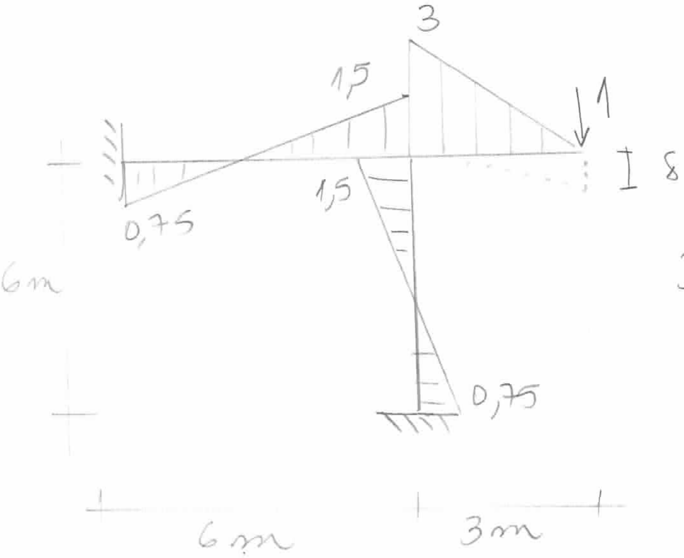
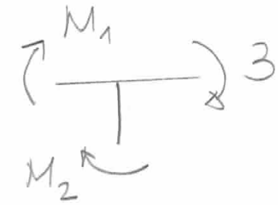
1) Solução



DMF

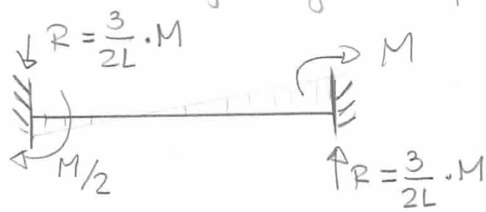
Obs. do Processo de Cross:



"O momento aplicado num nó se distribui em parcelas proporcionais à rigidez"

mesma rigidez $\Rightarrow M_1 = M_2 = 1,5$

Na barra bi-engastada:



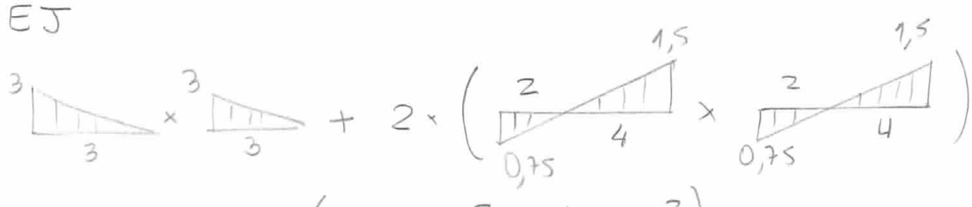
"transmissão do momento" $= -\frac{1}{2}$

Cálculo de δ:

P.T.V.: $W_{ext} = W_{int}$

$$\bar{P} \cdot \delta = \int \frac{M \bar{M}}{EJ} dx$$

$\Rightarrow 1 \cdot \delta \cdot EJ =$



$\Rightarrow$

$$\Rightarrow EJ \cdot \delta = \frac{3}{3} \cdot 3 \cdot 3 + 2 \times \left(+ \frac{2}{3} \times 0,75^2 + \frac{4}{3} \times 1,5^2 \right) = 15,75$$

$$\delta = 1,94 \cdot 10^{-4} \, m$$

$$k = \frac{1}{\delta} = 5143 \, kN/m$$

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{5143}{2,5}} = 45,4 \text{ rad/s}$$

$$f = \frac{\omega}{2\pi} = \boxed{7,22 \text{ Hz}}$$

$$T = \frac{1}{f} = 0,138 \text{ s}$$

$$x(t) = \rho \cos(\omega t + \phi) =$$

$$= \rho(\cos \omega t \cdot \cos\phi + \text{sen}\, \omega t \cdot \text{sen}\phi)$$

mas $x_0 = x(0) = \rho \cos\phi$

e $v_0 = x'(0) = \omega \rho \,\text{sen}\,\phi$

$$\Rightarrow \boxed{x(t) = x_0 \cos(45,4t) + \frac{v_0}{\omega} \text{sen}(45,4t)}$$

