

### GABARITO - EXERCÍCIO PARA A VF #1

a) Mostre que se  $[K] = [F]^{-1}$ , então  $\left\{ \frac{1}{\omega^2} I - [F][M] \right\} \phi = 0$ ;

$$\begin{aligned} & \{[K] - \omega^2[M]\} \phi = 0 \\ \Rightarrow & \frac{1}{\omega^2} [K]^{-1} \{[K] - \omega^2[M]\} \phi = 0 \\ \Rightarrow & \left\{ \frac{1}{\omega^2} I - [K]^{-1}[M] \right\} \phi = 0 \\ \Rightarrow & \left\{ \frac{1}{\omega^2} I - [F][M] \right\} \phi = 0 \end{aligned}$$

b) As frequências naturais, em Hz;

$$[F] = \frac{1}{768} \cdot \frac{L^3}{EI} \begin{bmatrix} 9 & 11 & 7 \\ 11 & 16 & 11 \\ 7 & 11 & 9 \end{bmatrix} = \begin{bmatrix} 10,125 & 12,375 & 7,850 \\ 12,375 & 18,000 & 12,375 \\ 7,850 & 12,375 & 10,125 \end{bmatrix} \cdot 10^{-4} m/kN$$

$$[M] = \begin{bmatrix} 4/3 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 4/3 \end{bmatrix} \cdot 10^3 kg$$

$$[F] \cdot [M] = \begin{bmatrix} 1,3500 & 1,2375 & 1,0500 \\ 1,6500 & 1,8000 & 1,6500 \\ 1,0500 & 1,2375 & 1,3500 \end{bmatrix} \cdot 10^{-3}$$

$$\|\lambda \cdot I - [F] \cdot [M]\| = 0 \quad \left\| \begin{array}{ccc} \lambda - 0,00135 & -1,2375 \cdot 10^{-3} & -1,0500 \cdot 10^{-3} \\ -1,65 \cdot 10^{-3} & \lambda - 0,0018 & -1,65 \cdot 10^{-3} \\ -1,0500 \cdot 10^{-3} & -1,2375 \cdot 10^{-3} & \lambda - 0,00135 \end{array} \right\| = 0$$

$$\Rightarrow \lambda^3 - 0,0045\lambda^2 + 1,496\lambda \cdot 10^{-6} - 7,087 \cdot 10^{-11} = 0 \quad (\text{no Matlab: função } poly)$$

$$\Rightarrow \begin{cases} \lambda_1 = 0,00414 \\ \lambda_2 = 0,000300 \\ \lambda_3 = 0,0000570 \end{cases} \quad (\text{no Matlab, função } roots \text{ ou função } eig)$$

$$\Rightarrow \begin{cases} \omega_1 = 15,5 \\ \omega_2 = 57,7 \\ \omega_3 = 132,4 \end{cases} \quad rad/s \Rightarrow \begin{cases} f_1 = 2,47 \\ f_2 = 9,19 \\ f_3 = 21,1 \end{cases} \quad Hz$$

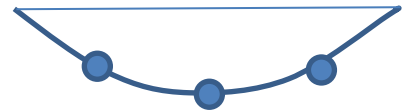
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c) As formas modais.

$$\begin{bmatrix} \lambda - 0,00135 & -1,2375 \cdot 10^{-3} & -1,0500 \cdot 10^{-3} \\ -1,65 \cdot 10^{-3} & \lambda - 0,0018 & -1,65 \cdot 10^{-3} \\ -1,0500 \cdot 10^{-3} & -1,2375 \cdot 10^{-3} & \lambda - 0,00135 \end{bmatrix} \begin{Bmatrix} X_1 \\ X_2 \\ X_3 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix}$$

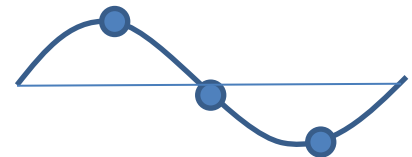
$$\Rightarrow \lambda_1 = 0,00414 \Rightarrow \begin{bmatrix} \lambda - 0,00135 & -1,2375 \cdot 10^{-3} & -1,0500 \cdot 10^{-3} \\ -1,65 \cdot 10^{-3} & \lambda - 0,0018 & -1,65 \cdot 10^{-3} \\ -1,0500 \cdot 10^{-3} & -1,2375 \cdot 10^{-3} & \lambda - 0,00135 \end{bmatrix} \begin{Bmatrix} X_1 \\ X_2 \\ X_3 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix}$$

$$\Rightarrow \phi_1 = \begin{Bmatrix} 1,00 \\ 1,409 \\ 1,00 \end{Bmatrix} \Rightarrow \phi_1 = \begin{Bmatrix} 0,71 \\ 1,00 \\ 0,71 \end{Bmatrix}$$



$$\Rightarrow \lambda_2 = 0,000300 \Rightarrow \begin{bmatrix} \lambda - 0,00135 & -1,2375 \cdot 10^{-3} & -1,0500 \cdot 10^{-3} \\ -1,65 \cdot 10^{-3} & \lambda - 0,0018 & -1,65 \cdot 10^{-3} \\ -1,0500 \cdot 10^{-3} & -1,2375 \cdot 10^{-3} & \lambda - 0,00135 \end{bmatrix} \begin{Bmatrix} X_1 \\ X_2 \\ X_3 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix}$$

$$\Rightarrow \phi_2 = \begin{Bmatrix} 1,00 \\ 0,0 \\ -1,00 \end{Bmatrix}$$



$$\Rightarrow \lambda_3 = 0,0000570 \Rightarrow \begin{bmatrix} \lambda - 0,00135 & -1,2375 \cdot 10^{-3} & -1,0500 \cdot 10^{-3} \\ -1,65 \cdot 10^{-3} & \lambda - 0,0018 & -1,65 \cdot 10^{-3} \\ -1,0500 \cdot 10^{-3} & -1,2375 \cdot 10^{-3} & \lambda - 0,00135 \end{bmatrix}$$

$$\Rightarrow \phi_3 = \begin{Bmatrix} 1,00 \\ -1,89 \\ 1,00 \end{Bmatrix} \Rightarrow \phi_3 = \begin{Bmatrix} -0,528 \\ 1,000 \\ -0,528 \end{Bmatrix}$$

