Recalques de apoio

$$\{F\} = [K] \cdot \{U\}$$

$$\begin{Bmatrix} R \\ F \end{Bmatrix} = \begin{bmatrix} K_{RR} & K_{RL} \\ K_{LR} & K \end{bmatrix} \cdot \begin{Bmatrix} U_R \\ U \end{Bmatrix}$$

$$\Rightarrow \{F\} = [K_{LR}] \cdot \{U_R\} + [K] \cdot \{U\}$$

$$\Rightarrow \{U\} = [K]^{-1} \left(\{F\} - [K_{LR}] \cdot \{U_R\} \right)$$

Recalques de apoio

implementação numérica

$$\begin{Bmatrix} R_1 \\ R_2 \\ F_1 \\ F_2 \\ \vdots \end{Bmatrix} = \begin{bmatrix} k_{11} & k_{12} & k_{13} & k_{14} & \cdots \\ k_{21} & k_{22} & k_{23} & k_{24} & \cdots \\ k_{31} & k_{32} & k_{33} & k_{34} & \cdots \\ k_{41} & k_{42} & k_{43} & k_{44} & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{bmatrix} \cdot \begin{Bmatrix} U_{R1} \\ U_{R2} \\ U_1 \\ U_2 \\ \vdots \end{Bmatrix}$$

$$\begin{Bmatrix} R_1 \\ R_2 \\ F_1 \\ F_2 \\ \vdots \end{Bmatrix} = \begin{bmatrix} k_{11} & k_{12} & k_{13} & k_{14} & \cdots \\ k_{21} & k_{22} & k_{23} & k_{24} & \cdots \\ k_{31} & k_{32} & k_{33} & k_{34} & \cdots \\ k_{41} & k_{42} & k_{43} & k_{44} & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{bmatrix} \cdot \left(\begin{Bmatrix} U_{R1} \\ U_{R2} \\ 0 \\ 0 \\ \vdots \end{Bmatrix} + \begin{Bmatrix} 0 \\ 0 \\ U_1 \\ U_2 \\ \vdots \end{Bmatrix} \right)$$

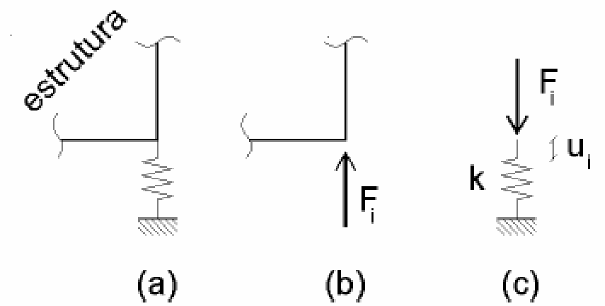
$$\begin{Bmatrix} R_1 - k_{11}U_{R1} - k_{12}U_{R2} \\ R_2 - k_{21}U_{R1} - k_{22}U_{R2} \\ F_1 - k_{31}U_{R1} - k_{32}U_{R2} \\ F_2 - k_{41}U_{R1} - k_{42}U_{R2} \\ \vdots \end{Bmatrix} = \begin{bmatrix} k_{11} & k_{12} & k_{13} & k_{14} & \cdots \\ k_{21} & k_{22} & k_{23} & k_{24} & \cdots \\ k_{31} & k_{32} & k_{33} & k_{34} & \cdots \\ k_{41} & k_{42} & k_{43} & k_{44} & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{bmatrix} \cdot \begin{Bmatrix} 0 \\ 0 \\ U_1 \\ U_2 \\ \vdots \end{Bmatrix}$$

desacoplamento!

$$\begin{Bmatrix} U_{R1} \\ U_{R2} \\ F_1 - k_{31}U_{R1} - k_{32}U_{R2} \\ F_2 - k_{41}U_{R1} - k_{42}U_{R2} \\ \vdots \end{Bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & \cdots \\ 0 & 1 & 0 & 0 & \cdots \\ 0 & 0 & k_{33} & k_{34} & \cdots \\ 0 & 0 & k_{43} & k_{44} & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{bmatrix} \cdot \begin{Bmatrix} U_{R1} \\ U_{R2} \\ U_1 \\ U_2 \\ \vdots \end{Bmatrix}$$

Técnica dos
“zeros e um”
diferenciada

Apoios Elásticos



(a) – Visualização do apoio elástico;
(b) – Diagrama do Corpo Livre (DCL) da estrutura;
(c) – DCL do apoio (mola).

O que matricialmente pode ser expresso por:

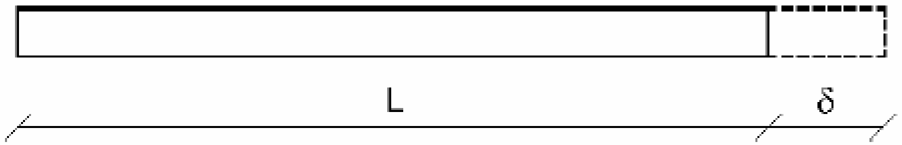
$$\begin{Bmatrix} \vdots \\ \vdots \\ -k \cdot u_i \\ \vdots \end{Bmatrix} = \begin{bmatrix} \ddots & & \vdots & \\ & \ddots & \vdots & \\ \cdots & \cdots & k_{ii} & \cdots \\ & & \vdots & \ddots \end{bmatrix} \cdot \begin{Bmatrix} \vdots \\ \vdots \\ u_i \\ \vdots \end{Bmatrix}$$

e equivale a somar k ao coeficiente de rigidez de diagonal k_{ii} :

$$\begin{Bmatrix} \\ 0 \\ \\ \end{Bmatrix} = \begin{bmatrix} \ddots & & \\ & \ddots & \\ & k_{ii} + k & \\ & & \ddots \end{bmatrix} \cdot \begin{Bmatrix} \\ u_i \\ \\ \end{Bmatrix}$$