2) Solução

$$\delta \equiv \underset{\text{logarítmico}}{\text{decremento}} = \ln \frac{x_0}{x_1} = \ln \frac{0{,}506}{0{,}406} = 0{,}22$$

$$\xi \cong \frac{\delta}{2\pi} = 0{,}0355 \Rightarrow 3{,}55\%$$

$$T = 1{,}4\,s \Rightarrow \omega_D = \frac{2\pi}{1{,}4} = 4{,}49 \text{ rad/s}$$

$$\omega_D = \omega \cdot \sqrt{1-\xi^2} = \omega \cdot \sqrt{0{,}999} \simeq \omega$$

$$\Rightarrow \omega = 4{,}49 \text{ rad/s} \quad (a)$$

$$c = \xi \cdot 2 m \omega = 277 \frac{t}{s} = 282 \text{ kgf} \cdot \frac{s}{cm}$$

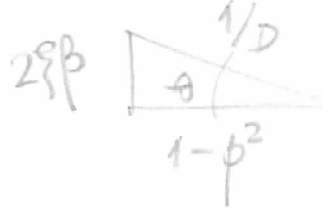
onde $m = \frac{k}{\omega^2} = \frac{1}{4{,}49^2} \cdot \frac{9.072}{0{,}508} = 885{,}8 \underbrace{\frac{\text{kgf} \cdot s^2}{cm}}_{kg \cdot 9{,}81 \frac{m}{s^2} \cdot \frac{s^2}{10^{-2} m}} = 869 \text{ ton}$

$$\delta = \frac{1}{6} \ln \frac{x_0}{x_6} = \frac{1}{6} \cdot \ln \frac{0{,}506}{x_6}$$

$$\Rightarrow x_6 = 0{,}506 \cdot e^{-6\delta} = 0{,}135 \text{ cm} \quad (c)$$

3) solução (Obs: $\rho = \frac{p_0}{k} \cdot D = \frac{p_0}{k} \frac{\cos\theta}{1-\beta^2}$)

$$\rho(\theta) = \frac{P_0}{k} \cdot \frac{\cos\theta}{1-\beta^2}$$



$$\Rightarrow k(1-\beta^2) = k - \overline{\omega}^2 m = \frac{p_0 \cos\theta}{\rho}$$

$$\Rightarrow \begin{bmatrix} 1 & -16^2 \\ 1 & -25^2 \end{bmatrix} \cdot \begin{Bmatrix} k \\ m \end{Bmatrix} = 226{,}8 \begin{Bmatrix} \frac{0{,}966}{18{,}3\times10^{-3}} \\ \frac{0{,}574}{36{,}8\times10^{-3}} \end{Bmatrix}$$

$$\Rightarrow k = 17{,}8 \times 10^3 \ kgf/cm$$

$$m = 22{,}95 \ kgf \cdot s^2/cm = 22{,}5 \ t$$

$$\omega = \sqrt{\frac{k}{m}} = 27{,}9 \ rad/s$$

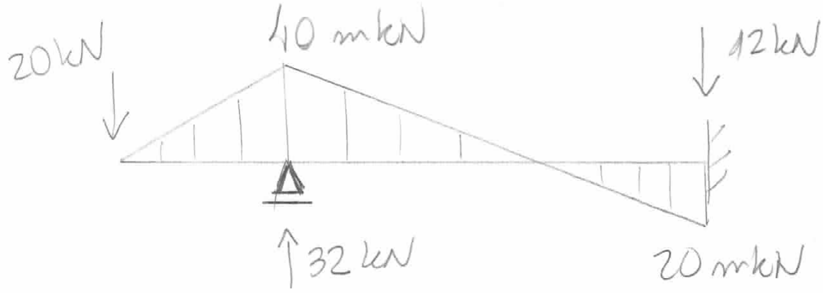
mas $\xi = \dfrac{p_0 \operatorname{sen}\theta}{2\beta k \rho} = \dfrac{p_0 \operatorname{sen}\theta}{c_c \overline{\omega} \rho}$

$$\Rightarrow c = \xi \cdot c_c = \frac{p_0 \operatorname{sen}\theta}{\overline{\omega}\rho} = 200{,}9 \ kgf \cdot s/cm$$

$$\xi \equiv \frac{c}{c_c} = \frac{c}{2m\omega} = \frac{c}{2k/\omega} = 15{,}7\ \%$$

4) Solução

DMF estático:



(transmissibilidade estática)

