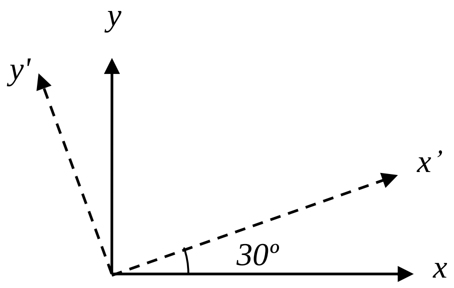
Teoria da Elasticidade – Prova 1/2014 – GABARITO

3ª Questão (3,0):

Seja o estado de deformação de uma chapa em um determinado ponto dado por:

$$\varepsilon_{xyz} = \begin{bmatrix} 40 & -15 & 0 \\ -15 & -10 & 0 \\ 0 & 0 & 0 \end{bmatrix} \cdot 10^{-6}$$



Pede-se:

a. (1,0) As deformações principais;

$$det\left(\varepsilon_{xyz} - \varepsilon_e I\right) = \begin{vmatrix} 40-\varepsilon_e & -15 & 0 \\ -15 & -10-\varepsilon_e & 0 \\ 0 & 0 & -\varepsilon_e \end{vmatrix} = 0$$

$$-\varepsilon_e\left[(40-\varepsilon_e)(-10-\varepsilon_e)-15^2\right] = 0$$

$$\Rightarrow \varepsilon_e\left[\varepsilon_e^2 - 30\varepsilon_e - 625\right] = 0$$

$$\Rightarrow \begin{cases} \varepsilon_e = 0 \\ \varepsilon_e = \dfrac{30 \pm \sqrt{30^2 + 4\cdot 625}}{2} \end{cases}$$

$$\Rightarrow \begin{cases} \varepsilon_e = 0 \\ \varepsilon_e = 44{,}15\cdot 10^{-6} \\ \varepsilon_e = -14{,}15\cdot 10^{-6} \end{cases}$$

b. (1,0) o alongamento unitário (deformação linear específica) nas direções x' e y'

$$\varepsilon_{x'} = \varepsilon_x \ell^2 + \varepsilon_y m^2 + \varepsilon_z n^2 + \gamma_{xy}\ell m + \gamma_{xz}\ell n + \gamma_{yz} mn$$

onde: $\begin{cases} \ell = \cos(x'x) = \cos(30°) = \dfrac{\sqrt{3}}{2} \\ m = \cos(x'y) = \cos(60°) = \dfrac{1}{2} \\ n = \cos(x'z) = \cos(90°) = 0 \end{cases}$ e $\begin{cases} \varepsilon_x = 40 \\ \varepsilon_y = -10 \\ \varepsilon_{xy} = \dfrac{\gamma_{xy}}{2} = -15 \\ \varepsilon_z = \varepsilon_{xz} = \varepsilon_{yz} = 0 \end{cases} \cdot 10^{-6}$

$$\Rightarrow \varepsilon_{x'} = \left[40 \cdot \left(\frac{\sqrt{3}}{2}\right)^2 - 10 \cdot \left(\frac{1}{2}\right)^2 + 0 - 30\left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{2}\right) + 0 + 0 \right] \cdot 10^{-6}$$

$$\Rightarrow \varepsilon_{x'} = \left[30 - 2{,}5 - 30\frac{\sqrt{3}}{4} \right] \cdot 10^{-6}$$

$\Rightarrow \varepsilon_{x'} = 14{,}51 \cdot 10^{-6}$ m/m

$$\varepsilon_{y'} = \varepsilon_x \ell^2 + \varepsilon_y m^2 + \varepsilon_z n^2 + \gamma_{xy}\ell m + \gamma_{xz}\ell n + \gamma_{yz} mn$$

onde: $\begin{cases} \ell = \cos(y'x) = -\cos(60°) = -\dfrac{1}{2} \\ m = \cos(y'y) = \cos(30°) = \dfrac{\sqrt{3}}{2} \\ n = \cos(y'z) = \cos(90°) = 0 \end{cases}$ e $\begin{cases} \varepsilon_x = 40 \\ \varepsilon_y = -10 \\ \varepsilon_{xy} = \dfrac{\gamma_{xy}}{2} = -15 \\ \varepsilon_z = \varepsilon_{xz} = \varepsilon_{yz} = 0 \end{cases} \cdot 10^{-6}$

$$\Rightarrow \varepsilon_{y'} = \left[40 \cdot \left(-\frac{1}{2}\right)^2 - 10 \cdot \left(\frac{\sqrt{3}}{2}\right)^2 + 0 - 30\left(-\frac{1}{2}\right)\left(\frac{\sqrt{3}}{2}\right) + 0 + 0 \right] \cdot 10^{-6}$$

$$\Rightarrow \varepsilon_{y'} = \left[10 - 7{,}5 + 30\frac{\sqrt{3}}{4} \right] \cdot 10^{-6}$$

$\Rightarrow \varepsilon_{y'} = 15{,}50 \cdot 10^{-6}$ m/m

c. (1,0) a distorção no plano $x'y'$.

$$\gamma_{x'y'} = 2\varepsilon_x \ell_{x'} \ell_{y'} + 2\varepsilon_y m_{x'} m_{y'} + 2\varepsilon_z n_{x'} n_{y'} + \gamma_{xy}\left(\ell_{x'} m_{y'} + m_{x'} \ell_{y'}\right) + \\ + \gamma_{xz}\left(\ell_{x'} n_{y'} + n_{x'} \ell_{y'}\right) + \gamma_{yz}\left(m_{x'} n_{y'} + n_{x'} m_{y'}\right)$$

Para o estado plano observado:

$$\Rightarrow \gamma_{x'y'} = 2\varepsilon_x \ell_{x'} \ell_{y'} + 2\varepsilon_y m_{x'} m_{y'} + \gamma_{xy}\left(\ell_{x'} m_{y'} + m_{x'} \ell_{y'}\right)$$

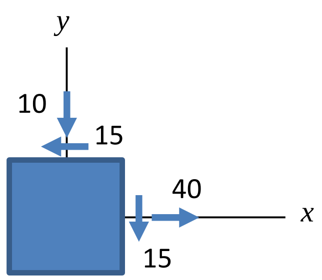
$$\Rightarrow \gamma_{x'y'} = \left[2\cdot(40)\cdot\left(\frac{\sqrt{3}}{2}\right)\cdot\left(-\frac{1}{2}\right) + 2\cdot(-10)\cdot\left(\frac{1}{2}\right)\cdot\left(\frac{\sqrt{3}}{2}\right) + (-30)\cdot\left(\left(\frac{\sqrt{3}}{2}\right)\cdot\left(\frac{\sqrt{3}}{2}\right) + \left(\frac{1}{2}\right)\cdot\left(-\frac{1}{2}\right)\right)\right]\cdot 10^{-6}$$

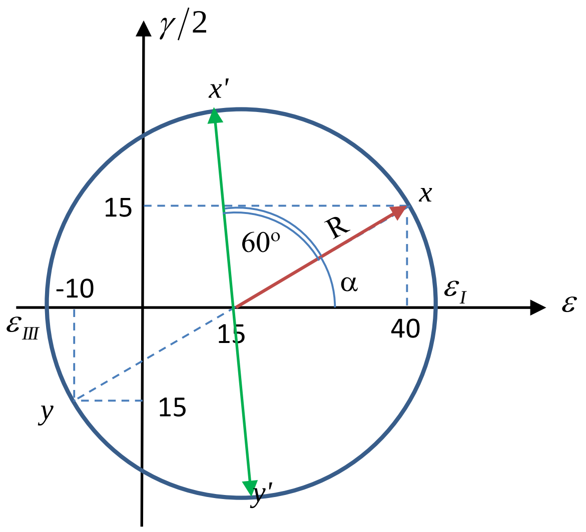
$$\Rightarrow \gamma_{x'y'} = \left[-20\sqrt{3} - 5\sqrt{3} - 15\right]\cdot 10^{-6}$$

$$\Rightarrow \gamma_{x'y'} = -58{,}30\cdot 10^{-6} \text{ rad}$$

Solução gráfica:

$$\Rightarrow R = \sqrt{15^2 + 25^2} = 29{,}15$$
$$\Rightarrow \varepsilon_I = 15 + 29{,}15 = 44{,}15\cdot 10^{-6}$$
$$\Rightarrow \varepsilon_{II} = 0$$
$$\Rightarrow \varepsilon_{III} = 15 - 29{,}15 = -14{,}15\cdot 10^{-6}$$
$$\Rightarrow \alpha = \arctan(0{,}6) = 30{,}96°$$
$$\Rightarrow \varepsilon_{x'} = 15 - 29{,}15\cdot\sin(0{,}96°) = 14{,}51\cdot 10^{-6}$$
$$\Rightarrow \varepsilon_{y'} = 15 + 29{,}15\cdot\sin(0{,}96°) = 15{,}49\cdot 10^{-6}$$
$$\Rightarrow \gamma_{x'y'} = 2\cdot 29{,}15\cdot\cos(0{,}96°) = 58{,}29\cdot 10^{-6}$$





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4ª Questão (1,0):

Se $T_{ij} = -T_{ji}$ e $S_{ij} = S_{ji}$, utilizando a notação indicial mostre que $T_{kl}S_{kl} = 0$

Para o caso $j = i$, tem-se $T_{ii} = -T_{ii}$, ou seja: $T_{11} = T_{22} = T_{33} = 0$

Logo, desenvolvendo o monômio, tem-se:

$$T_{kl}S_{kl} = \quad T_{k1}S_{k1} + T_{k2}S_{k2} + T_{k3}S_{k3} =$$

$$= T_{11}S_{11} + T_{21}S_{21} + T_{31}S_{31} + T_{12}S_{12} + T_{22}S_{22} + T_{32}S_{32} + T_{13}S_{13} + T_{23}S_{23} + T_{33}S_{33} =$$

$$= \cancel{0 \cdot S_{11}} + T_{21}S_{21} + T_{31}S_{31} + T_{12}S_{12} + \cancel{0 \cdot S_{22}} + T_{32}S_{32} + T_{13}S_{13} + T_{23}S_{23} + \cancel{0 \cdot S_{33}} =$$

$$= T_{21}S_{21} + T_{31}S_{31} + T_{12}S_{12} + T_{32}S_{32} + T_{13}S_{13} + T_{23}S_{23} =$$

$$= \cancel{T_{21}S_{21}} + \cancel{T_{31}S_{31}} - \cancel{T_{21}S_{21}} + \cancel{T_{32}S_{32}} - \cancel{T_{31}S_{31}} - \cancel{T_{32}S_{32}} =$$

$$= 0$$

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Formulário - Deformações:

$$\begin{cases} \varepsilon_x = \dfrac{\partial u}{\partial x} \\ \varepsilon_y = \dfrac{\partial v}{\partial y} \\ \varepsilon_z = \dfrac{\partial w}{\partial z} \end{cases} \qquad \begin{cases} \gamma_{xy} = \dfrac{\partial u}{\partial y} + \dfrac{\partial v}{\partial x} \\ \gamma_{xz} = \dfrac{\partial u}{\partial z} + \dfrac{\partial w}{\partial x} \\ \gamma_{yz} = \dfrac{\partial v}{\partial z} + \dfrac{\partial w}{\partial y} \end{cases}$$

$$\varepsilon_s = \varepsilon_x \ell^2 + \varepsilon_y m^2 + \varepsilon_z n^2 + \gamma_{xy} \ell m + \gamma_{xz} \ell n + \gamma_{yz} mn$$

$$\gamma_{st} = 2\varepsilon_x \ell_s \ell_t + 2\varepsilon_y m_s m_t + 2\varepsilon_z n_s n_t + \gamma_{xy}\left(\ell_s m_t + m_s \ell_t\right) + \\ + \gamma_{xz}\left(\ell_s n_t + n_s \ell_t\right) + \gamma_{yz}\left(m_s n_t + n_s m_t\right)$$

$$J_1 = \varepsilon_x + \varepsilon_y + \varepsilon_z$$

$$J_2 = \begin{vmatrix} \varepsilon_x & \varepsilon_{xy} \\ \varepsilon_{yx} & \varepsilon_y \end{vmatrix} + \begin{vmatrix} \varepsilon_x & \varepsilon_{xz} \\ \varepsilon_{zx} & \varepsilon_z \end{vmatrix} + \begin{vmatrix} \varepsilon_y & \varepsilon_{yz} \\ \varepsilon_{zy} & \varepsilon_z \end{vmatrix}$$

$$J_3 = \begin{vmatrix} \varepsilon_x & \varepsilon_{xy} & \varepsilon_{xz} \\ \varepsilon_{yx} & \varepsilon_y & \varepsilon_{yz} \\ \varepsilon_{zx} & \varepsilon_{zy} & \varepsilon_z \end{vmatrix}$$

$$\begin{cases} \dfrac{\partial^2 \varepsilon_x}{\partial y^2} + \dfrac{\partial^2 \varepsilon_y}{\partial x^2} - \dfrac{\partial^2 \gamma_{xy}}{\partial x \partial y} = 0 \\ \dfrac{\partial^2 \varepsilon_x}{\partial z^2} + \dfrac{\partial^2 \varepsilon_z}{\partial x^2} - \dfrac{\partial^2 \gamma_{xz}}{\partial x \partial z} = 0 \\ \dfrac{\partial^2 \varepsilon_y}{\partial z^2} + \dfrac{\partial^2 \varepsilon_z}{\partial y^2} - \dfrac{\partial^2 \gamma_{yz}}{\partial y \partial z} = 0 \end{cases} \qquad \begin{cases} 2\dfrac{\partial^2 \varepsilon_x}{\partial y \partial z} = \dfrac{\partial^2 \gamma_{xz}}{\partial x \partial y} + \dfrac{\partial^2 \gamma_{xy}}{\partial x \partial z} - \dfrac{\partial^2 \gamma_{yz}}{\partial x^2} \\ 2\dfrac{\partial^2 \varepsilon_y}{\partial x \partial z} = \dfrac{\partial^2 \gamma_{yx}}{\partial y \partial z} + \dfrac{\partial^2 \gamma_{yz}}{\partial y \partial x} - \dfrac{\partial^2 \gamma_{xz}}{\partial y^2} \\ 2\dfrac{\partial^2 \varepsilon_z}{\partial x \partial y} = \dfrac{\partial^2 \gamma_{zx}}{\partial z \partial y} + \dfrac{\partial^2 \gamma_{zy}}{\partial z \partial x} - \dfrac{\partial^2 \gamma_{xy}}{\partial z^2} \end{cases}$$