On the Performance of Constrained Adaptive Algorithms for Combined Beamforming and AOA Tracking of a Moving Target

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Abstract— **In this paper, we investigate the performance of constrained adaptive algorithms in a beamforming system that is also responsible for the estimation of the angle-of-arrival (AOA) of a moving source. The merits of a number of linearly constrained adaptive algorithms are verified in an assumed far-field condition: the Constrained Least Mean Square (CLMS) algorithm, the Constrained Affine Projection algorithm (CAPA), the Constrained Conjugate Gradient (CCG) algorithm, and the Constrained Recursive Least Square (CRLS) algorithm. In order to have a more suitable scenario for the simulations, a fixed AOA estimation method is used and a moving target model is proposed and tested in our experiments.**

Index Terms— **angle-of-arrival, Capon, adaptive beamformers, tracking, moving target**

I. Introduction

Tracking the angle-of-arrival (AOA) of moving targets is a procedure of great interest in many fields, including: communications, air traffic control, and defense operations [1]-[2]. In communications systems, for instance, an array of antennas can be used in an attempt to effectively reject undesired signals from other directions, while keeping the peak of the radiation pattern pointed to the direction of the impinging main signal. In order to do that, a simple and intuitive approach may be based on AOA estimation methods with an array of sensors, followed by an adaptive beamforming procedure.

AOA estimation by array processing is widely known in the literature. Along the last years, it has gathered even more attention, due to the use of smart antenna techniques for performance improvement in mobile communications (as in the 3G systems, for instance). The main AOA estimation methods comprise different techniques, viz, beamforming approaches [3]-[4], subspace approaches [5]-[7], and maximum likelihood (ML) approaches [8].

The problem that may arise, when the previously mentioned AOA-based tracking procedure is adopted for moving targets, is the time needed for accurate AOA estimations. Most of the AOA estimation methods are based on the so-called "array model" [3], that takes at least a few snapshots of the signal transmitted by the target (corrupted by noise and interference) to provide an acceptable estimate. If the target is moving fast, relatively to the AOA estimation velocity, tracking procedure may present poor performance. Actually, based on such premise, more refined tracking approaches may be found in the literature. In [9], for example, a more elaborated ML approach was introduced as an amendment to overcome alleged deteriorated performance of the traditional techniques when dealing with moving targets.

Despite an adequate tracking of fast moving targets may not be achieved with the AOA-based approach, it is assumed that at least for slow targets that procedure may behave well enough. This work assumes that premise to present simulation results of a beamformer that tracks the corrupted signal of a (slowly) moving target, based on the estimation of the corresponding AOA. The results are analyzed aiming the performance of the beamformer, deployed using four different linearly constrained adaptive algorithms: the Constrained Least Mean Square (CLMS) [12] algorithm, the Constrained Affine Projection algorithm (CAPA) [14], the Constrained Conjugate Gradient (CCG) [15] algorithm, and the Constrained Recursive Least Squares (CRLS) [13] algorithm. Since the focus has been chosen to be on the beamforming, a single AOA estimation method was adopted: Capon [4].

This paper is organized as follows: Section II lists some typical AOA estimation methods. Section III briefly presents a description of the linearly constrained adaptive filtering adopted in this work for beamforming. In the next section, the simulated system setup is described, comprising the application scenario details as well as the beamformer structure. The tracking ability of the simulated structure is pointed out in section V, where a qualitative assessment of the beamformers performance is presented. Conclusions are summarized in section VI.

II. Spatial Spectrum Estimation Techniques

As before mentioned, there are several AOA estimation methods available in the literature. They can be seen as specializations of the generic spectral estimation problem, in which the searched "frequency" is a parameter associated to spatial information sampling, measured with

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multiple sensors properly arranged [3]. The simplest algorithms are the Fourier based ones, like the so-called "conventional" beamforming [1] or Bartlett method [10]. Finer approaches examples are Capon [4], MUSIC (MUltiple SIgnal Classification) [5], and Esprit methods [6].

Bartlett and Capon are among the simplest and widest used non-parametric algorithms. On the other hand, MUSIC is perhaps the most popular among the parametric methods [3]. The advantage of those two non-parametric methods is that they do not assume anything about the signals statistical properties. However, in cases where such information is available or at least when it is likely that those properties may be partially assumed, parametric methods present better performances than the non-parametric ones.

III. Adaptive Beamforming Techniques

In adaptive beamforming [11], an array of sensors (or antennas) is used to provide the maximum reception in a specified direction, based on an estimate of the impinging signal. This "desired" signal is corrupted by noise and usually also by signals at the same frequency, arriving from different directions. The beamforming technique is also able to reject such undesirable interferences; this is carried out by matching radiation pattern nulls to their directions. The weights of each sensor used in the array is then adapted for that purpose at each iteration.

This adaptive approach is efficient in tracking the direction of the interferers but it usually requires some *a priori* information or properties of the desired signal. The use of a "reference" signal, which presents good correlation to the desired one, is a common practice in some systems, for example. The correlation between desired and reference signals influences the beamforming algorithm performance, specially regarding its accuracy and convergence.

Since a reference signal is not always available, an alternative that presents good results is the application of linear restrictions to the weighting vector. Such is the case of the so-called LCMV (Linearly Constrained Minimum Variance) beamformer, which chooses its weights to minimize the output variance of the filter, subject to linear constraints. The LCMV beamformer may be implemented in many different ways, depending on the type of tradeoff that is chosen for the beamformer performance. For example, the Constrained Least Mean Square (CLMS) approach, which was first introduced by Frost in [12], does not require re-initialization and incorporates the constraints into the solution. On the other hand, the Constrained Recursive Least Squares (CRLS) algorithm is a solution that tries to overcome the slow convergence problem experienced by the CLMS algorithm when the input signal is strongly correlated [13]. Besides the CLMS and the CRLS algorithms, intermediary performance algorithms will be used in this work, namely: the Constrained Affine Projection algorithm (CAPA) [14] and the Constrained Conjugate Gradient (CCG) algorithm [15].

[1]It is worth mentioning that the word beamforming is also used to refer to the procedure used to modify the radiation pattern of a sensors array, as can be noticed in the following section.

IV. System Setup

The application scenario thought for the tracking analysis in this work is illustrated in Fig. 1. The main target moves with uniform speed (v) along a linear path, as indicated. Two fixed distinct co-channel interferers were also considered in the scenario. The type of array used was the so-called ULA (Uniform Linear Array). The far-field condition hypothesis was assumed in all simulations. Although such hypothesis is restrictive for some typical application scenarios, it has been pointed out that AOA estimation under that consideration does not present significant errors, contrary to what might be expected [16].

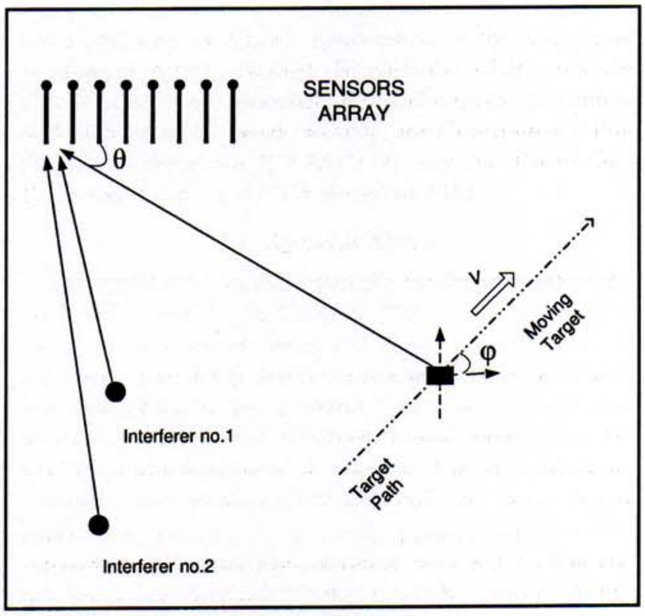

Fig. 1. Moving target AOA tracking scenario (the array is out of scale in the drawing).

The structure implemented to carry out the tracking of the signal emitted by moving target is depicted in Fig. 2. As it can be seen, an array of N sensors carries out a spatial sampling of the impinging signal transmitted from the target (corrupted by noise and interferences). The vector thus generated feeds the AOA estimation block, which computes the main AOAs present (from the target and from the interferers), after a few (time) snapshots of the impinging signal are available. These AOA estimates help to build the constrained matrix needed in the adaptive beamforming algorithm, which will be responsible for lining up the main beam of the array beampattern with the target's AOA. Since the constrained matrix also includes the interferer's AOAs presence, the beamformer is able to minimize their undesirable contribution, putting nulls at those angles in the beampattern.

Regarding the corruption of the signal, it was assumed the presence of additive noise (zero mean white Gaussian) and non-moving interferers operating in the same frequency band of the target signal, (co-channel). It was also assumed that the signal was narrowband, that is, the potential variations of the channel impulse response along the time affected almost equally the spectral content of the signal received in each sensor (flat fading) [1]. Another assumed hypothesis was that the received signal was in baseband.

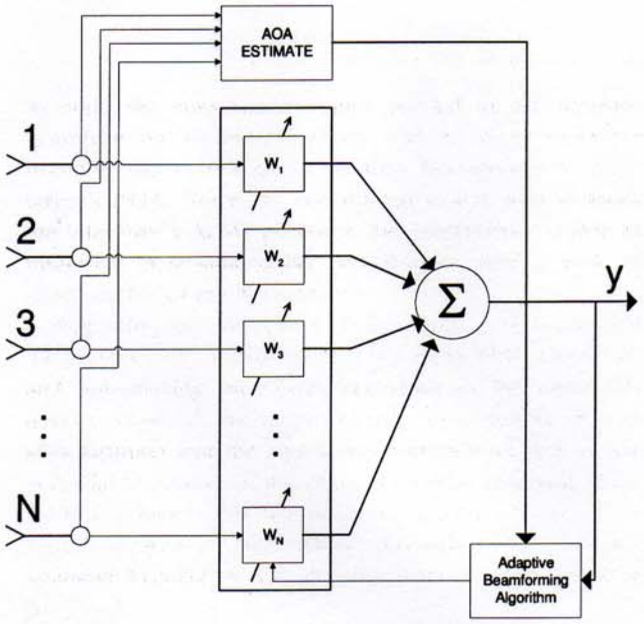

Fig. 2. Analyzed system structure.

A complete simulation run comprised the use of the beamforming structure in Fig. 2 along a limited time period. In this work, a signal "time windowing" scheme was adopted, which may be described as follows. After the algorithm starts, a first window is defined in order to carry out the spatial spectrum (AOA) estimation (time duration ***n1*** in number of samples). In the following instant, the beamformer algorithm is executed, having as reference the AOA estimate of the desired signal, extracted from that spatial spectrum previously processed. The beamforming is kept iterating, with no AOA reference modification, during a second time window (time duration ***n2***), long enough for achieving convergence. This second window is longer than the one needed for the AOA estimation (***n2*** > ***n1***). Immediately after the end of this second window, an AOA estimation is carried out again, in order to update the beamformer AOA reference, and so the cycle goes on, until the end of the specified total time period of the simulation run.

V. Performance Analysis

To exemplify the behavior of the structure, a ULA with 8 sensors with element spacing equal half wavelength was considered. The signal transmitted by the moving target was a 1 kHz tone, sampled with a 8 kHz rate (more than enough to prevent aliasing). Both interferers also transmitted 1 kHz tones, but with different phase shifts from each other and from the target signal (the phase shifts were chosen to be uniformly distributed random variables). The total signal impinging the array was comprised by the sum of the three signals plus white noise. Relatively low values of signal-to-noise ratios (SNR) have been chosen: 10 dB for the main signal and 5 dB for the interferer's signals. The signal-to-interference ratio (SIR) set to was thus 5 dB.

Since the target was moving and a plane wave propagation mechanism was assumed (far-field condition), the motion effect was represented as a time variant additional phase shift in the main signal. This phase shift was calculated as a function of time, of the target speed and of geometric parameters of the adopted scenario.

Regarding the windowing procedure, a tradeoff relationship had to be considered in order to define the two windows lengths (***n1*** and ***n2***). For the AOA estimation, better estimates are expected if a great number of snapshots is available. On the other hand, since the AOA spectrum is (slowly) time variant, the window should not be that large, otherwise the tracking accuracy would be compromised, not to mention the increasing computational burden. For the situations analyzed in the present work, this AOA estimation window width adopted was ***n1*** = 20 (samples or snapshots).

The beamforming window also presents a low width limit in order to perform well. More specifically, the main issue here is the beamforming algorithm convergence, specially for the CLMS approach [12]. On the other hand, a high width limit was also desirable, in order to make the structure capable of tracking even abrupt changes in the path orientation (φ). The beamforming time window adopted for the examples presented in this work was ***n2*** = 500 samples.

In order to illustrate the beamformer behavior, it was assumed a scenario where two target speeds were tested (1 m/s and 10 m/s) and the inteferer's AOAs were 100^o and 120^o. The moving target path began 5m away from the array center, that is, with an initial AOA of 90^o, and presented an inclination with respect to an axis parallel to the array line (φ) of 15^o. Taking the fastest case (10 m/s) as the first example, Fig. 3 presents the 3D estimated spatial spectrum around 2 s duration simulation run. As it can be noticed, both interferer's AOAs estimations presented good agreement with the actual values. The same good estimation performance has been achieved for the main signal's AOA, with estimation errors no greater than 2^o.

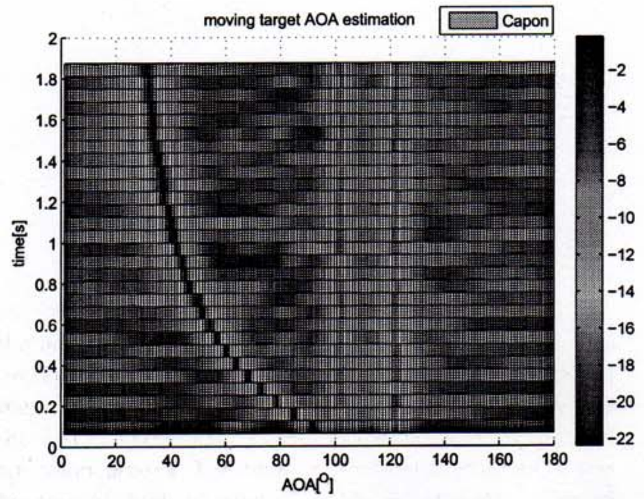

Fig. 3. Top view of a 3D spatial spectrum estimated for a 2s simulation run (v=10m/s)

In the adopted procedure, a beampattern is calculated every instant after the AOA estimation is available. However, it suffices to show one beampattern per window in order to illustrate the tracking capacity of the simulated structure. Fig. 4 and 5 present 3D views of beampatterns calculated around 2s duration simulation run, taking only the last pattern of each window. The top view illustrated in Fig. 5 makes it easier to verify that the beamforming was able to track the main signal, while nulling the interferences along the simulation time.

In the overall, all the constrained algorithms tested performed well regarding tracking accuracy. Only slight differences were observed among the beampatterns, except for the CLMS, as exemplified in Figs. 6 and 7. The poorer CLMS performance was somehow expected, due to its inherent slow convergence. The corresponding spatial spectra have been inserted in those figures in order to illustrate the good accuracy performance of the beampatterns.

VI. Final Remarks

In this work, a beamforming structure was proposed and simulated for tracking the AOA of a slowly moving target. Four different linearly constrained adaptive algorithms have been tested: CLMS ($\mu = 0.005$), CAPA (3

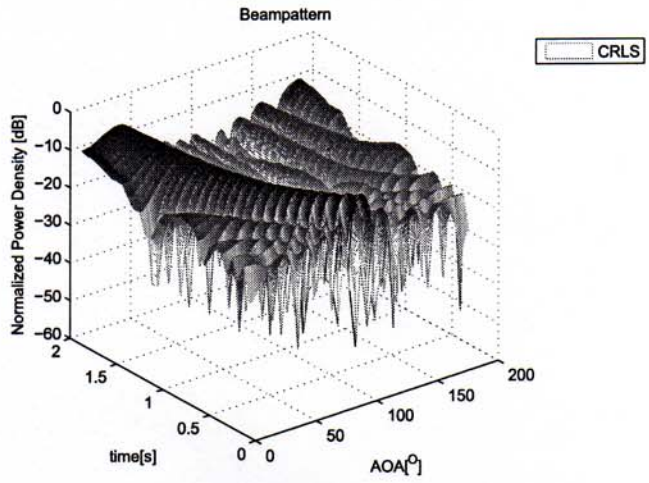

Fig. 4. 3D CRLS beampattern for a 2s simulation run (v=10 m/s).

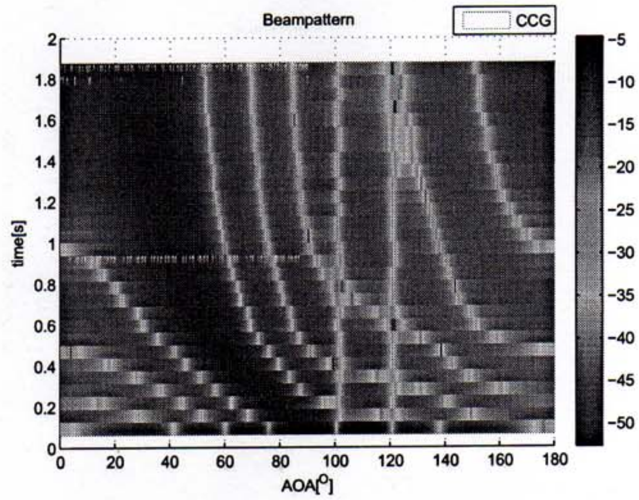

Fig. 5. Top view of a 3D CCG beampattern for a 2s simulation run (v=10m/s).

hyperplanes), CCG ($\eta = 0.98$ and $\xi = 10^{-4}$) and CRLS ($\lambda = 0.99$). The constrained matrix needed in the algorithms was generated based on spatial spectrum estimates derived from Capon's method. The performance of those algorithms has been assessed for a typical scenario.

In the overall, all algorithms presented good behavior regarding stability, convergence and accuracy. The CLMS was the one with the poorest accuracy performance, yet the results were still acceptable. The other three algorithms presented almost no difference in accuracy.

Since accuracy was not a critical issue, stability, convergence speed, and computational burden should be taken into account in order to evaluate the performance of the

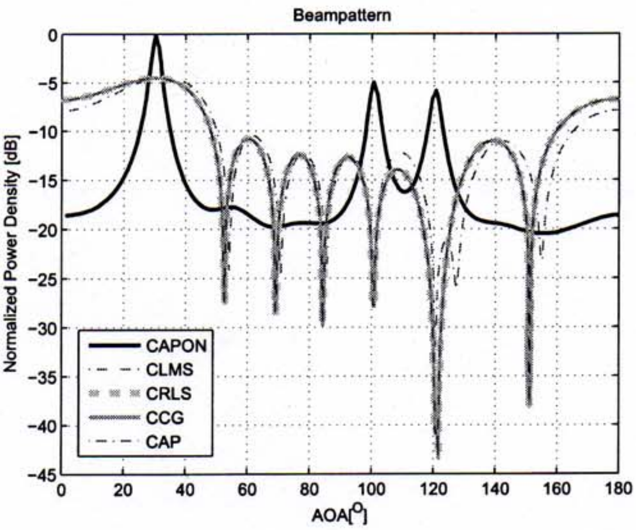

Fig. 6. AOA spectrum and beampatterns at the end of a 2s simulation run (v=10m/s).

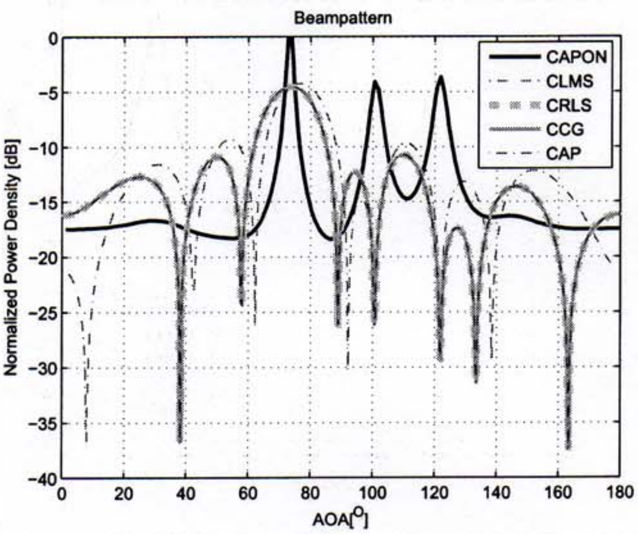

Fig. 7. AOA spectrum and beampatterns at the end of a 2s simulation run (v=1m/s).

algorithms. In this sense, the CCG algorithm would be an attractive choice for its numerical stability and its near RLS speed of convergence. Concerning its computational complexity, we can say that it is usually between the RLS and the APA. Please note that the CRLS algorithm is not stable as seen in [17]. It is also worth mentioning that the CAP algorithm presents intermediary behaviors in terms of computational complexity and speed of convergence by varying the number of hyperplanes [14].

Despite the good results achieved with the proposed technique, an alternative tracking procedure using a sliding window procedure (as opposed to the jumping window approach used in this paper) is currently under investigation. Also, the case of non-linear motion is a topic of further research. Finally, it is worth mentioning that the corresponding GSC (Generalized Sidelobe Canceller) versions of the algorithms used would lead to similar conclusions.

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