen WT2¹⁴; (b) model orientated at the theory of elasticity; (c) crack pattern from test; (d) model adjusted to the failure mechanism

must be adapted to the real behaviour (Fig 15(c)) by shifting the compression chord to the upper end of the deep beam (Fig 15(d)). If further the rupture strength of the main reinforcement is introduced ($T_{1u} = 117.1 \text{ kN}$) and if also the mild steel mesh reinforcement is taken into account (assuming $\sigma_s = 340 \text{ N/mm}^2$, $T_{2u} = 53.4 \text{ kN}$), 94% of the real ultimate load is explained. The rest can be attributed to friction in the supports.

This example shows that, with strut-and-tie models, the real behaviour of cracked structures can be analysed much better than by the theory of elasticity and that considerable 'redistribution' is possible in deep beams. Nevertheless, it is recommended not to depart too much from the theory of elasticity with respect to crack width in the serviceability limit state.

To complete the check of the tested deep beam, also the compression struts and the nodes have to be looked at.

Strut C_1 can easily be chosen as a prismatic stress field deep enough not to exceed $\sigma_c = f_c$.

Following the earlier description, the bearing pressure $0.8 f_{cd}$ in the support allows an ultimate design load

$$F = 2 A = 2 t a \sigma_{cA} = 2 \times 0.10 \times 0.16 \times 0.8 \times 30 \cdot 2 = 0.773 \text{ MN}$$

which is only 65% of the failure load in the test. This can be explained by transverse compression in the concrete due to friction in the bearing plates and to the reinforcement loops. These loops also provide safe anchorage over the support.

Concrete stresses in the support node boundary adjacent to strut C_3 are smaller than σ_{cA} since the reinforcement is very well distributed in the node region over a considerable height u .

Beams with openings in the web (Fig 16)

The truss model clearly shows where openings in the webs may be placed, and how much reinforcement is necessary for a given reinforcement layout. A check of concrete stresses ($\leq 0.6 f_{cd}$) in the remaining cross-sectional area of the struts between the opening completes the safety check. It is obvious that the standard 'shear design' in such cases is just nonsense.

Openings in walls and slabs

In Fig 17(b), (c) the strut-and-tie models of wall regions with a rectangular opening are given for the two cases of uniform compression resp. tension, applied to opposite boundaries. If these walls are considered as the upper resp. lower layer of a slab with constant moment (Fig 17(a)), the two models reveal as well the necessary reinforcement in the slab due to the opening.

Frame corner with closing moment

The simple model in Fig 18(a) is acceptable only if the dimensions of the column and beam do not differ too much and if the whole chord reinforcement is bent continuously around the corner according to Fig 18(b).

If such a model is applied also to the frame corner in Fig 18(c) with different chord forces $T_2 \gg T_1$, not only is the orientation at the theory of elasticity rather poor but also equilibrium is no more possible for the individual reinforcement bars, as shown in Fig 18(d).

Instead, the difference of chord forces $\Delta T = T_2 - T_1$, which is anchored within the depth of the girder, calls for horizontal ties T_3 ,

$$\Sigma T_3 = \Delta T \text{ according to Fig 18(e), (f).}$$

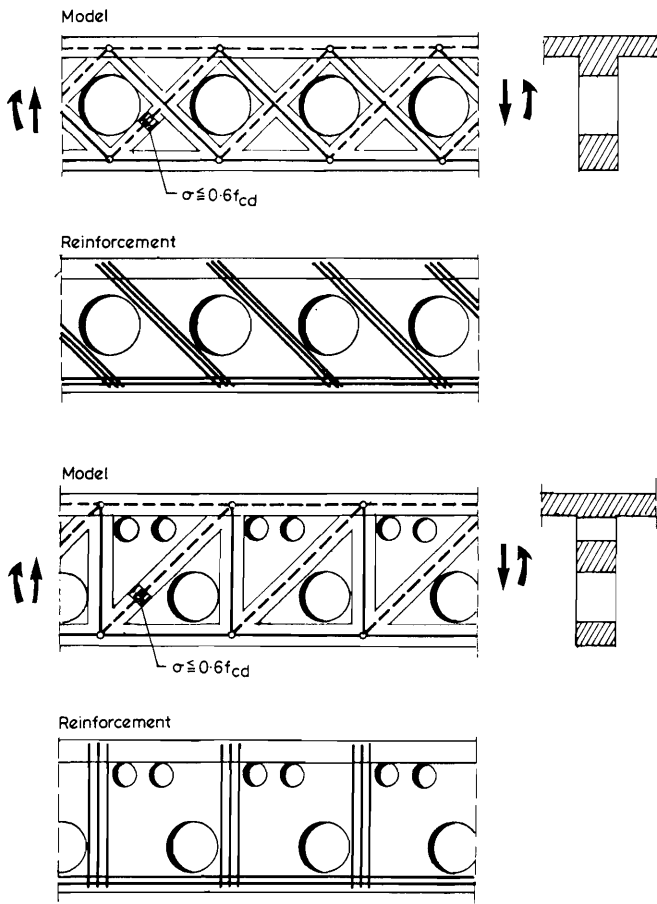


Fig 16. Beams with small openings in the web: models and corresponding reinforcement

Block-foundation

The problem is given in Fig 19(a). Following the load path method, the pressures applied to the foundation from above and below are subdivided and connected as shown in Fig 19(b). The component F_1 represents that part of the column load which is diverted to the left-hand side of the foundation, while F_2 and F_4 are diverted to the other side, where the load path $F_3 = T$ (forming a U-turn) has to be arranged in between F_2 and F_4 in order to avoid a crossing of the load paths. The model is easily completed by the horizontal compression and tension chords C_o and T_o . Their maximum force amounts to

$$C_o = T_o = F_1 \cot \theta_1.$$

The column compression forces F_1 , F_2 and F_4 are distributed also in the transverse direction over the width of the foundation and thereby create transverse tension, T_1 , T_2 , T_4 , e.g.

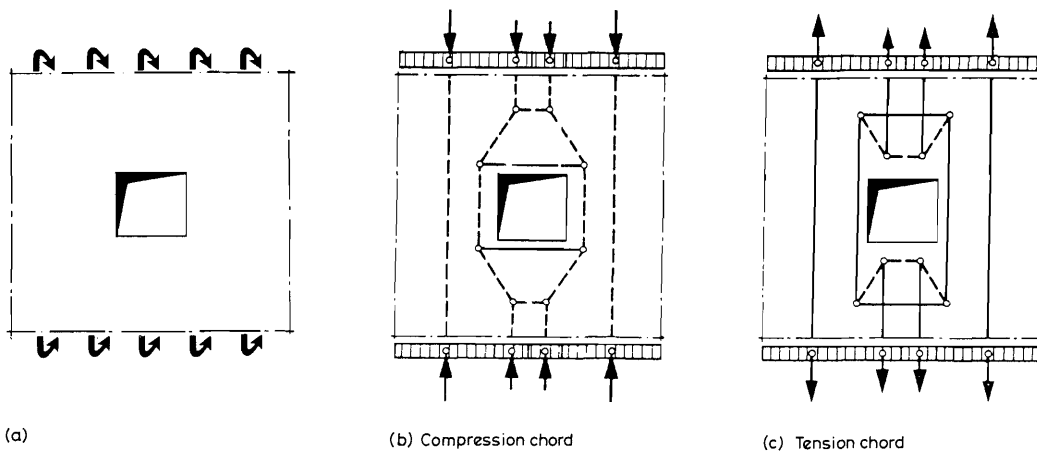


Fig 17. Opening in slab with constant moment

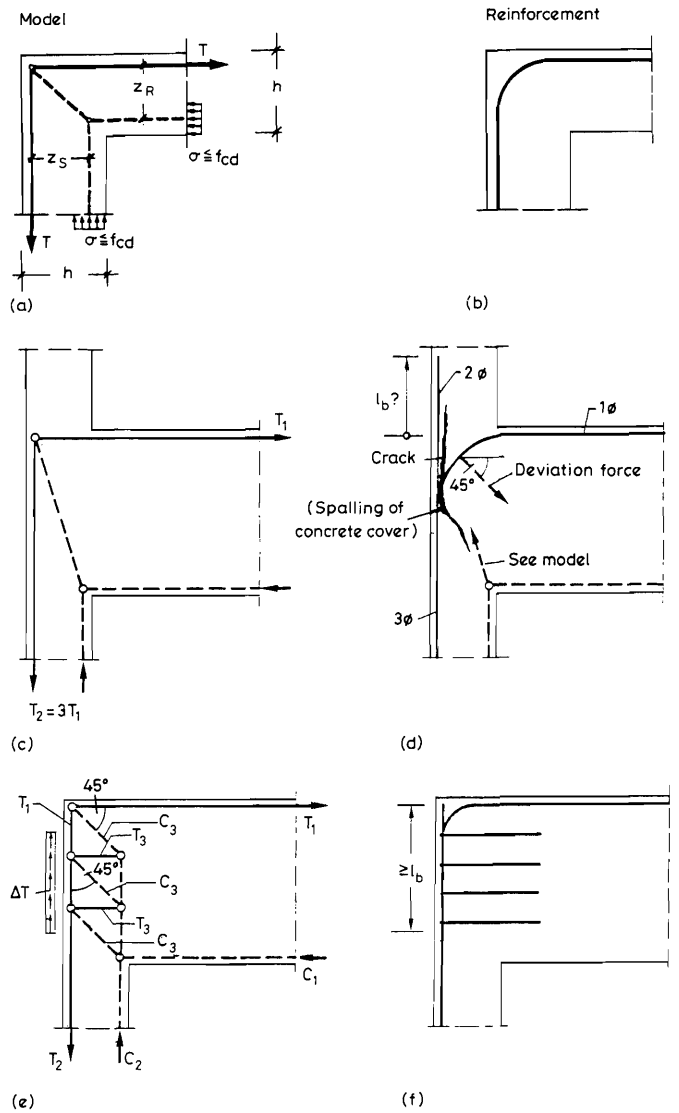


Fig 18. Frame corner with closing moment: (a), (b) model and reinforcement for similar dimensions of column and beam; (c), (d) bad model and reinforcement for dissimilar dimensions of column and beam; (e), (f) good model and reinforcement for dissimilar dimensions of column and beam

$$T_1 = 0.5 F_1 \cot \theta_2.$$

as can be seen from Fig 19(c).

Since the tension force T in the column reinforcement is carried to the bottom of the foundation and combines there with the load path of F_3

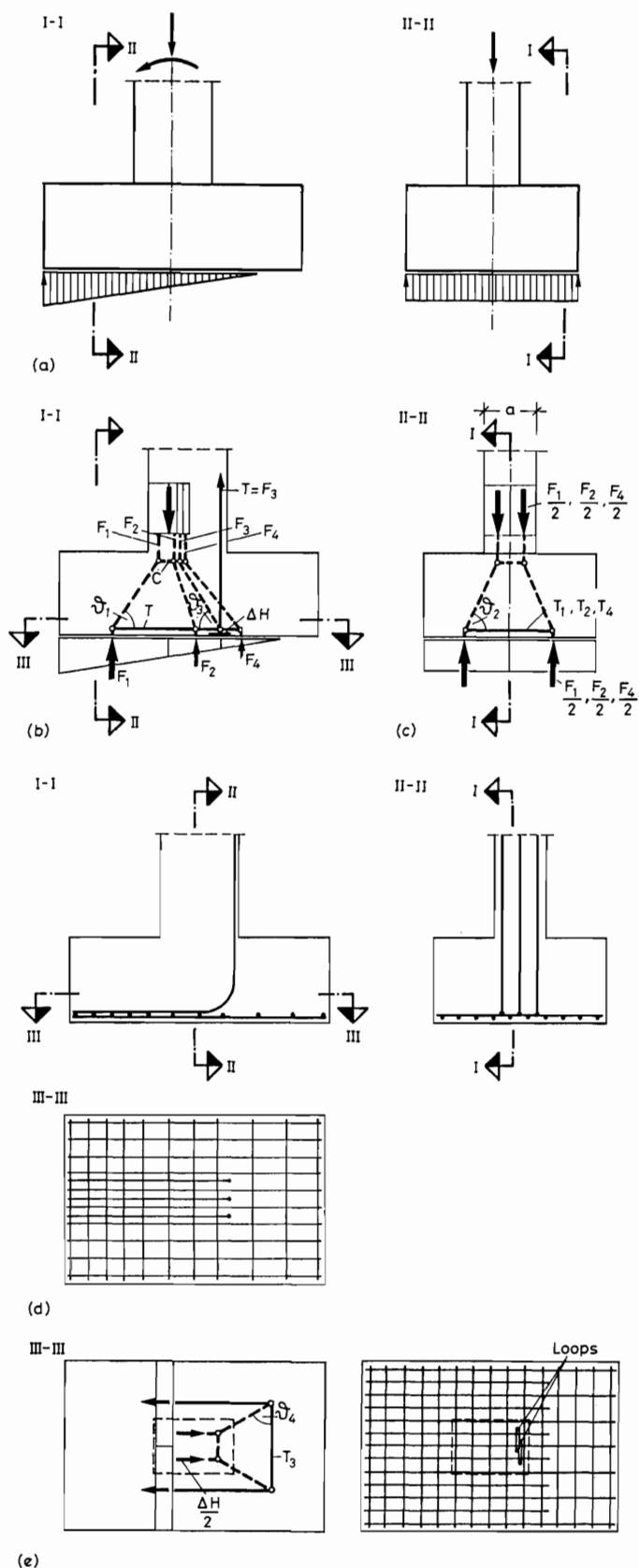


Fig 19. Block foundation: (a) layout and applied forces; (b), (c) strut-and-tie models; (d) corresponding reinforcement; (e) model for the distribution of the horizontal forces ΔH and combined reinforcement if this model is applied

within the width a of the column, its contribution to the tensile chord T_0

$$\Delta H = F_3 \cot \theta_3,$$

must be arranged within this width as well, e.g. by horizontal legs of the column reinforcement (Fig 19(d)).

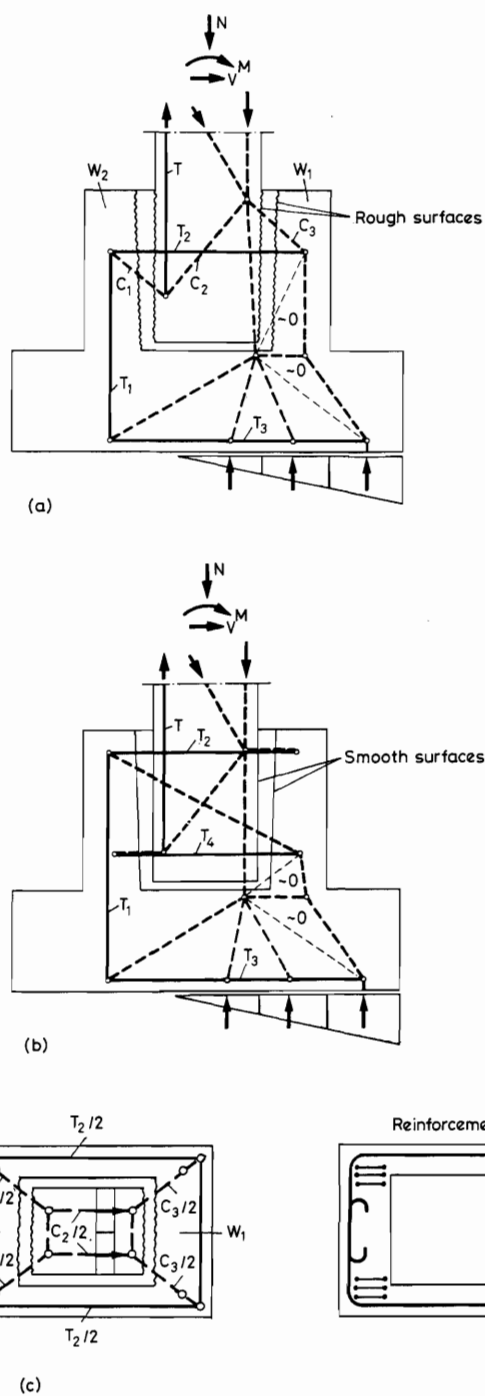


Fig 20. Hole footings: (a) model for a footing with rough joints; (b) model for a footing with smooth joints; (c) simplified model for the walls in horizontal projection and corresponding reinforcement

These legs can be avoided and the longitudinal reinforcement can be evenly distributed over the whole foundation width if, in accordance with Fig. 19(e), the additional transverse tension

$$T_3 = 0.5 \Delta H \cot \theta_4$$

is also covered by reinforcement. Loops would be necessary in this case for the anchorage of the column reinforcement T near the bottom of the foundation.

All tensile forces have to be covered by reinforcement. The reinforcement for T_0 must be extended to the left end of the foundation and the transverse reinforcement for T_1, T_2, T_3 and T_4 over the whole width. No hooks are necessary for better anchorage of horizontal bars if approximately two-thirds of the necessary anchorage length of the Code is provided 'behind' the nodes of the model.

The compression strut C_0 needs no special check, if the compression node immediately below the column is assumed deep enough (see typical node N2, Fig 10).

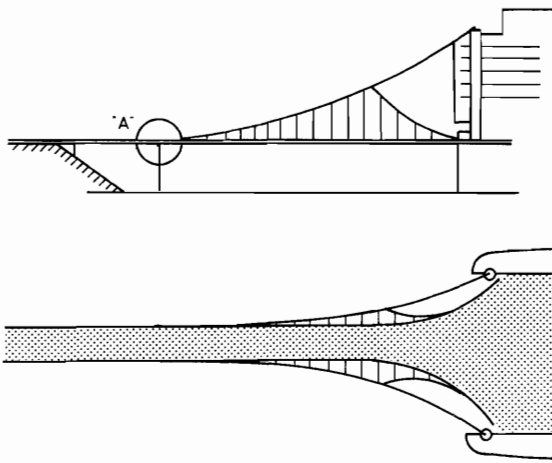


Fig 21. Layout of a cable bridge for pedestrians in Stuttgart

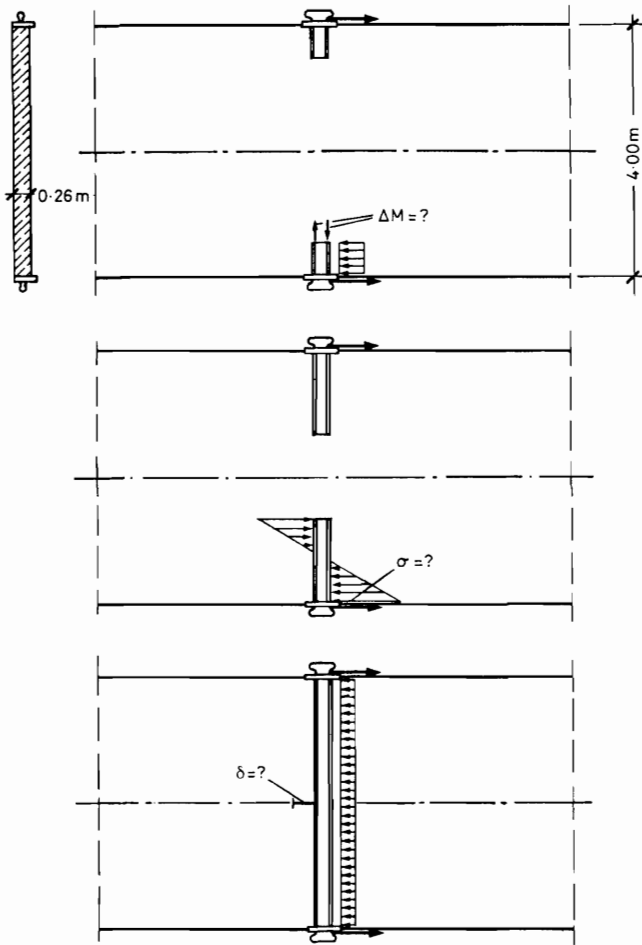
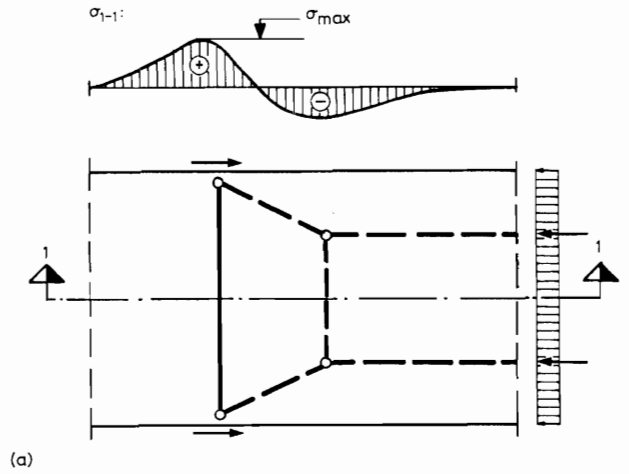


Fig 22. Rejected proposals for the anchorage of the cables in the bridge deck (detail A of Fig 21)

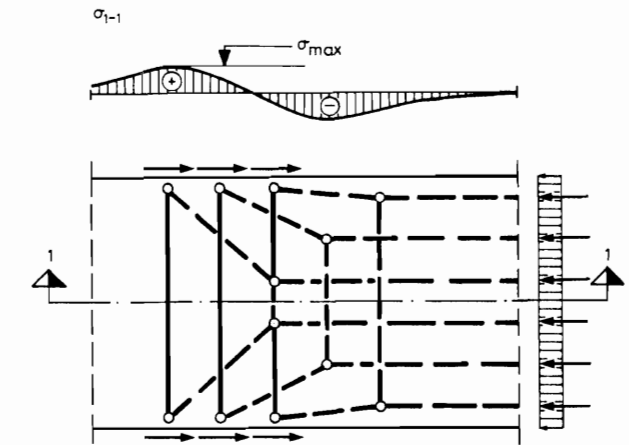
Hole footings

In Figs 20(a) and (b) models are given for two cases. In the first case, rough (profiled) concrete surfaces are assumed for the joint of column and foundation and, in the other, smooth surfaces which do not permit an inclined strut to cross the joint.

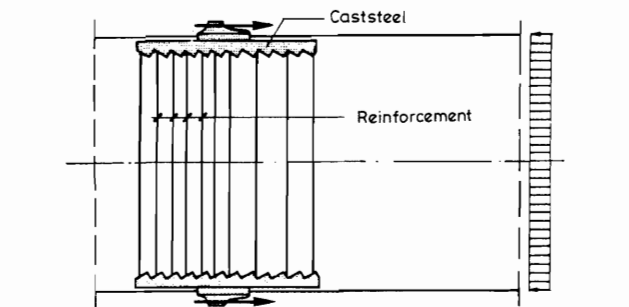
If the joint is rough (Fig 20(a)), the column tie T is overlapped via inclined compression C_1 with vertical ties T_1 in the footing. Thereby horizontal forces are applied to the footing walls W1 and W2, which must be transferred laterally into the side walls, as e.g. shown in Fig 20(c) by a very simplified model. The walls W1 and W2 can, depending on their slenderness, also be treated like short beams following the Code. The ties $T_2/2$ in this model correspond to the horizontal tie T_2 in Fig 20(a). Their forces can be covered by horizontal reinforcement on both the inner and outer sides of the foundation side walls, if the model is adjusted accordingly.



(a)



(b)



(c)

Fig 23. Anchorage of the cable forces (detail A of Fig 21): (a) basic model of the forces in the bridge deck and corresponding stress distribution; (b) refined model; (c) cast steel components and reinforcement according to model (b)

The vertical tie force T_1 is deviated into the lower layer of reinforcement of the foundation like the chord in a frame corner. It balances there with the forces proceeding from the column's compression chord.

The model for the foundation with smooth joints (Fig 20(b)) leads to approximately 1.7 times larger horizontal forces T_2 and an additional tensile force T_4 of similar magnitude in the side walls. Also the diagonal compression forces in the column and walls are increased accordingly.

These examples show the usefulness of strut-and-tie models not only for dimensioning but also for the conceptual design of a structure or detail. This will be shown more clearly in the next example.

Detail of a pedestrian bridge

Fig 21 gives an impression of a cable bridge in Stuttgart, which is suspended from an existing building. We shall develop a design of the nodes where the cables are anchored and introduce their horizontal forces into the concrete bridge deck.

Preliminary proposals for this detail using steel girders embedded in the concrete (Fig 22) were rejected after the strut-and-tie model (Fig 23(a)) showed how the cable forces could be introduced immediately without any bending of embedded members and the associated high stresses and deformations. The compression forces are applied directly in the direction of the diagonal model struts via profiled cast-steel components without major disturbance of the thin concrete deck (Fig 23(c)). Transverse reinforcing bars, welded to the cast steel components, carry the tie forces of the model. The stress distribution is further improved by smearing the node A over a greater length, as suggested by the refined model of Fig 23(b).

The check of the concrete compression stress applied by the cast steel needs no further explanation.

If the cables are inclined at a considerable angle in order to support the bridge deck, as is usually the case, a bottom flange must be attached to the steel member for vertical support of the deck.

Conclusions

Strut-and-tie models can be used for tracing the internal forces in complicated details. They are very helpful in the conceptual design of a detail, leading the designer intuitively to simple and sound solutions. Strut-and-tie models are also a basis for the quantitative check of details and whole structures. However, the method also requires some engineering knowledge and training to which this paper is intended to contribute with a summary of principles and a few applications of the method.

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Synopsis

The following is a precis of a paper that is filed in the Institution's library. Copies of the full text may be borrowed by members in the UK or may be consulted in the library.

$P-\delta$ analysis of pitched roof frames

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The buckling loads, buckling lengths as well as the $P-\delta$ effects for pitched roof frames with different loading arrangements are examined. The results were compared to some common and internationally known Code results. It was found that the loading distribution affects the buckling loads significantly and hence the buckling lengths and the magnified (or added) moments. This paper clarifies a defect in some national building Codes, and encourages the use of computers in determining the buckling loads and/or magnified moments. Among the conclusions are the following: (1) It is not sufficient that the national building Codes divide structures into braced and unbraced, since unbraced structures can deform in different mode shapes.

(2) The magnification factor (or the added moment) in columns of braced structures is very much affected by the magnitude and distribution of loads. This effect is ignored in the national building Codes.

(3) The magnification factor using the American Code, which is based on the buckling load, is more reliable than calculating this factor (or the added moments) using the British Code, which is based on the buckling length. (Ref. X104.)