SEÇÃO DE ENSINO DE ENGENHARIA DE FORTIFICAÇÃO E CONSTRUÇÃO

Dinâmica das Estruturas

Aula #5

Carregamento Súbito, impulsivo e qualquer

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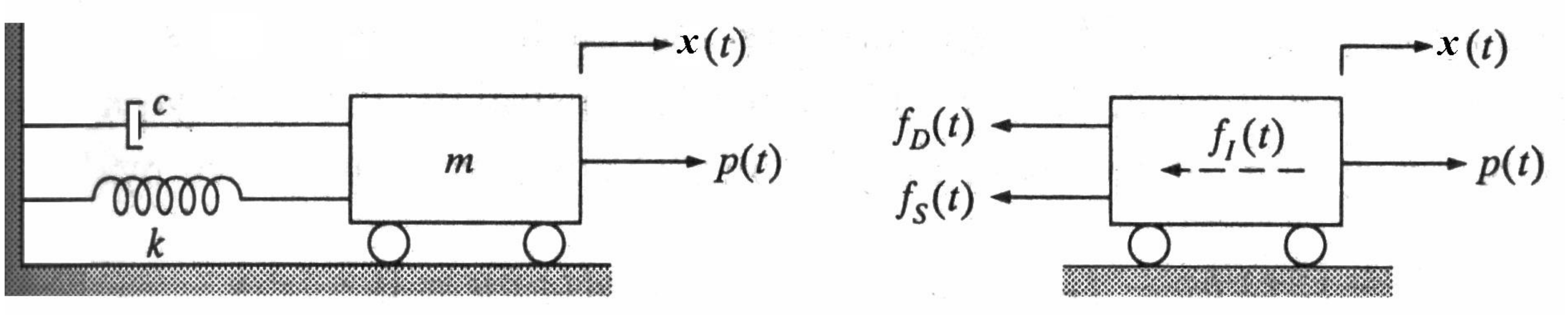
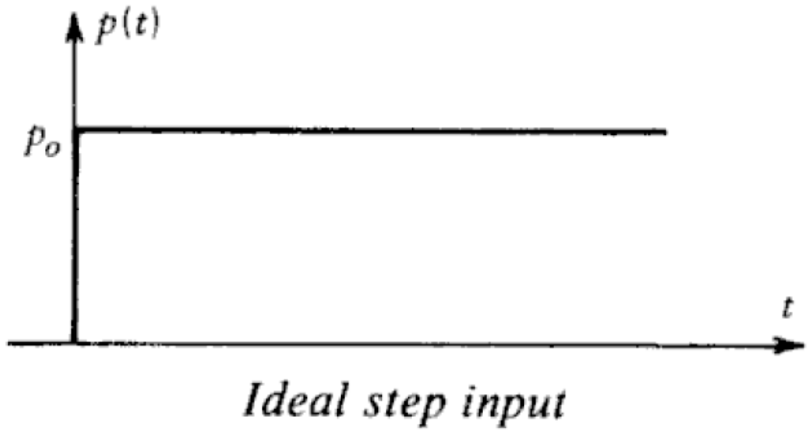
Equação do Equilíbrio Dinâmico



$$f_I + f_D + f_S = p$$

Carregamento: $p(t) = p_0$



Resposta a um carregamento súbito

$$m\,\ddot{x}(t) + c\,\dot{x}(t) + k\,x(t) = p_0$$

1) Solução homogênea: vibração livre amortecida

$$x_h(t) = (A\cos\omega_D\ t + B\sin\omega_D\ t)\,e^{-\xi\,\omega\,t}$$

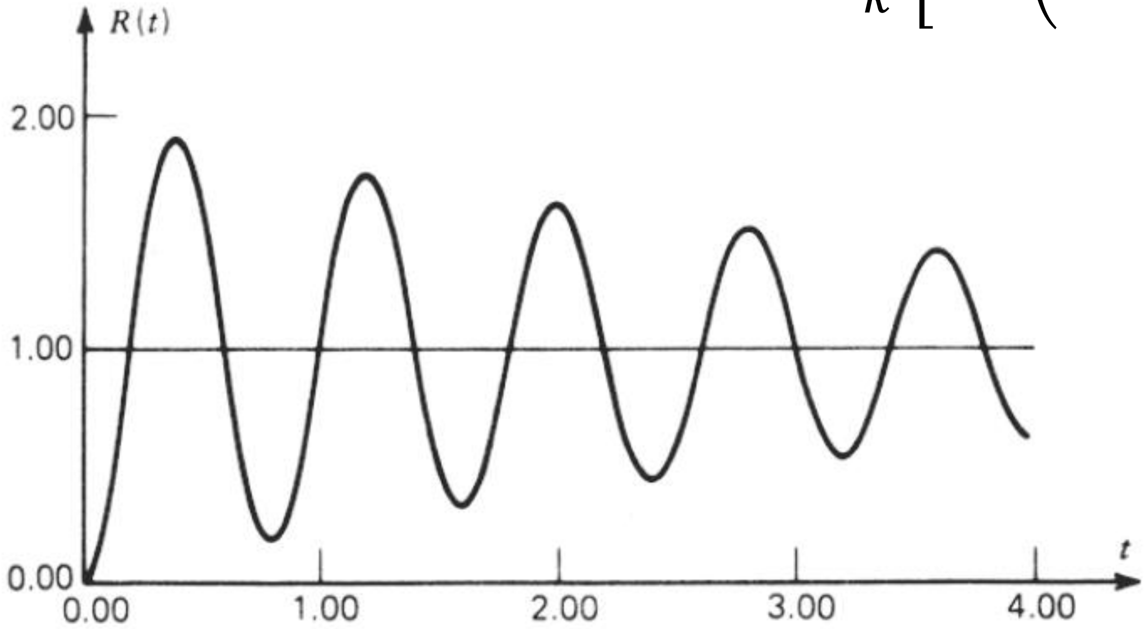
2) Solução particular: consiste da deflexão estática:

$$x_p(t) = \frac{p_0}{k}$$

Resposta a um carregamento súbito

Solução geral para $x(0) = \dot{x}(0) = 0$:

$$x(t) = x_h(t) + x_p(t) = \frac{p_0}{k}\left[1 - \left(\cos\omega_D\ t + \frac{\xi\omega}{\omega_D}\,\text{sen}\,\omega_D\ t\right)e^{-\xi\omega t}\right]$$



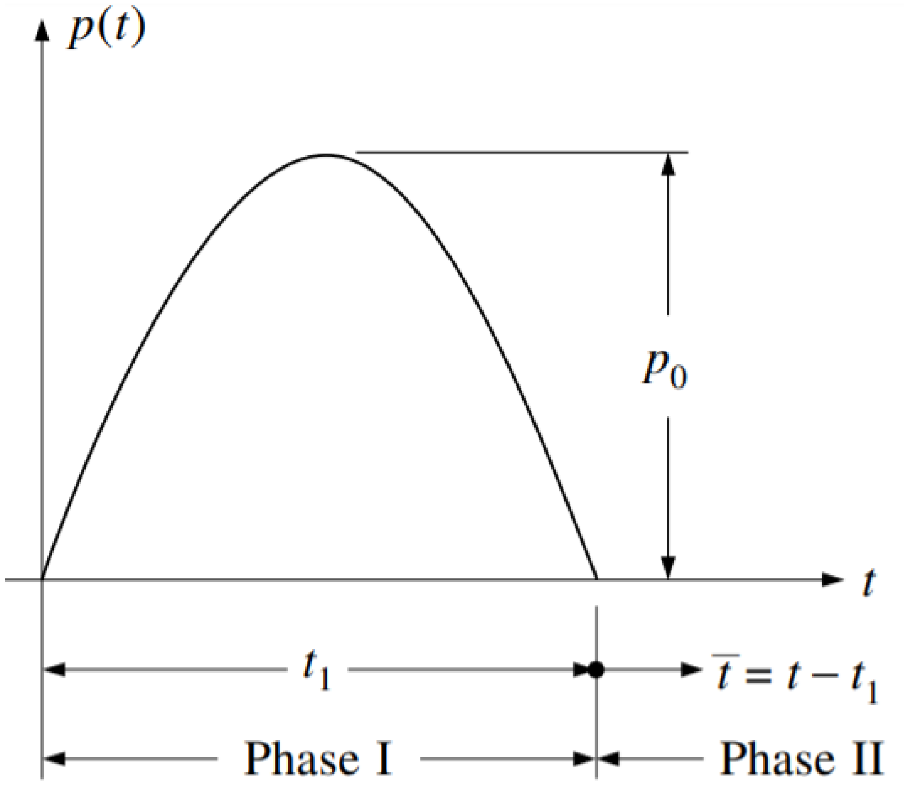
Response ratio plot for a step input.

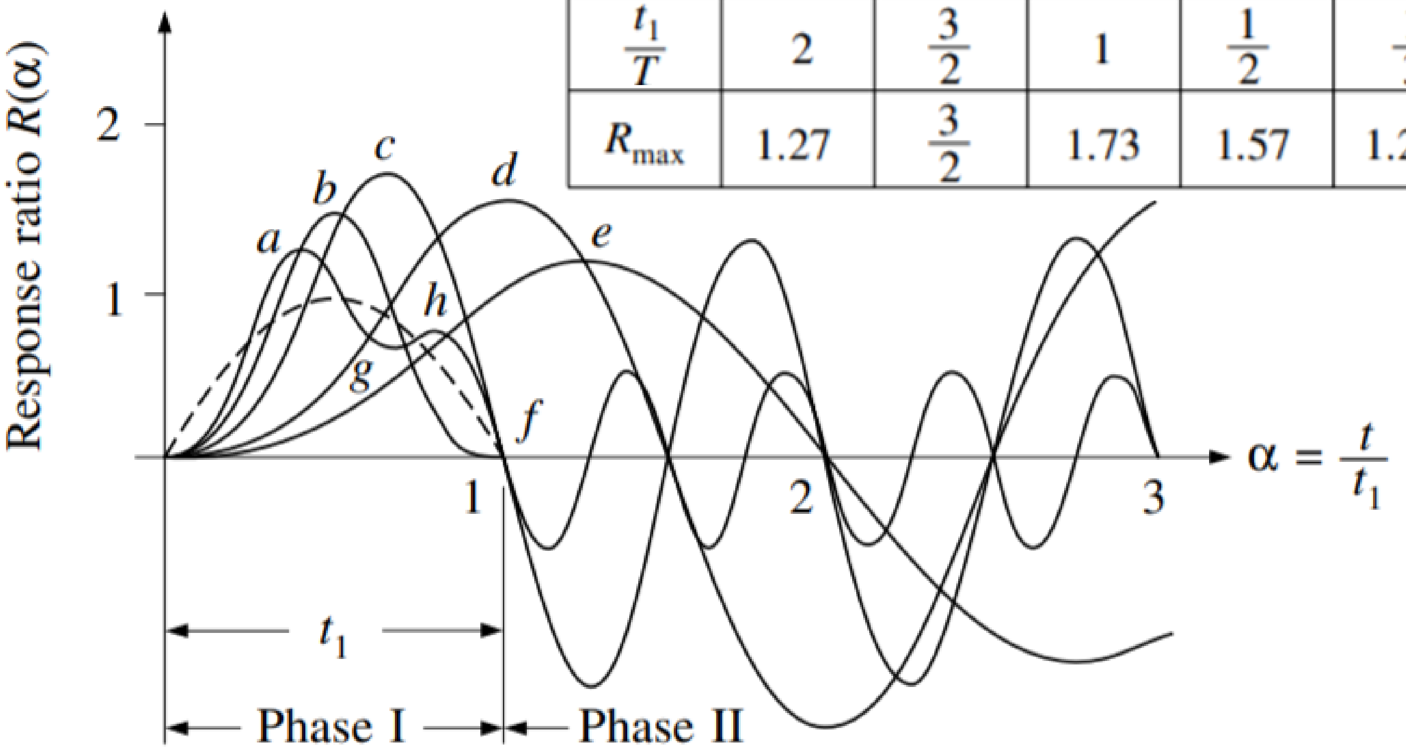
Resposta NÃO-AMORTECIDA:

$$x(t) = \frac{p_0}{k}[1 - \cos\omega_D\ t]$$

$$x_{max} = 2\delta_{est}$$

Resposta a um carregamento impulsivo





pt.	a	b	c	d	e
$\frac{t_1}{T}$	2	$\frac{3}{2}$	1	$\frac{1}{2}$	$\frac{1}{3}$
R_{max}	1.27	$\frac{3}{2}$	1.73	1.57	1.20

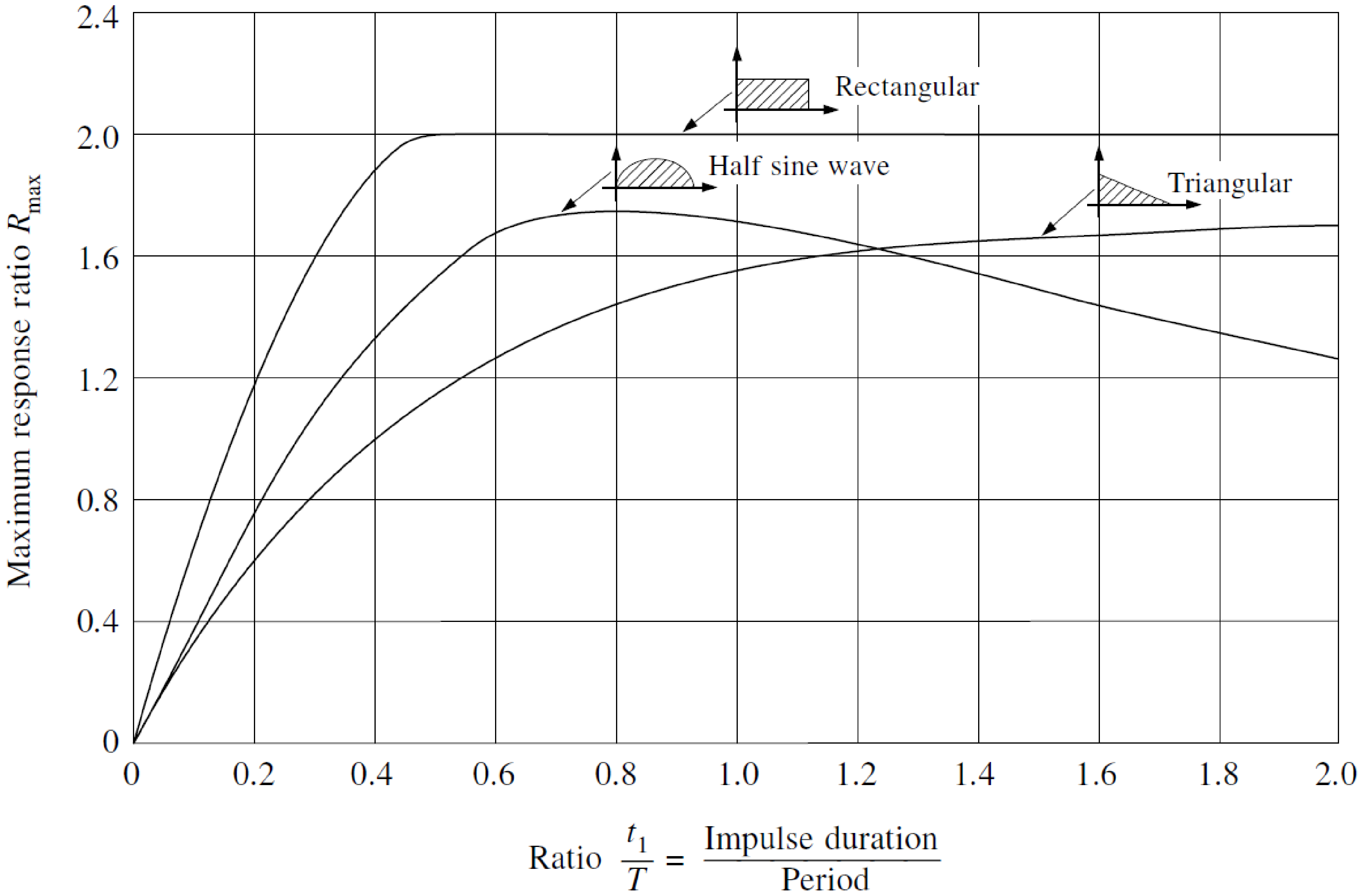
Response ratios due to half-sine pulse.

$$R(t) \equiv \frac{v(t)}{v_{st}} = \frac{v(t)}{p_o/k}$$

Ref.: Clough, R. W., Penzien, J., "Dynamics of Structures", McGraw-Hill, 2nd Ed., 1993.

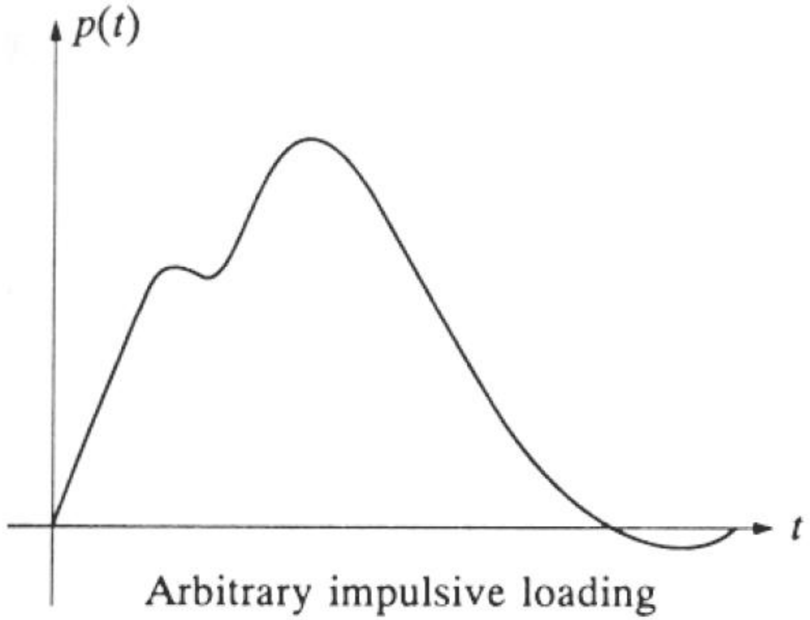
Resposta a um carregamento impulsivo

$$R(t) \equiv \frac{v(t)}{v_{st}} = \frac{v(t)}{p_o/k}$$



Ref.: Clough, R. W., Penzien, J., "Dynamics of Structures", McGraw-Hill, 2nd Ed., 1993.

Resposta a um carregamento impulsivo



Arbitrary impulsive loading

- Resposta máxima atingida muito rapidamente;
- As forças de amortecimento não absorvem energia significativa do sistema;
- Considera-se este tipo de resposta como não-amortecida;
- Aplicando-se o Teorema do Impulso:

$$m\Delta\dot{x} = \int_0^{t_1} [p(t) - kx(t)]\, dt$$

Resposta a um carregamento impulsivo

$$m\Delta\dot{x} = \int_0^{t_1} [p(t) - kx(t)]\, dt \qquad \Rightarrow m\Delta\dot{x} = \int_0^{t_1} p(t)\, dt - \int_0^{t_1} kx(t)\, dt$$

$$t_1 \longrightarrow 0 \quad \Rightarrow m\Delta\dot{x} = \int_0^{t_1} p(t)\, dt - \int_0^{t_1} kx(t)\, dt \qquad \Longrightarrow m\Delta\dot{x} \cong \int_0^{t_1} p(t)\, dt$$

$para\ t_1\ curto$

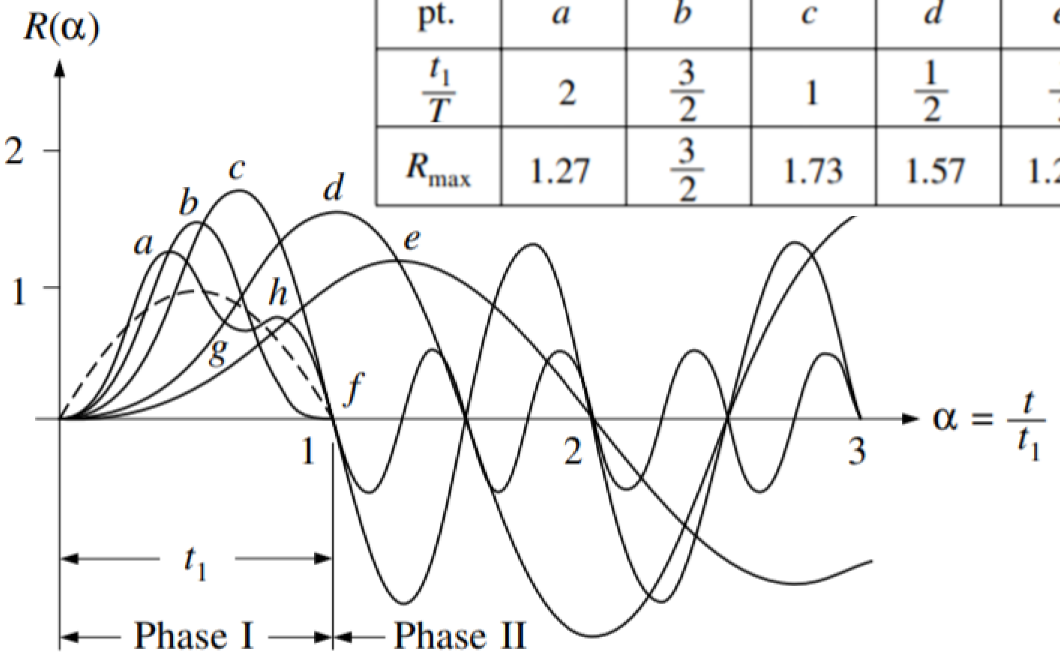
$$\bar{t} = t - t_1 \qquad \Longrightarrow \boxed{x(\bar{t}) \cong \frac{1}{m\omega}\left(\int_0^{t_1} p(t)dt\right)\sin(\omega t)}$$

Resposta a um carregamento impulsivo

(1) For long-duration loadings, for example, $t_1/T > 1$, the dynamic magnification factor depends principally on the rate of increase of the load to its maximum value. A step loading of sufficient duration produces a magnification factor of 2; a very gradual increase causes a magnification factor of 1.

(2) For short-duration loads, for example, $t_1/T < 1/4$, the maximum displacement amplitude $v_{\max}$ depends principally upon the magnitude of the applied impulse $I = \int_0^{t_1} p(t)\ dt$ and is not strongly influenced by the form of the loading impulse. The maximum response ratio $R_{\max}$ is, however, quite dependent upon the form of loading because it is proportional to the ratio of impulse area to peak-load amplitude, as may be noted by comparing the curves of Fig. 5-6 in the short-period range. Thus $v_{\max}$ is the more significant measure of response.

pt.	a	b	c	d	e
$\frac{t_1}{T}$	2	$\frac{3}{2}$	1	$\frac{1}{2}$	$\frac{1}{3}$
$R_{\max}$	1.27	$\frac{3}{2}$	1.73	1.57	1.20



$$R(t) \equiv \frac{v(t)}{v_{st}} = \frac{v(t)}{p_o/k}$$

Ref.: Clough, R. W., Penzien, J., "Dynamics of Structures", McGraw-Hill, 2nd Ed., 1993.

Carregamento impulsivo - exemplo

Example E5-2. As an example of the use of this approximate formula, consider the response of the structure shown in Fig. E5-2 to the impulsive loading indicated. In this case, $\omega = \sqrt{kg/W} = 3.14\ rad/sec$ and $\int_0^{t_1} p(t)\, dt = 10\ kip \cdot sec$. The response then is approximately

$$v(\bar{t}) = \frac{10\ (386)}{2,000\ (3.14)}\ \sin \omega \bar{t}$$

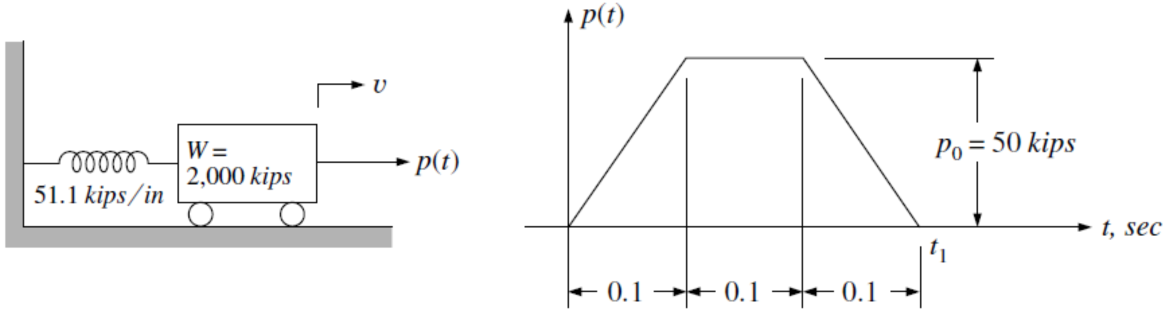


FIGURE E5-2
Approximate impulse-response analysis.

in which the acceleration of gravity is taken as $g = 386\ in/sec^2$ $[980.7\ cm/sec^2]$. The maximum response results when $\sin \omega \bar{t} = 1$, that is,

$$v_{\max} \doteq 0.614\ in \quad [1.56\ cm]$$

The maximum elastic force developed in the spring, which is of major concern to the structural engineer, is

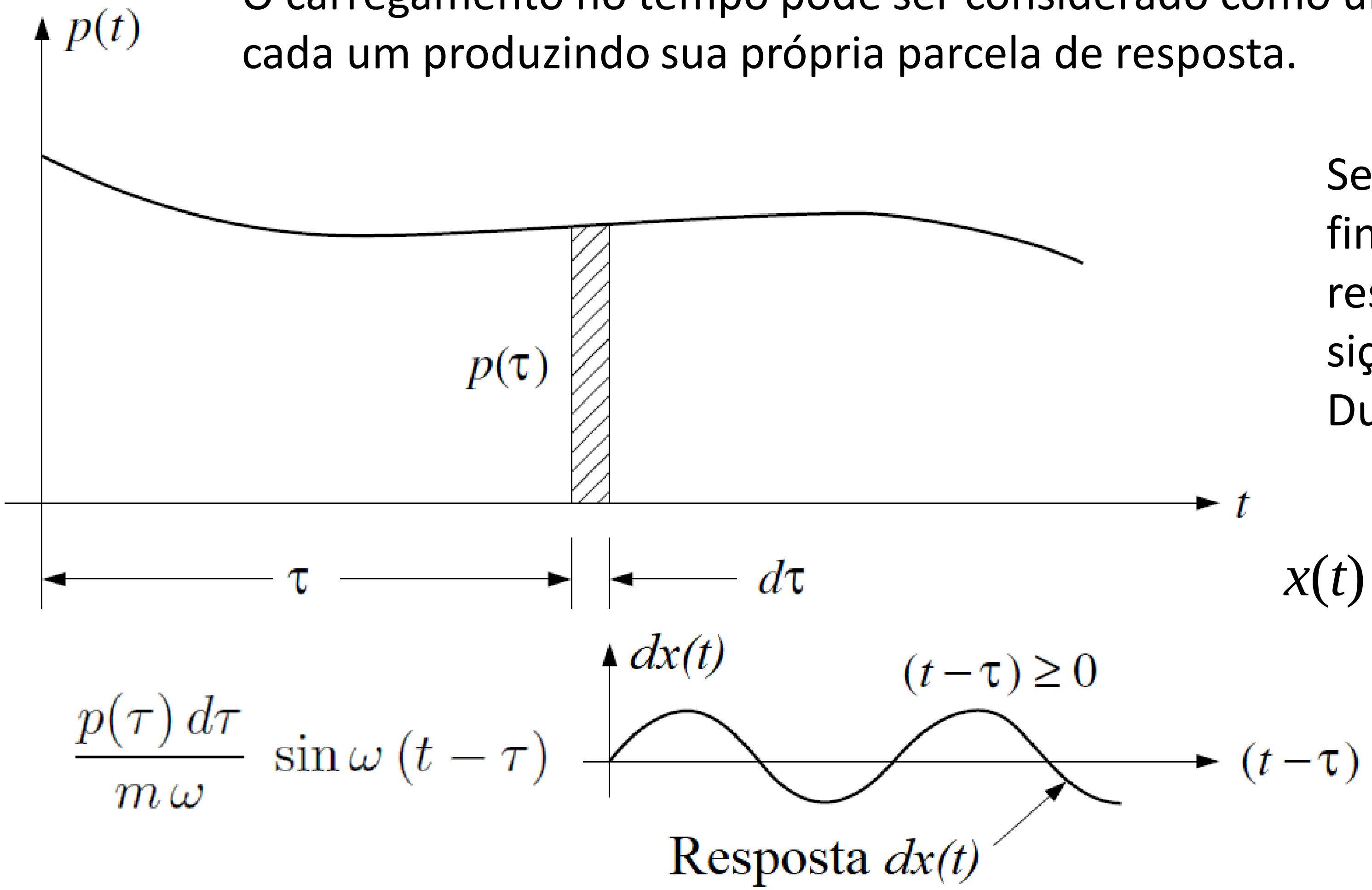
$$f_{S,\max} = k\ v_{\max} = 51.1\ (0.614) = 31.4\ kips \quad [14,240\ kg]$$

Since the period of vibration of this system is $T = 2\pi/\omega = 2\ sec$, the ratio of load duration to period is $t_1/T = 0.15$; thus, the approximate analysis in this case is quite accurate. In fact, the exact maximum response determined by direct integration of the equation of motion is $0.604\ in$ $[1.53\ cm]$, and so the error in the approximate result is less than 2 percent.

Ref.: Clough, R. W., Penzien, J., "Dynamics of Structures", McGraw-Hill, 2nd Ed., 1993.

Carregamento qualquer – Integral de Duhamel

O carregamento no tempo pode ser considerado como uma sucessão de pequenos impulsos, cada um produzindo sua própria parcela de resposta.



Sendo o regime elástico e linear, a resposta final pode ser obtida pela soma de todas as respostas diferenciais (método da superposição), obtendo-se a expressão da Integral de Duhamel (Convolução):

$$x(t) = \frac{1}{m\,\omega} \int_0^t p(\tau)\ \sin\omega\,(t-\tau)\ d\tau$$

Carregamento qualquer – Integral de Duhamel

$$x(t) = \frac{1}{m\,\omega} \int_0^t p(\tau)\;\sin\omega\,(t-\tau)\;d\tau$$

$$t \geq 0$$

Considerando:

$$\sin(\omega t - \omega\tau) = \left[\sin\omega t\;\cos\omega\tau - \cos\omega t\;\sin\omega\tau\right]$$

Então:

$$x(t) = \sin\omega t\left[\frac{1}{m\,\omega}\int_0^t p(\tau)\;\cos\omega\tau\;d\tau\right] - \cos\omega t\left[\frac{1}{m\,\omega}\int_0^t p(\tau)\;\sin\omega\tau\;d\tau\right]$$

$$x\,(t) = \left[\overline{A}(t)\;\sin\omega t - \overline{B}(t)\;\cos\omega t\right]$$

$$\overline{A}(t) \equiv \frac{1}{m\,\omega}\int_0^t p(\tau)\;\cos\omega\tau\;d\tau$$

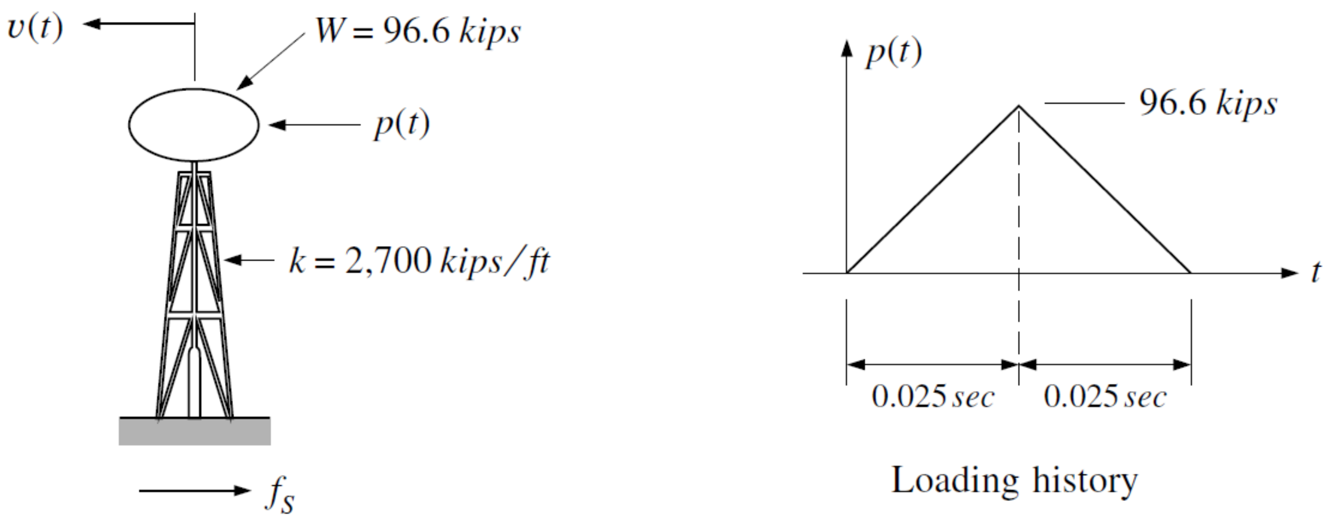
$$\overline{B}(t) \equiv \frac{1}{m\,\omega}\int_0^t p(\tau)\;\sin\omega\tau\;d\tau$$

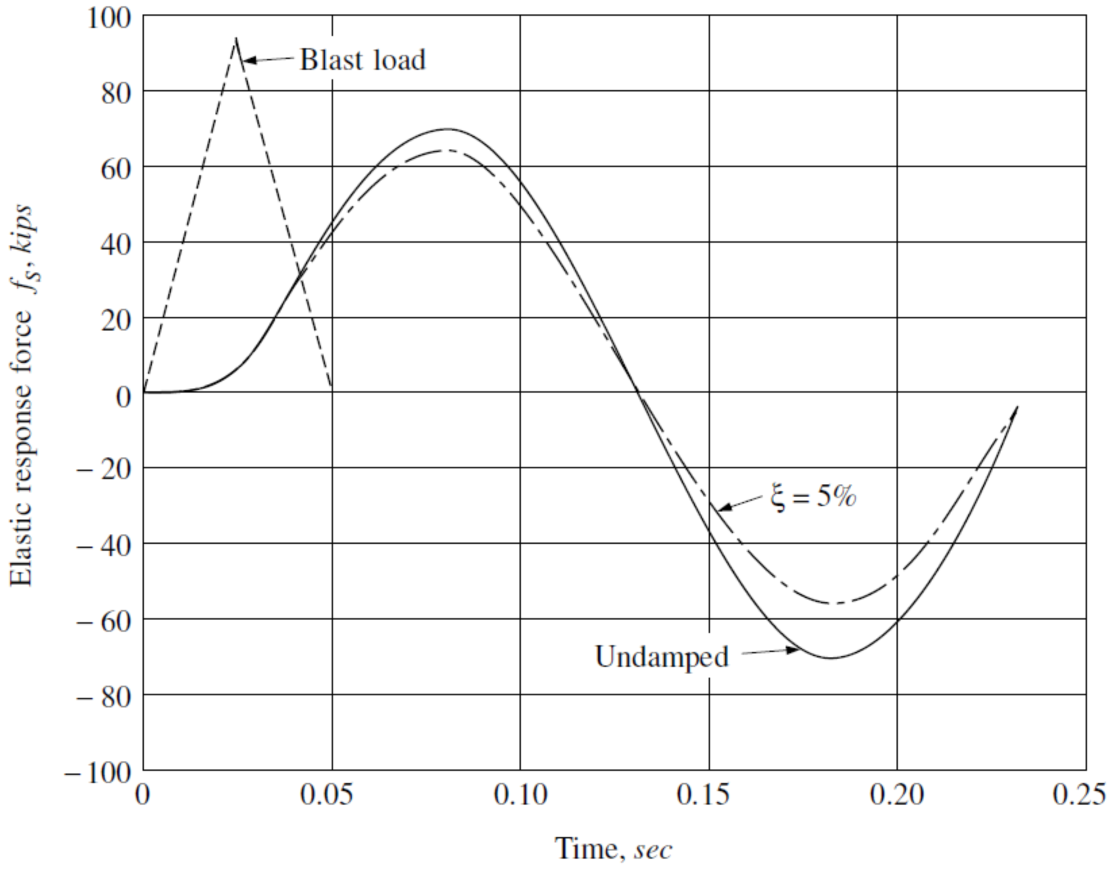
Geralmente faz-se uso de métodos numéricos para a resolução desta integral.

Carregamento qualquer - Exemplo

Example E6-1. The dynamic response of a water tower subjected to a blast loading will now be presented to illustrate the above numerical procedure for obtaining undamped response through the time domain in accordance with Eq. (6-14). The idealizations of the structure and blast loading are shown in Fig. E6-1. For this system, the vibration frequency and period are

$$\omega = \sqrt{\frac{kg}{W}} = \sqrt{\frac{2,700\ (32.2)}{96.6}} = 30\ rad/sec \qquad T = \frac{2\,\pi}{\omega} = 0.209\ sec$$





Ref.: Clough, R. W., Penzien, J., "Dynamics of Structures", McGraw-Hill, 2nd Ed., 1993.

Carregamento qualquer – Exemplo

Solução numérica, realizada no aplicativo MathCad®:

$k := 39400$ $m := 43.8$

$\omega := \sqrt{\frac{k}{m}}$ $\omega = 29.99$

$$p(t) := \begin{cases} \left(\frac{430}{0.025}\cdot t\right) & \text{if } t < 0.025 \\ \left(860 - \frac{430}{0.025}\cdot t\right) & \text{if } 0.025 \le t \le 0.05 \\ 0 & \text{otherwise} \end{cases}$$

$\xi := 0.0$

$\omega d := \omega\cdot\sqrt{1-\xi^2}$ $\omega d = 29.99$

$$A(t) := \frac{1}{m\cdot\omega d\cdot e^{\xi\cdot\omega\cdot t}}\cdot\int_0^t p(\tau)\cdot e^{\xi\cdot\omega\cdot\tau}\cdot\cos(\omega d\cdot\tau)\,d\tau$$

$$B(t) := \frac{1}{m\cdot\omega d\cdot e^{\xi\cdot\omega\cdot t}}\cdot\int_0^t p(\tau)\cdot e^{\xi\cdot\omega\cdot\tau}\cdot\sin(\omega d\cdot\tau)\,d\tau$$

$$x(t) := A(t)\cdot\sin(\omega d\cdot t) - B(t)\cdot\cos(\omega d\cdot t)$$

$\xi := 0.05$

$\omega d := \omega\cdot\sqrt{1-\xi^2}$ $\omega d = 29.95$

$$A(t) := \frac{1}{m\cdot\omega d\cdot e^{\xi\cdot\omega\cdot t}}\cdot\int_0^t p(\tau)\cdot e^{\xi\cdot\omega\cdot\tau}\cdot\cos(\omega d\cdot\tau)\,d\tau$$

$$B(t) := \frac{1}{m\cdot\omega d\cdot e^{\xi\cdot\omega\cdot t}}\cdot\int_0^t p(\tau)\cdot e^{\xi\cdot\omega\cdot\tau}\cdot\sin(\omega d\cdot\tau)\,d\tau$$

$$xd(t) := A(t)\cdot\sin(\omega d\cdot t) - B(t)\cdot\cos(\omega d\cdot t)$$