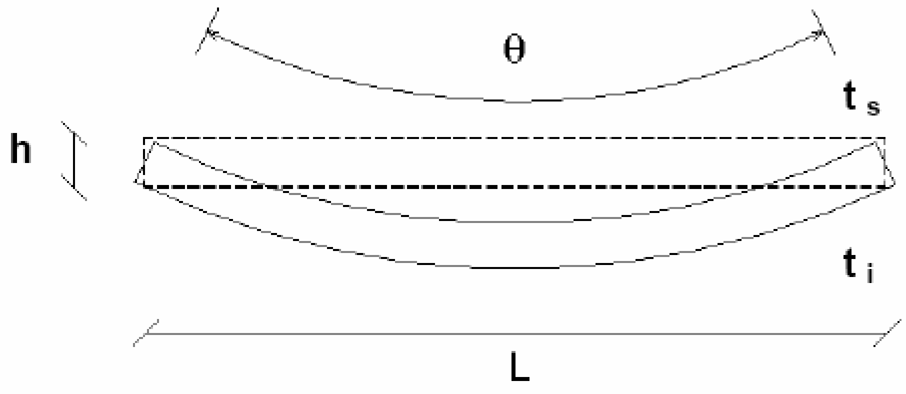
Variação de Temperatura

$$\delta = \alpha \cdot L \cdot \Delta t$$

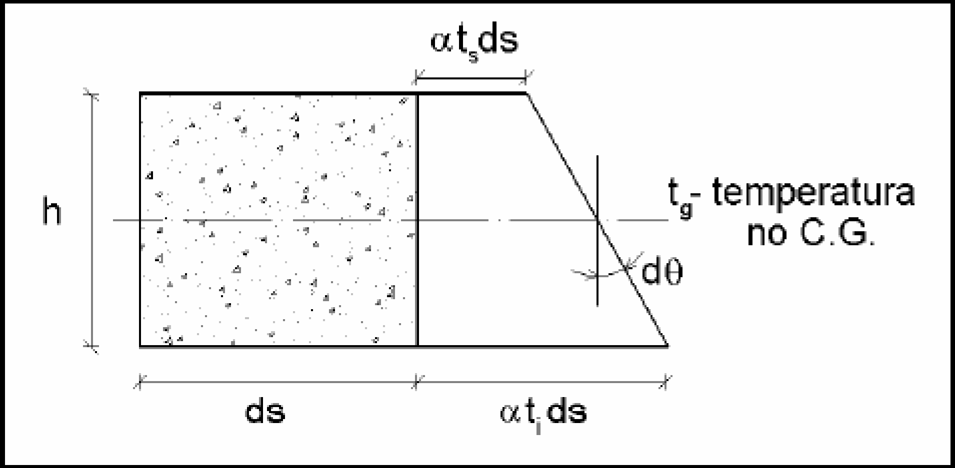


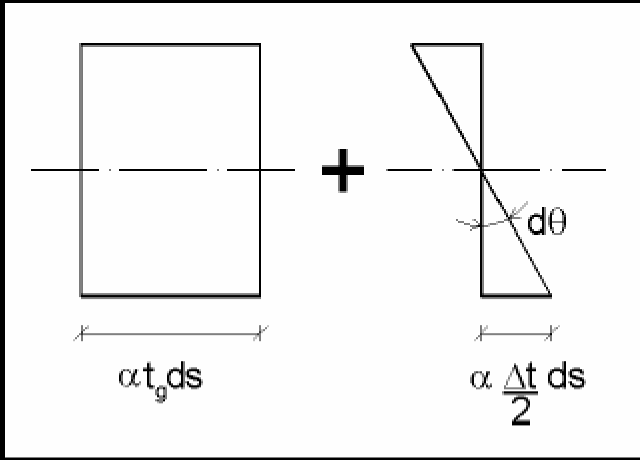
$$\Rightarrow \sigma = E\varepsilon \Rightarrow F = EA \cdot \frac{\delta}{L} = E \cdot A \cdot \alpha \cdot \Delta t$$

$$\theta = \alpha \cdot (t_s - t_i) \cdot \frac{L}{h}$$

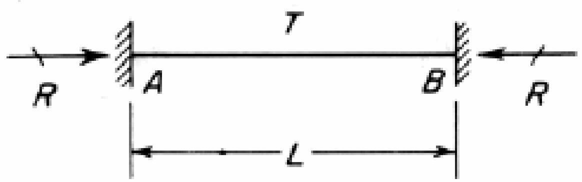
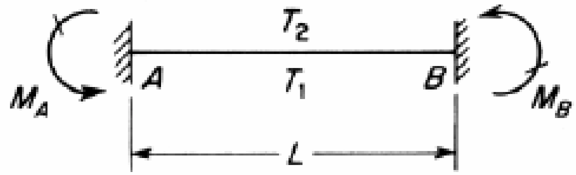


Variação de Temperatura





Variação de Temperatura

1 Aumento uniforme de temperatura	2 Diferença de temperatura
	
$R = EA\alpha T$	$M_A = -M_B = \dfrac{\alpha EI(T_1 - T_2)}{d}$
E = módulo de elasticidade	I = momento de inércia
A = área da seção transversal	T_1 = temperatura na parte inferior da viga
α = coeficiente de dilatação térmica	T_2 = temperatura na parte superior da viga
T = aumento de temperatura	d = largura da viga