

# Chapter 3

## Conventional and Inverse QRD-RLS Algorithms

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**Abstract** This chapter deals with the basic concepts used in the recursive least-squares (RLS) algorithms employing conventional and inverse QR decomposition. The methods of triangularizing the input data matrix and the meaning of the internal variables of these algorithms are emphasized in order to provide details of their most important relations. The notation and variables used herein will be exactly the same used in the previous introductory chapter. For clarity, all derivations will be carried out using real variables and the final presentation of the algorithms (tables and pseudo-codes) will correspond to their complex-valued versions.

### 3.1 The Least-Squares Problem and the QR Decomposition

We start by introducing the weighted least-squares (WLS) filtering problem for the identification of a linear system [1]. To this end, we consider two sets of variables,  $d(\ell)$  and  $x(\ell)$ , and the errors  $\bar{e}(\ell)$ , for  $0 \leq \ell \leq k$ . The set  $d(\ell)$  is the response of an unknown system at time-instant  $\ell$  when the input is the set of variables  $x(\ell)$ , with  $x(\ell) = 0$  for  $\ell < 0$ . The error at time-instant  $\ell$  is defined as  $\bar{e}(\ell) = d(\ell) - \mathbf{w}^T(k)\mathbf{x}(\ell)$ ,  $\mathbf{w}(k)$  being the filter coefficient vector. These errors and the variables  $d(\ell)$  and  $x(\ell)$  are attenuated by the factor  $\lambda^{(k-\ell)/2}$ ,  $0 \ll \lambda < 1$ .

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