Sistema de 1GL: $k = \frac{P}{\delta} = \frac{20}{10^{-3}} = 20 \cdot 10^{3}$ kN/m

Considerando $\xi = 0$ $\rightarrow$ $\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{20 \cdot 10^3}{2}} = 100$ rad/s $\cong \omega_D$

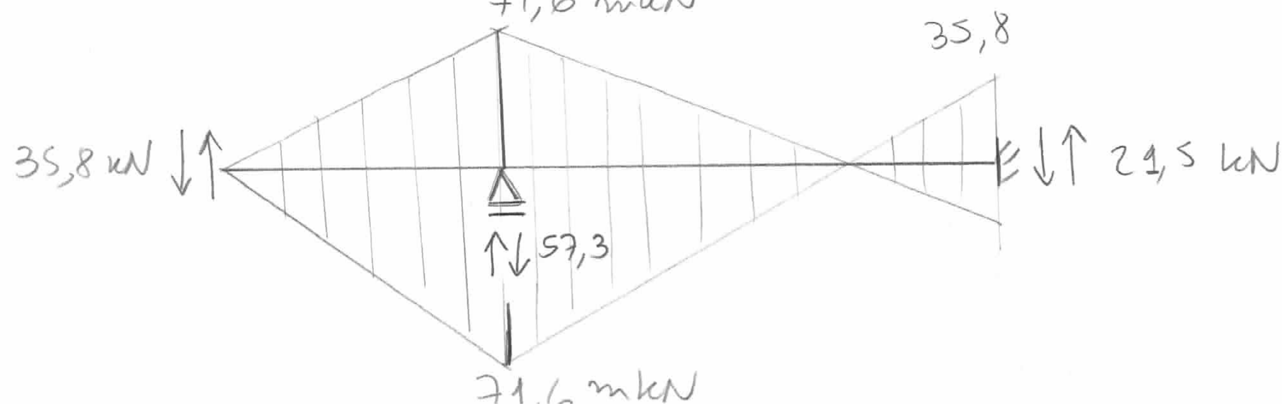
$\bar{\omega} = \frac{900 \cdot 2\pi}{60} = 94$ rad/s

$\beta = \frac{94}{100} = 0{,}94$ (razão entre frequências)

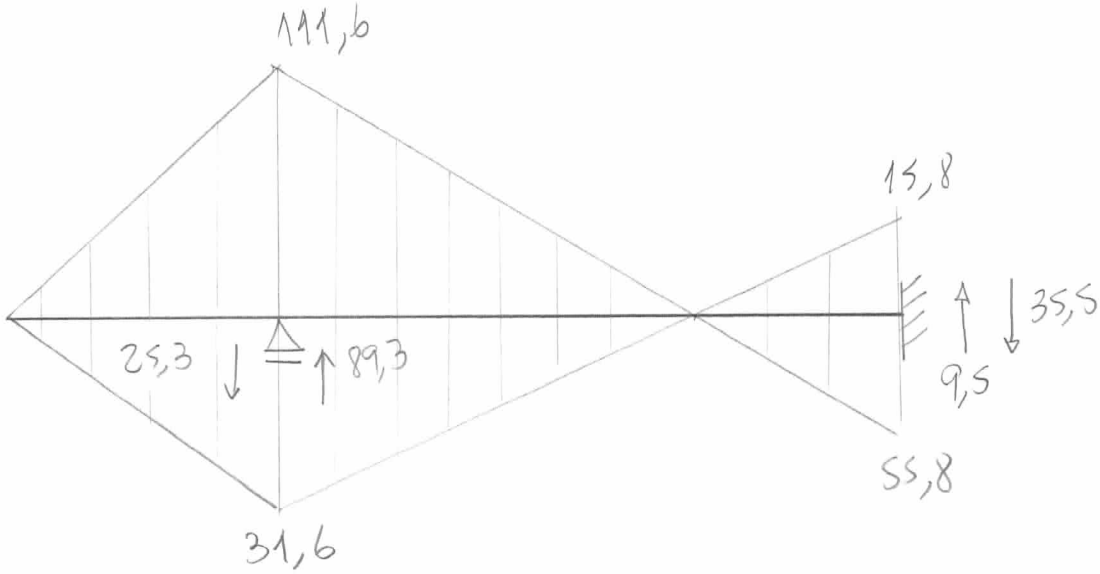
$D = \frac{1}{1-\beta^2} = 8{,}95$ (fator de amplif. din.)

$x_{est} = \frac{P_0}{k} = \frac{4}{20 \cdot 10^3} = 2 \times 10^{-4}$ m (deflexão estática)

$F_D \equiv$ amplitude da força dinâmica $\overset{= k \cdot x_{max}}{=} P_0 \times D = 4 \times 8{,}95 = 35{,}8$ kN



Envoltória final:



5) Solução:

$$x = \bar{x} + x_s$$

Equação do movimento livre não-amortecido:

$$\underset{\substack{\uparrow \\ \text{absoluto}}}{M \cdot x''} + k \underset{\substack{\uparrow \\ \text{relativo}}}{\bar{x}} = 0 \qquad \text{mas } x'' = \bar{x}'' + x_s''$$

$$\Rightarrow M\bar{x}'' + Mx_s'' + k\bar{x} = 0$$

$$\Rightarrow M\bar{x}'' + k\bar{x} = -Mx_s''$$

onde $x_s = X_s \operatorname{sen} \bar{\omega}t \Rightarrow x_s'' = -\bar{\omega}^2 X_s \operatorname{sen} \bar{\omega}t$

$$\Rightarrow M\bar{x}'' + k\bar{x} = \underbrace{M\bar{\omega}^2 X_s}_{P_0} \operatorname{sen} \bar{\omega}t$$

(solução idêntica a carregamento harmônico)

$$\bar{\omega} = 500 \times \frac{2\pi}{60} = 52{,}36 \text{ rad/s}$$

$$X_s = 0{,}8 \times 10^{-3} \text{ m}$$

$$P_0 = M\bar{\omega}^2 X_s = 2 \cdot 52{,}36^2 \cdot 0{,}8 \times 10^{-3} = 4{,}39 \text{ kN}$$

$$\beta = \frac{\bar{\omega}}{\omega} = \frac{52{,}36}{60{,}0} = 0{,}873$$

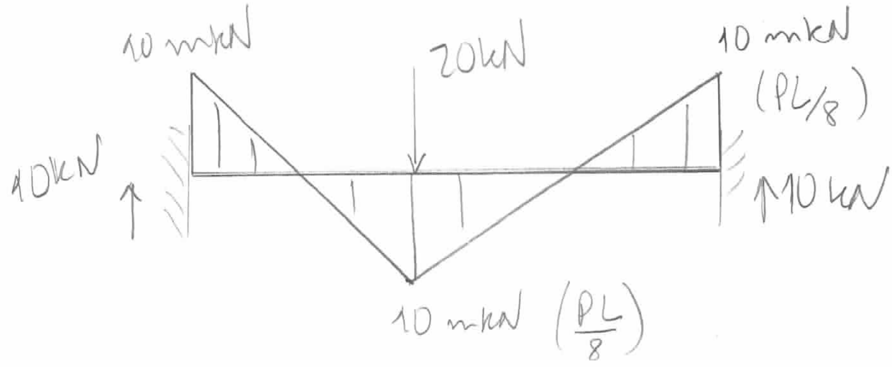
onde $\omega = \sqrt{\frac{k}{M}} = \sqrt{\frac{7200}{2}} = 60 \text{ rad/s}$

onde $k = \dfrac{20 \text{ kN}}{2{,}778 \times 10^{-3} \text{ m}} = 7200 \text{ kN/m}$

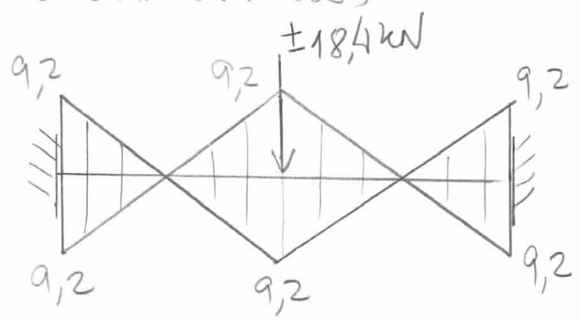
$D \equiv$ fator de amplif dinâmica $(\xi = 0)$ $= \dfrac{1}{1-\beta^2} = 4,2$

$F_D \equiv$ Amplitude da força dinâmica $= p_0 \cdot D = 4,39 \times 4,2 = 18,4$ kN

DMF estático (relações da barra bi engastada)



DMF efeitos dinâmicos



Envoltória final: