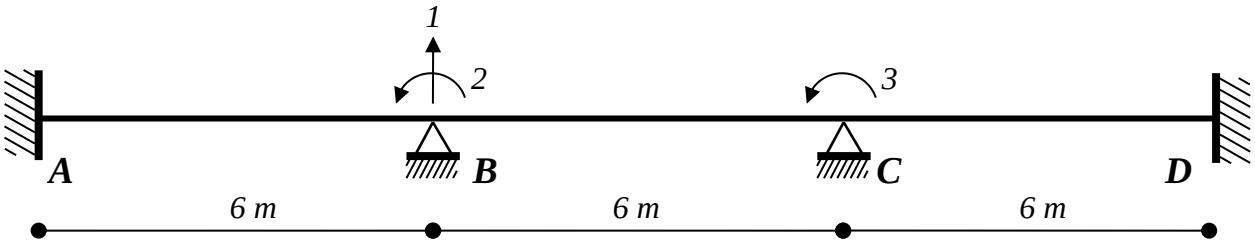
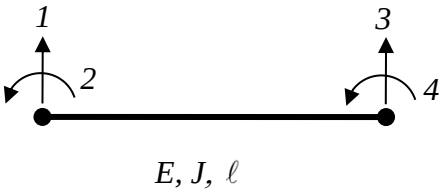
1ª QUESTÃO

Uma viga contínua de um pavimento é apresentada a seguir. Utilizando o processo da rigidez direta e a discretização sugerida, pede-se:



$$EJ = 4 \cdot 10^4 \, kNm^2$$



$$[k_e] = EJ \begin{bmatrix} \frac{12}{\ell^3} & \frac{6}{\ell^2} & -\frac{12}{\ell^3} & \frac{6}{\ell^2} \\ \frac{6}{\ell^2} & \frac{4}{\ell} & -\frac{6}{\ell^2} & \frac{2}{\ell} \\ -\frac{12}{\ell^3} & -\frac{6}{\ell^2} & \frac{12}{\ell^3} & -\frac{6}{\ell^2} \\ \frac{6}{\ell^2} & \frac{2}{\ell} & -\frac{6}{\ell^2} & \frac{4}{\ell} \end{bmatrix}$$

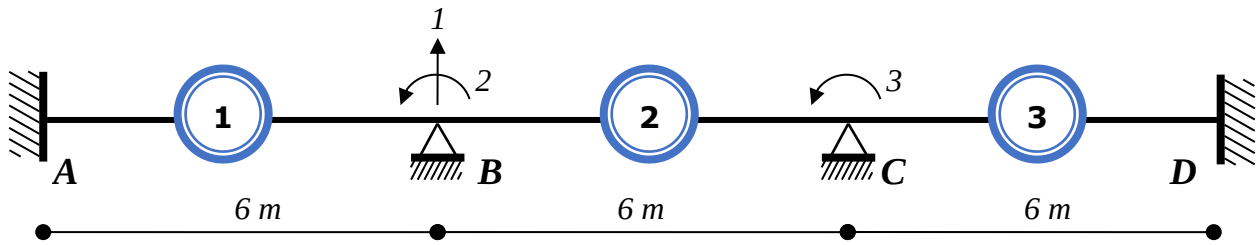
a) A matriz de rigidez global (com 3 GL);

b) O DMF e o DEC da viga quando submetida a um carregamento uniformemente distribuído de *24 kN/m*;

c) O DMF da viga quando submetida a um carregamento uniformemente distribuído de *24 kN/m* e admitindo-se ainda que o apoio B se comporta verticalmente como um apoio elástico, recalcando 1mm para cada 10 kN de carga aplicada;

d) O DMF da viga quando solicitada por um recalque vertical de 2 cm no apoio B.

a) A matriz de rigidez global (com 3 GL):



Elmtos 1, 2 e 3 (L = 6 m, rigidez EJ=40.000 kNm²):

Elmto 1:	X	X	1	2
Elmto 2:	1	2	X	3
Elmto 3:	X	3	X	X

$$\Rightarrow [k_{e1}]=[k_{e2}]=[k_{e3}]=\frac{40}{6^3}\begin{bmatrix} 12 & 36 & -12 & 36 \\ 36 & 144 & -36 & 72 \\ -12 & -36 & 12 & -36 \\ 36 & 72 & -36 & 144 \end{bmatrix}\cdot 10^3 \qquad \begin{matrix}X\\3\\X\\X\end{matrix}\quad\begin{matrix}1\\2\\X\\3\end{matrix}\quad\begin{matrix}X\\X\\1\\2\end{matrix}$$

$$\Rightarrow [K]=0{,}1852\begin{bmatrix} 12+12 & -36+36 & 36 \\ -36+36 & 144+144 & 72 \\ 36 & 72 & 144+144 \end{bmatrix}\cdot 10^7$$

$$\Rightarrow [K]=0{,}1852\begin{bmatrix} 24 & 0 & 36 \\ 0 & 288 & 72 \\ 36 & 72 & 288 \end{bmatrix}\cdot 10^3 \Rightarrow [K]=\begin{bmatrix} 0{,}4444 & 0 & 0{,}6667 \\ 0 & 5{,}3333 & 1{,}3333 \\ 0{,}6667 & 1{,}3333 & 5{,}3333 \end{bmatrix}\cdot 10^4$$

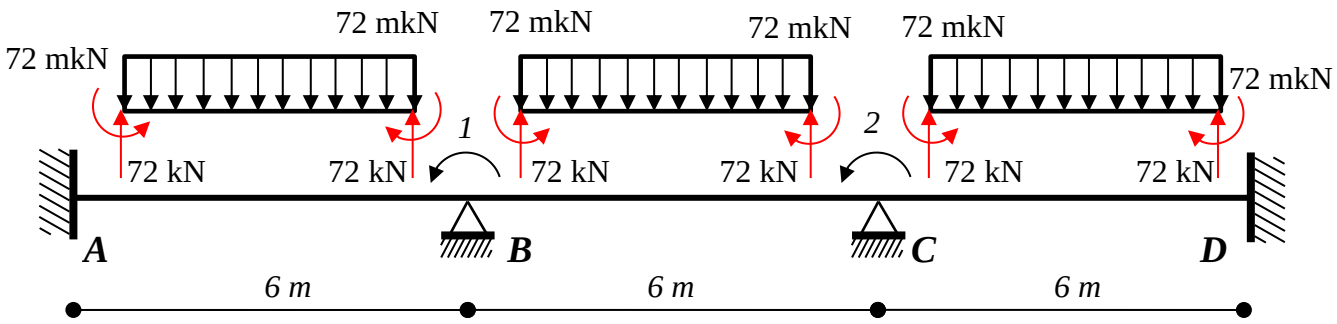
SCRIPT – MATLAB

```
EJ=40000;L=6;

k1=[12*EJ/L^3     6*EJ/L^2     -12*EJ/L^3      6*EJ/L^2;
    6*EJ/L^2      4*EJ/L       -6*EJ/L^2       2*EJ/L;
    -12*EJ/L^3   -6*EJ/L^2      12*EJ/L^3      -6*EJ/L^2;
    6*EJ/L^2      2*EJ/L       -6*EJ/L^2       4*EJ/L]

k2=k1;k3=k1;
```

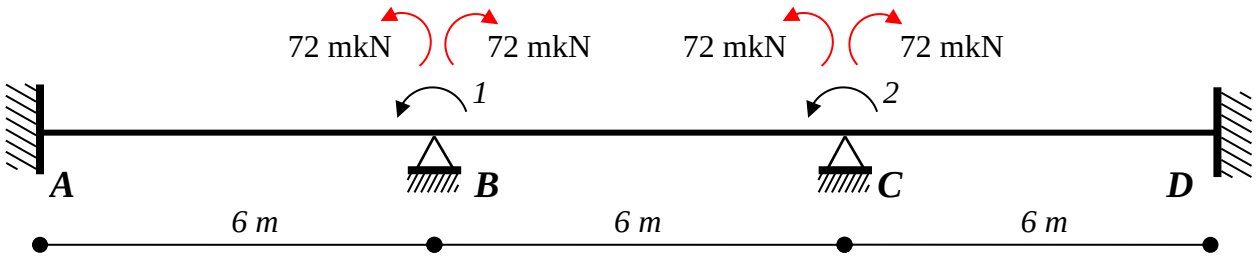
```
Kg=[k2(1,1)+k1(3,3)     k2(1,2)+k1(3,4)     k2(1,4);
    k2(2,1)+k1(4,3)     k2(2,2)+k1(4,4)     k2(2,4);
    k2(4,1)             k2(4,2)             k2(4,4)+k3(2,2)]

Kg = 1.0e+04 *

    0.4444         0         0.6667
    0              5.3333    1.3333
    0.6667         1.3333    5.3333
```

b) O DMF e o DEC da viga quando submetida a um carregamento uniformemente distribuído de *24 kN/m*;

Reações de Fixação:



Carregamento Nodal Equivalente



Reações de Fixação:

$$\{S_0\}_{e1} = \{S_0\}_{e2} = \{S_0\}_{e3} = \begin{Bmatrix} 72 \\ 72 \\ 72 \\ -72 \end{Bmatrix}$$

CNE:

$$\{F\} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

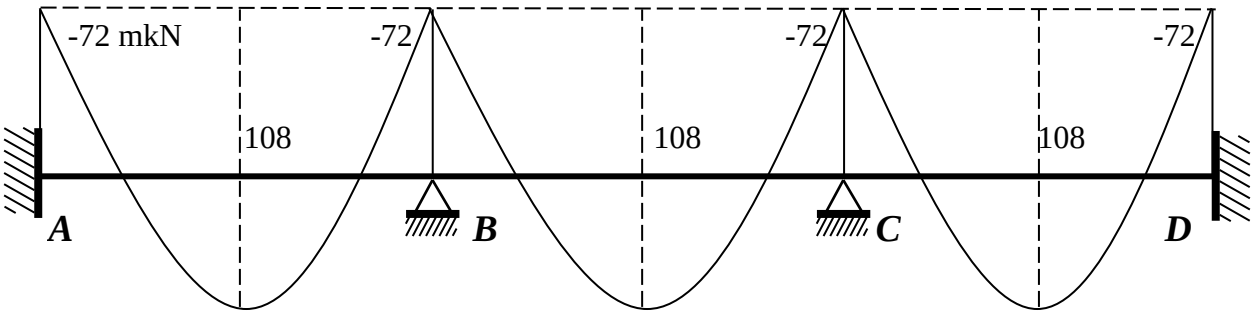
Equilíbrio:

$$\{F\}=[K]\cdot\{r\} \quad \Rightarrow \begin{Bmatrix}0\\0\end{Bmatrix}=\begin{bmatrix}0{,}4444 & 0 & 0{,}6667\\ 0 & 5{,}3333 & 1{,}3333\\ 0{,}6667 & 1{,}3333 & 5{,}3333\end{bmatrix}\cdot 10^4\cdot\{r\} \quad \Rightarrow \{r\}=\begin{Bmatrix}0\\0\end{Bmatrix}$$

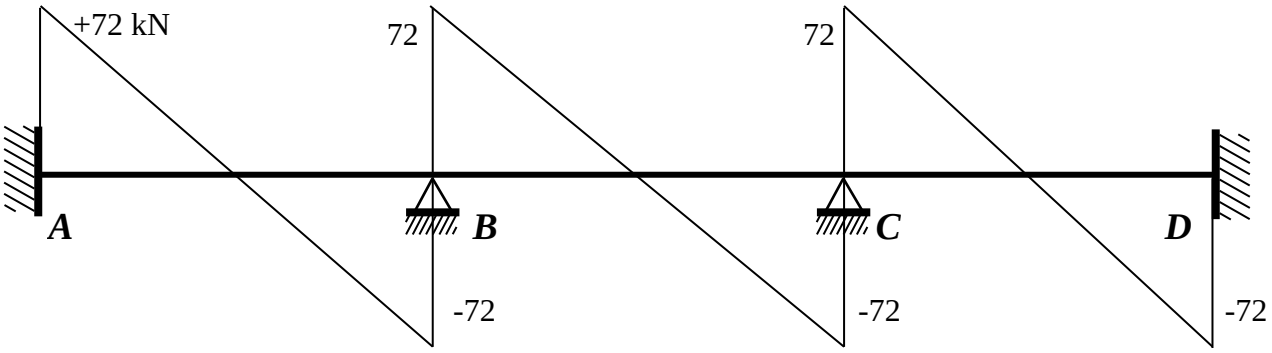
Esforços

$$\{S\}_{e1}=\{S_0\}_{e1}+[k_{e1}]\{u_{e1}\}=\begin{Bmatrix}72\\72\\72\\-72\end{Bmatrix}+[k_{e1}]\begin{Bmatrix}0\\0\\0\\0\end{Bmatrix} \Rightarrow \{S\}_{e2}=\{S\}_{e3}=\{S\}_{e1}$$

DMF:



DEC:

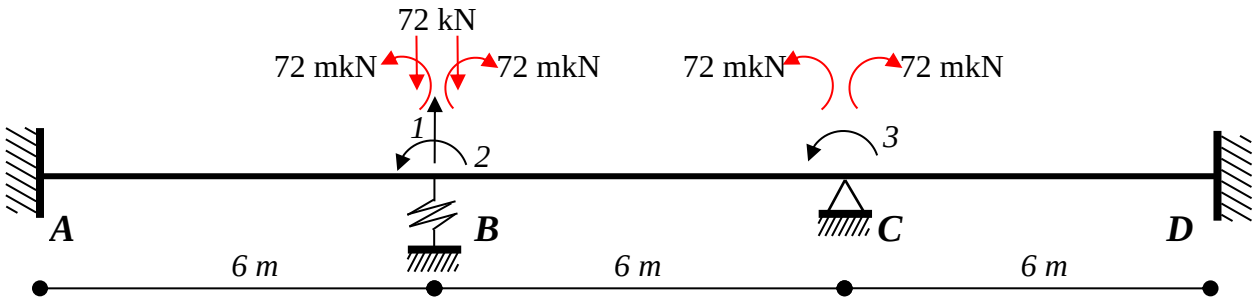


c) O DMF da viga quando submetida a um carregamento uniformemente distribuído de *24 kN/m* e admitindo-se ainda que o apoio B se comporta verticalmente como um apoio elástico, recalcando 1mm para cada 10 kN de carga aplicada;

$$\Rightarrow k = 10/0{,}001 = 1{,}0 \cdot 10^4\, kN/m$$

$$\Rightarrow [K] = \begin{bmatrix} 0{,}4444+k & 0 & 0{,}6667 \\ 0 & 5{,}3333 & 1{,}3333 \\ 0{,}6667 & 1{,}3333 & 5{,}3333 \end{bmatrix} \cdot 10^4 = \begin{bmatrix} 1{,}4444 & 0 & 0{,}6667 \\ 0 & 5{,}3333 & 1{,}3333 \\ 0{,}6667 & 1{,}3333 & 5{,}3333 \end{bmatrix} \cdot 10^4$$

Carregamento Nodal Equivalente



Reações de Fixação:

$$\{S_0\}_{e1} = \{S_0\}_{e2} = \{S_0\}_{e3} = \begin{Bmatrix} 72 \\ 72 \\ 72 \\ -72 \end{Bmatrix}$$

CNE:

$$\{F\} = \begin{Bmatrix} -144 \\ 0 \\ 0 \end{Bmatrix}$$

Equilíbrio:

$$\{F\} = [K] \cdot \{r\} \quad \Rightarrow \begin{Bmatrix} -144 \\ 0 \\ 0 \end{Bmatrix} = \begin{bmatrix} 1{,}4444 & 0 & 0{,}6667 \\ 0 & 5{,}3333 & 1{,}3333 \\ 0{,}6667 & 1{,}3333 & 5{,}3333 \end{bmatrix} \cdot 10^4 \cdot \{r\} \quad \Rightarrow \{r\} = \begin{Bmatrix} -106 \\ -3{,}54 \\ 14{,}2 \end{Bmatrix} \cdot 10^{-4}$$

SCRIPT – MATLAB

```
>> k=10/0.001;
>> Kg_c=Kg+[k 0 0;0 0 0;0 0 0];
>> F_c=[-144 0 0]';
>> r_c=inv(Kg_c)*F_c;
>> r_c*10^4
ans =
   -106.2295
   -3.5410
   14.1639
```

Esforços

$$\{S\}_{e1} = \{S_0\}_{e1} + [k_{e1}]\{u_{e1}\} = \begin{Bmatrix} 72 \\ 72 \\ 72 \\ -72 \end{Bmatrix} + [k_{e1}] \begin{Bmatrix} 0 \\ 0 \\ -106 \\ -3{,}54 \end{Bmatrix} \cdot 10^{-4}$$

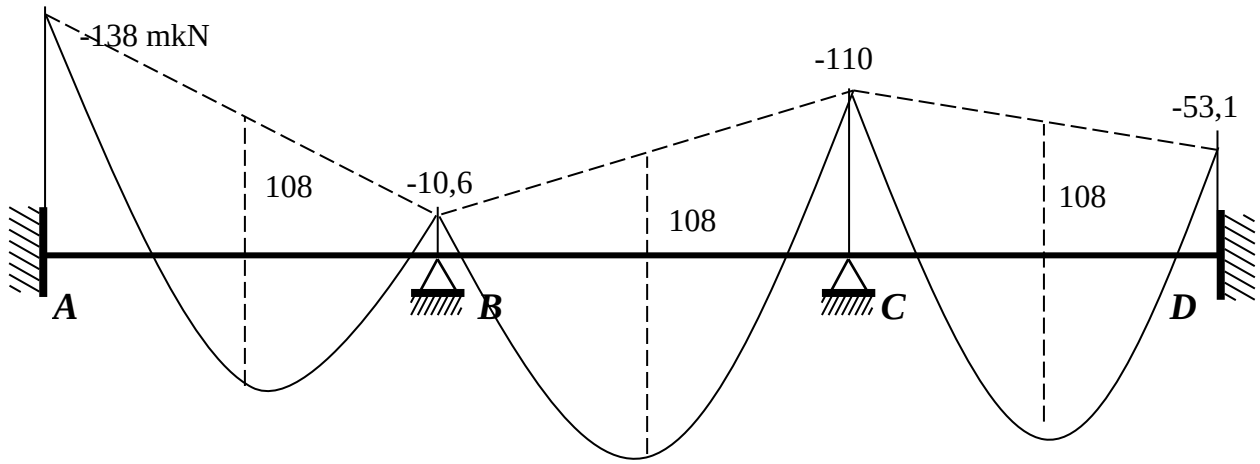
$$\{S\}_{e2} = \{S_0\}_{e2} + [k_{e2}]\{u_{e2}\} = \begin{Bmatrix} 72 \\ 72 \\ 72 \\ -72 \end{Bmatrix} + [k_{e2}] \begin{Bmatrix} -106 \\ -3{,}54 \\ 0 \\ 14{,}2 \end{Bmatrix} \cdot 10^{-4}$$

$$\{S\}_{e3} = \{S_0\}_{e3} + [k_{e3}]\{u_{e3}\} = \begin{Bmatrix} 72 \\ 72 \\ 72 \\ -72 \end{Bmatrix} + [k_{e3}] \begin{Bmatrix} 0 \\ 14{,}2 \\ 0 \\ 0 \end{Bmatrix} \cdot 10^{-4}$$

SCRIPT – MATLAB

```
>> S0=[72 72 72 -72]';
>> S1=S0+k1*[0 0 r_c(1) r_c(2)]'
S1 =
   93.2459
  138.0984
   50.7541
  -10.6230
>> S2=S0+k2*[r_c(1) r_c(2) 0 r_c(3)]'
S2 =
   55.4754
   10.6230
   88.5246
 -109.7705
>> S3=S0+k3*[0 r_c(3) 0 0 ]'
S3 =
   81.4426
  109.7705
   62.5574
  -53.1148
```

DMF:



d) O DMF da viga quando solicitada por um recalque vertical de 2 cm no apoio B.

Equilíbrio:

$$\{F\}=[K]\cdot\{r\} \qquad \Rightarrow \begin{Bmatrix} R_1 \\ 0 \\ 0 \end{Bmatrix} = \begin{bmatrix} 0{,}4444 & 0 & 0{,}6667 \\ 0 & 5{,}3333 & 1{,}3333 \\ 0{,}6667 & 1{,}3333 & 5{,}3333 \end{bmatrix} \cdot 10^4 \cdot \begin{Bmatrix} -0{,}02 \\ r_2 \\ r_3 \end{Bmatrix}$$

$$\Rightarrow \{r\} = \begin{Bmatrix} r_2 \\ r_3 \end{Bmatrix} = \begin{bmatrix} 5{,}3333 & 1{,}3333 \\ 1{,}3333 & 5{,}3333 \end{bmatrix}^{-1} \left(\begin{Bmatrix} 0 \\ 0 \end{Bmatrix} - \begin{bmatrix} 0 \\ 0{,}6667 \end{bmatrix} \cdot \{0{,}02\} \right) = \begin{Bmatrix} 6{,}67 \\ -26{,}7 \end{Bmatrix} \cdot 10^{-4}$$

SCRIPT – MATLAB

```
>> r_d=inv(Kg(2:3,2:3))*(-Kg(2:3,1)*(-0.02));
>> r_d*10^4
ans =
   -6.6667
   26.6667
```

Esforços

$$\{S\}_{e1} = \{S_0\}_{e1} + [k_{e1}]\{u_{e1}\} = \begin{Bmatrix} 72 \\ 72 \\ 72 \\ -72 \end{Bmatrix} + [k_{e1}] \begin{Bmatrix} 0 \\ 0 \\ -0{,}02 \\ -6{,}67 \cdot 10^{-4} \end{Bmatrix}$$

$$\{S\}_{e2} = \{S_0\}_{e2} + [k_{e2}]\{u_{e2}\} = \begin{Bmatrix} 72 \\ 72 \\ 72 \\ -72 \end{Bmatrix} + [k_{e2}] \begin{Bmatrix} -0{,}02 \\ -6{,}67 \cdot 10^{-4} \\ 0 \\ 26{,}7 \cdot 10^{-4} \end{Bmatrix}$$

$$\{S\}_{e3} = \{S_0\}_{e3} + [k_{e3}]\{u_{e3}\} = \begin{Bmatrix} 72 \\ 72 \\ 72 \\ -72 \end{Bmatrix} + [k_{e3}] \begin{Bmatrix} 0 \\ 26{,}7 \cdot 10^{-4} \\ 0 \\ 0 \end{Bmatrix}$$

SCRIPT – MATLAB

```
S1=k1*[0 0 -0.02 r_d(1)]'
S1 =
   40.0000
  124.4444
  -40.0000
  115.5556


S2=k2*[-0.02 r_d(1) 0 r_d(2)]'
S2 =
  -31.1111
 -115.5556
   31.1111
  -71.1111

S3=k3*[0 r_d(2) 0 0 ]'
S3 =

   17.7778
   71.1111
  -17.7778
   35.5556
```

DMF: