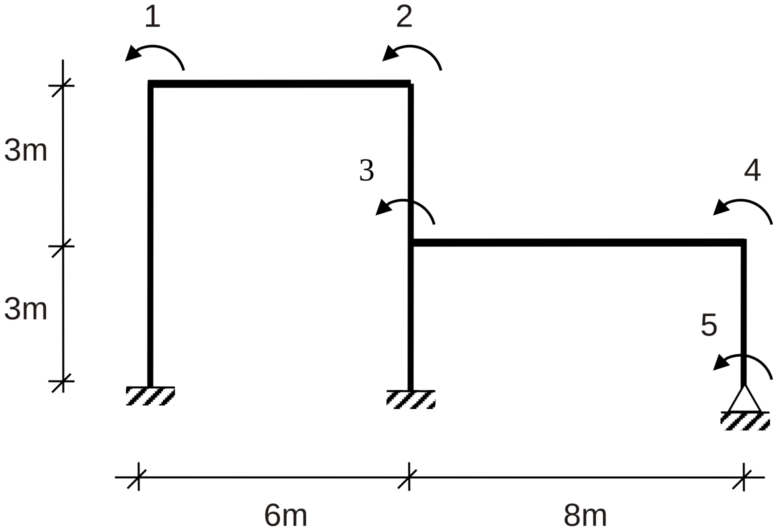
EXERCÍCIO DE MÉTODO DA RIGIDEZ – SOLUÇÃO GERAL

Um edifício foi modelado conforme desenho apresentado abaixo:



A partir da solução geral do método da rigidez pedem-se:

a) A matriz de rigidez global da estrutura;

b) O DMF para um carregamento uniformemente distribuído de 12kN/m aplicado sobre todas as vigas.

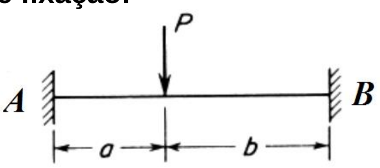
Dados:

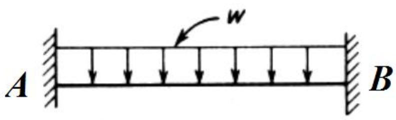
$$EJ_{VIGAS} = 4EJ_{PILARES}$$

$$[k_e] = \frac{2EJ}{L}\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$$

FORMULÁRIO

1) Reações de fixação:





$$M_A = \frac{Pab^2}{L^2} \quad M_B = -\frac{Pa^2b}{L^2}$$

$$R_A = \frac{Pb^2}{L^3}(3a+b) \quad R_B = \frac{Pa^2}{L^3}(a+3b)$$

$$M_A = -M_B = \frac{wL^2}{12}$$

$$R_A = R_B = \frac{wL}{2}$$

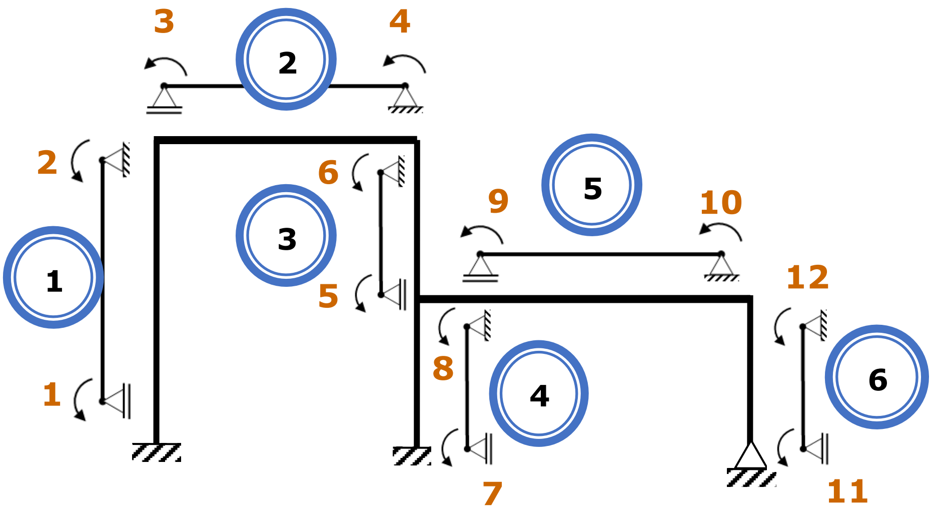
2) Formulação do método da rigidez:

Solução Geral:

$$[K] = [A]^t \cdot [K_\ell] \cdot [A] \qquad \{F\} = [K] \cdot \{r\} \qquad \{S\} = \{S_0\} + [K_\ell] \cdot [A] \cdot \{r\}$$

Solução:

1) Desmembramento do pórtico em elementos e coordenadas locais:



2) Determinação da matriz de compatibilidade cinemática:

Coor global 1 = Coor locais 2 e 3
$$[A]_{2,1} = [A]_{3,1} = 1$$

Coor global 2 = Coor locais 4 e 6
$$[A]_{4,2} = [A]_{6,2} = 1$$

Coor global 3 = Coor locais 5, 8 e 9
$$[A]_{5,3} = [A]_{8,3} = [A]_{9,3} = 1$$

Coor global 4 = Coor locais 10 e 12
$$[A]_{10,4} = [A]_{12,4} = 1$$

Coor global 5 = Coor locais 11
$$[A]_{11,5} = 1$$

$$\Rightarrow [A] = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

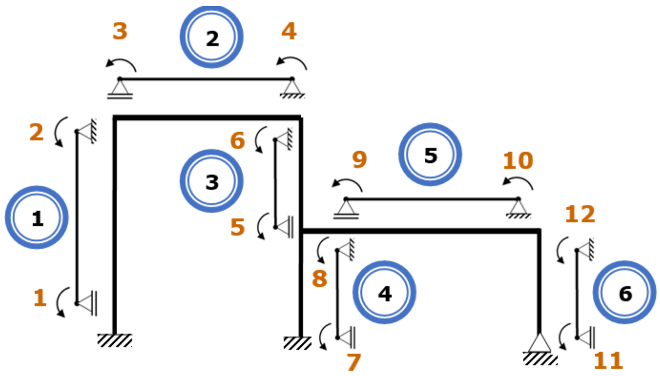
Command Window

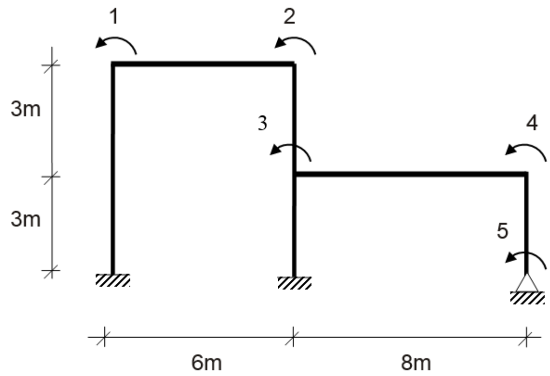
```
>> A(2,1)=1;A(3,1)=1;
A(4,2)=1;A(6,2)=1;
A(5,3)=1;A(8,3)=1;A(9,3)=1;
A(10,4)=1;A(12,4)=1;A(11,5)=1;
>> A'

ans =

     0     1     1     0     0     0     0     0     0     0     0     0
     0     0     0     1     0     1     0     0     0     0     0     0
     0     0     0     0     1     0     0     1     1     0     0     0
     0     0     0     0     0     0     0     0     0     1     0     1
     0     0     0     0     0     0     0     0     0     0     1     0
```

3) Determinação das matrizes de rigidez dos elementos:





$$[k_e] = \frac{2EJ}{L}\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$$

Fazendo: $EJ_{VIGAS} = 4EJ_{PILARES} = 4EJ$

Elmto 1 (L = 6 m, rigidez EJ):

$$\Rightarrow [k_{e1}] = \frac{2EJ}{6}\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} = \frac{EJ}{3}\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$$

Elmto 2 (L = 6 m, rigidez 4EJ):

$$\Rightarrow [k_{e2}] = \frac{8EJ}{6}\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} = \frac{4EJ}{3}\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$$

Elmtos 3, 4 e 6 (L = 3 m, rigidez EJ):

$$\Rightarrow [k_{e3}] = [k_{e4}] = [k_{e6}] = \frac{2EJ}{3}\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$$

Elmto 5 (L = 8 m, rigidez 4EJ):

$$\Rightarrow [k_{e5}] = \frac{8EJ}{8}\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} = EJ\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$$

Command Window

```
>> E=sym('E');J=sym('J');
>>
```

Command Window

```
>> k1=E*J/3*[2 1;1 2]

k1 =

[ (2*E*J)/3,    (E*J)/3]
[   (E*J)/3,  (2*E*J)/3]

>> k2=4*E*J/3*[2 1;1 2];
>> k3=2*E*J/3*[2 1;1 2];
>> k4=k3;
>> k5=E*J*[2 1;1 2];
>> k6=k3;
fx >>
```

4) Determinação da matriz de rigidez global:

$$[K_\ell]=\begin{bmatrix} [k_{e1}] & & & & \\ & [k_{e2}] & & & \\ & & [k_{e3}] & & \\ & & & [k_{e4}] & \\ & & & & [k_{e5}] \end{bmatrix}$$

$$[K]=[A]^t \cdot [K_\ell] \cdot [A]$$

Command Window

```
>> Kl=blkdiag(k1,k2,k3,k4,k5,k6)
```

```
Kl =

[ (2*E*J)/3,   (E*J)/3,         0,         0,         0,         0,         0,         0,     0,     0,         0,         0]
[   (E*J)/3, (2*E*J)/3,         0,         0,         0,         0,         0,         0,     0,     0,         0,         0]
[         0,         0, (8*E*J)/3, (4*E*J)/3,         0,         0,         0,         0,     0,     0,         0,         0]
[         0,         0, (4*E*J)/3, (8*E*J)/3,         0,         0,         0,         0,     0,     0,         0,         0]
[         0,         0,         0,         0, (4*E*J)/3, (2*E*J)/3,         0,         0,     0,     0,         0,         0]
[         0,         0,         0,         0, (2*E*J)/3, (4*E*J)/3,         0,         0,     0,     0,         0,         0]
[         0,         0,         0,         0,         0,         0, (4*E*J)/3, (2*E*J)/3,     0,     0,         0,         0]
[         0,         0,         0,         0,         0,         0, (2*E*J)/3, (4*E*J)/3,     0,     0,         0,         0]
[         0,         0,         0,         0,         0,         0,         0,         0, 2*E*J,   E*J,         0,         0]
[         0,         0,         0,         0,         0,         0,         0,         0,   E*J, 2*E*J,         0,         0]
[         0,         0,         0,         0,         0,         0,         0,         0,     0,     0, (4*E*J)/3, (2*E*J)/3]
[         0,         0,         0,         0,         0,         0,         0,         0,     0,     0, (2*E*J)/3, (4*E*J)/3]

fx >>
```

Command Window

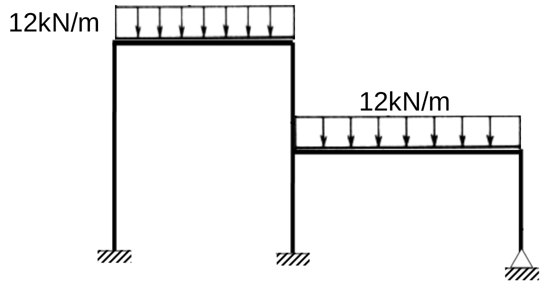
```
>> K=A'*Kl*A

K =

[ (10*E*J)/3, (4*E*J)/3,          0,          0,         0]
[  (4*E*J)/3,     4*E*J,  (2*E*J)/3,          0,         0]
[          0, (2*E*J)/3, (14*E*J)/3,        E*J,         0]
[          0,         0,        E*J, (10*E*J)/3, (2*E*J)/3]
[          0,         0,          0,  (2*E*J)/3, (4*E*J)/3]
```

5) Carregamento:

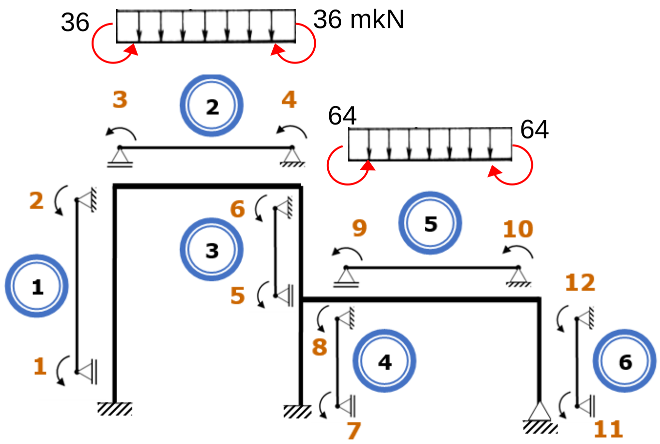
Carregamento distribuído:

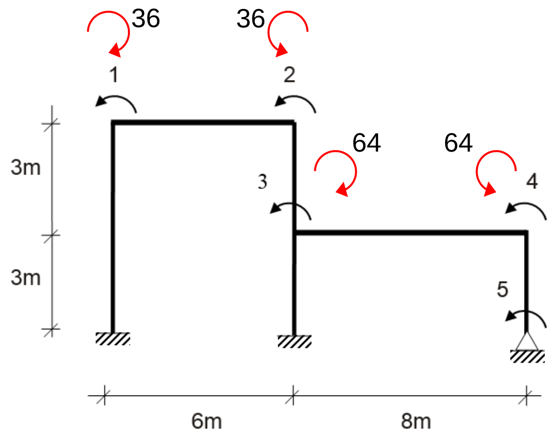


Reações de Fixação

+

Carregamento Nodal Equivalente





$$\{S_0\} = \begin{Bmatrix} 0 \\ 0 \\ 36 \\ -36 \\ 0 \\ 0 \\ 0 \\ 0 \\ 64 \\ -64 \\ 0 \\ 0 \end{Bmatrix} m \cdot KN$$

$$\{F\} = \begin{Bmatrix} -36 \\ 36 \\ -64 \\ 64 \\ 0 \end{Bmatrix} m \cdot KN$$

Command Window

```
>> F=[-36 36 -64 64 0]';S0=[0 0 36 -36 0 0 0 0 64 -64 0 0]';
>> F'

ans =

   -36    36   -64    64     0

>> S0'

ans =

     0     0    36   -36     0     0     0     0    64   -64     0     0
```

6) Equilíbrio:

$$\{F\} = [K] \cdot \{r\}$$

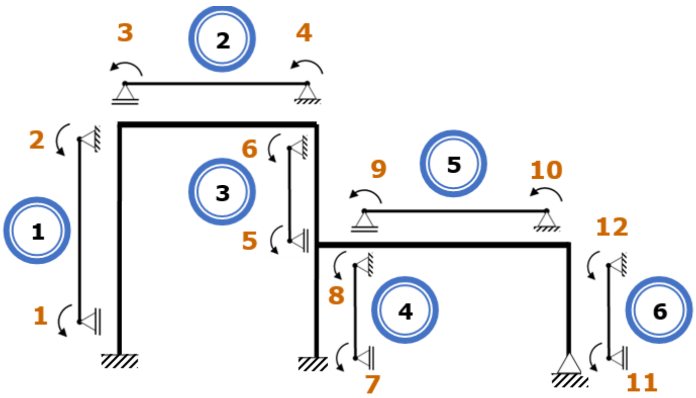
Command Window

```
>> r=inv(K)*F

r =

  -1505/(82*E*J)
   3097/(164*E*J)
  -1853/(82*E*J)
   2367/(82*E*J)
 -2367/(164*E*J)
```

7) Esforços:



$$\{S\} = \{S_0\} + [K_\ell] \cdot [A] \cdot \{r\}$$

Command Window

```
>> S=S0+K1*A*r;
>> S'*246

ans =

[ -1505, -3010, 3010, -2488, -4315, 2488, -3706, -7412, 11727, -7101, 0, 7101]
```