On an Efficient Implementation of the Multistage Wiener Filter Through Householder Reflections for DS-CDMA Interference Suppression

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***Abstract*— This paper describes the implementation of the multistage Wiener filter (MWF) through a series of nested Householder transformations applied to the input signal as a mean to form the analysis filter-bank part of the structure. The input signal is successively projected onto an appropriate subspace whereby the mutual information is maximized at each step, as required by the MWF structure. The method presented herein is described as a constrained optimization problem, where the constraints are imposed to the input signal, and not to the coefficients of the filter. The method is applicable to reduced-rank as well as to full-rank implementations of the MWF. The Householder transformation assures that the equivalent blocking matrices for all the stages are efficiently implemented via single reflections and that only unitary reflections are employed; robustness against deleterious finite-precision effects is, therefore, improved. Simulations of a DS-CDMA interference suppression receiver illustrates the robust behavior of the proposed scheme when implemented in finite precision as compared to the conventional MWF using non-orthogonal blocking matrices.**

I. INTRODUCTION

Since its introduction in [1], [2], [3], the multistage Wiener filter (MWF) has gained popularity and recognition due to its excellent performance in several applications, e.g., beamforming [3] and DS-CDMA interference suppression [4], [5], [6]. The MWF decomposition has the form of an analysis filter bank whose output signals have tri-diagonal correlation matrix. The analysis filter bank is followed by synthesis filter bank with nested scalar Wiener filters. In applications where reducing the rank of the Wiener filter is necessary, this particular structure may outperform other reduced-rank schemes [7]. The rank reduction is trivially implemented in this structure which is known to maximize the mutual information for a given rank [3].

The Householder transformation applied to linearly constrained filtering, recently presented [9], offers an efficient procedure to implement unitary transformation using nested Householder reflections and has a strong similarity with the MWF structure. This similarity is exploited in this paper in order to achieve an elegant and efficient decomposition scheme as an attractive option to the design of the MWF employing unitary transformations. Simulations for a DS-CDMA interference suppression receiver illustrate the robustness of the proposed structure when implemented in a finite precision environment.

II. MULTISTAGE WIENER FILTER

The MWF is an ingenious modification of the Wiener filter presented in Fig. 1, for which the following equations hold [8]:

$$e_0(n) = d_0(n) - \mathbf{w}_0^H \mathbf{x}_0(n)$$
$$\mathbf{w}_0 = \mathbf{R}_0^{-1}\mathbf{r}_0$$

where $e_0(n)$ is the output error, $d_0(n)$ is the reference signal, $\mathbf{x}_0(n)$ is an $M \times 1$ input-signal vector, $\mathbf{R}_0 = E[\mathbf{x}_0(n)\mathbf{x}_0^H(n)]$ is the autocorrelation function of the input signal, $\mathbf{r}_0 = E[\mathbf{x}_0(n)d_0^*(n)]$ is the cross-correlation vector between the input-signal vector and the reference signal, and $\mathbf{w}_0$ is the $M\times 1$ coefficient vector that minimizes the mean squared output error (MSE), also known as the Wiener filter. The observed signals $\mathbf{x}_0(n)$ and $d_0(n)$ are stationary, at least in the wide sense, and $\mathbf{x}_0(n)$ is persistently exciting at least of order M.

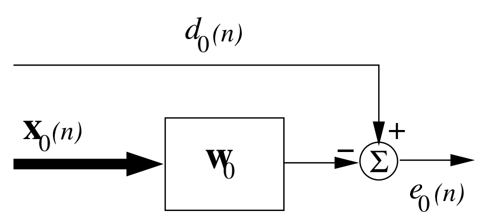

Fig. 1. The classical setup of a Wiener filter.

In the MWF implementation of the Wiener filter, the designer applies a series of transformations $\mathbf{T}_i$ to the input-signal vector such that the components collinear with the cross-correlation vector of each stage are identified and separated. For the first stage, this is accomplished by the transformation $\mathbf{T}_1$ of the form

$$\mathbf{T}_1 = \begin{bmatrix}\mathbf{h}_1^H \\ \mathbf{B}_1^H\end{bmatrix}$$

such that $\mathbf{w}_0 = \mathbf{T}_1^H \bar{\mathbf{w}}_0$. This transformation can be regarded as a GSC (Generalized Sidelobe Canceller)-like structure as depicted in Fig. 2.

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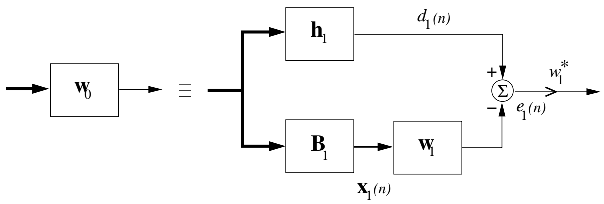

Fig. 2. The first stage transformation.

In the MWF, $\mathbf{h}_1$ is usually chosen as [3] $\mathbf{h}_1 = \mathbf{r}_0/\|\mathbf{r}_0\|$ and $\mathbf{B}_1$, as in the GSC, is such that $\mathbf{B}_1^H\mathbf{h}_1 = \mathbf{0}$.

Matrix $\mathbf{B}_1$ is called the blocking matrix, for it spans the null space of vector $\mathbf{h}_1$, which is collinear with the cross-correlation vector $\mathbf{r}_0$. The optimal filter $\mathbf{w}_1$, as suggested by the GSC-like Wiener filter in the transformed structure of Fig. 2, is given by [3]

$$\mathbf{w}_1 = \mathbf{R}_1^{-1}\mathbf{r}_1$$

where $\mathbf{R}_1 = E[\mathbf{x}_1(n)\mathbf{x}_1^H(n)] = \mathbf{B}_1^H\mathbf{R}_0\mathbf{B}_1$ is the autocorrelation matrix of the *input vector* for this stage $\mathbf{x}_1(n) = \mathbf{B}_1^H\mathbf{x}_0(n)$, and $\mathbf{r}_1 = E[\mathbf{x}_1(n)d_1^*(n)] = \mathbf{B}_1^H\mathbf{R}_0\mathbf{r}_0/\parallel \mathbf{r}_0 \parallel$ is the cross-correlation vector between $\mathbf{x}_1(n)$ and the *desired* signal for this stage $d_1(n) = \mathbf{h}_1^H\mathbf{x}_0(n)$.

The multistage structure arises when the procedure described above is applied recursively for all signal vectors $\mathbf{x}_i(n)$, where $\mathbf{x}_0(n)$ is the input-signal vector and the $M-i\times 1$ vectors $\mathbf{x}_i(n)$, for $i = 1, \ldots, M-1$, are obtained as [3] $\mathbf{x}_i(n) = \mathbf{B}_i^H\mathbf{x}_{i-1}(n), \quad i = 1, \ldots, M-1$ and all $\mathbf{B}_i$, for $i = 1, \ldots, M-1$, satisfy condition $\mathbf{B}_i^H\mathbf{h}_i = \mathbf{0}$.

Vectors $\mathbf{h}_i$ are always collinear with the cross-correlation vectors of the nested stages [3], i.e., $\mathbf{h}_i = \mathbf{r}_{i-1}/\|\mathbf{r}_{i-1}\|, \ i = 1, \ldots, M-1$ The MWF is illustrated in Fig. 3 for the case of $M = 4$.

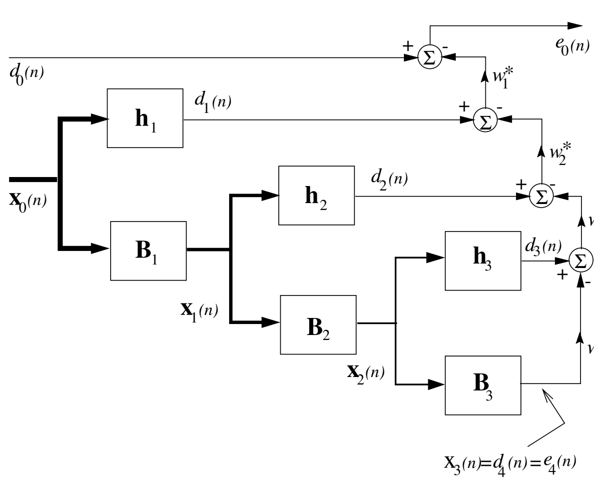

Fig. 3. An example of the MWF with M=4.

The overall transformation applied to the input-signal vector is described by matrix $\mathbf{T}$ given below [3]

$$\mathbf{T} = \begin{bmatrix} \mathbf{h}_1^H \\ \mathbf{h}_2^H\mathbf{B}_1^H \\ \vdots \\ \mathbf{h}_{M-1}^H\mathbf{B}_{M-2}^H\cdots\mathbf{B}_1^H \\ \mathbf{B}_{M-1}^H\mathbf{B}_{M-2}^H\cdots\mathbf{B}_1^H \end{bmatrix} = \begin{bmatrix} \mathbf{I}_{M-1} & \mathbf{0} \\ \mathbf{0} & \begin{bmatrix} \mathbf{h}_{M-1}^H \\ \mathbf{B}_{M-1}^H \end{bmatrix} \end{bmatrix} \cdots \begin{bmatrix} 1 & \mathbf{0} \\ \mathbf{0} & \begin{bmatrix} \mathbf{h}_2^H \\ \mathbf{B}_2^H \end{bmatrix} \end{bmatrix} \begin{bmatrix} \mathbf{h}_1^H \\ \mathbf{B}_1^H \end{bmatrix} = \mathbf{Q}_{M-1}\cdots\mathbf{Q}_2\mathbf{Q}_1 \quad (1)$$

Matrix $\mathbf{Q}_1$ above acts such that a signal vector in the direction of $\mathbf{h}_1$ will pass undistorted and, therefore, will produce as a result vector $[\delta_1\ 0\ \cdots\ 0]$. Matrix $\mathbf{Q}_2$ makes sure that whatever is in the subspace orthogonal to vector $\mathbf{h}_1$ and in the direction of vector $\mathbf{h}_2$ will pass undistorted, and so on.

The output error can be rewritten in terms of the coefficients of the transformed system as [3]

$$e_0(n) = d_0(n) - [w_1^* \ -w_1^*w_2^* \ \cdots \ (-1)^{M-1}w_1^*w_2^*\ldots w_M^*]\mathbf{T}\mathbf{x}_0(n)$$

where all coefficients w_i are scalars calculated as

$$w_i = \begin{cases} E[e_i(n)d_{i-1}^*]/E[e_i(n)e_i^*(n)], & i = 1, \ldots, M-1 \\ E[x_{M-1}(n)d_{M-1}^*]/E[x_{M-1}(n)x_{M-1}^*(n)], & i = M \end{cases}$$

Notice that, as a new stage is introduced, the order is reduced such that persistence of excitation is maintained despite the projection onto a null space performed by the blocking matrix.

III. HOUSEHOLDER-TRANSFORM MWF

In this section we shall examine how the HT proposed in [9] can be applied to implement efficiently the MWF. The approach to be described herein is different from that mentioned in passing in [3], where the utilization of the Householder transform applied to the autocorrelation matrix of the augmented vector $[d_0(n)\ \mathbf{x}_0(n)]$ is suggested as one possible implementation of the the analysis part of the MWF structure. In our approach we use the fact that the analysis part of the MWF projects the input signal onto a Krylov subspace whose bases can be specified as the M columns of a constraint matrix $\mathbf{C}$. The projection onto the Krylov space is a characteristic of the MWF, as already noted in [5]. However, the implementation following the method presented here provides insight to the overall transformation applied and also suggests an efficient procedure for implementation.

In [9], the linearly-constrained Wiener filter is efficiently implemented as a series of nested Householder transformations applied to the input-signal vector. The Householder vectors are chosen such that the filter coefficients satisfy a prescribed set of linear constraints given by a constraint matrix $\mathbf{C}$ and a gain vector $\mathbf{f}$, i.e.,

$$\mathbf{C}^H\mathbf{w} = \mathbf{f}$$

In the unconstrained MWF, $\mathbf{w}_0 = \mathbf{R}_0^{-1}\mathbf{r}_0$ and, therefore, $\mathbf{f} = \mathbf{C}^H\mathbf{R}_0^{-1}\mathbf{r}_0$.

Implementation of the last part of the transformation in (1) can be carried out by a Householder transformation that will preserve the direction of $\mathbf{h}_1 = \mathbf{r}_0/\|\mathbf{r}_0\|$. Therefore we may simply state that $\mathbf{Q}_1$ can be a Householder transformation onto $\mathbf{c}_1 = \mathbf{r}_0$. Verification that this transformation implies that the first row of $\mathbf{Q}_1$ is indeed $\mathbf{h}_1^H$ and that its lower partition spans the null space of $\mathbf{h}_1$ is naturally trivial. Vector $\mathbf{c}_1$ is the first column to be utilized when building the constraint matrix $\mathbf{C}$.

Now let us investigate the implementation of $\mathbf{Q}_2$, as given below:

$$\mathbf{Q}_2 = \begin{bmatrix} 1 & \mathbf{0} \\ \mathbf{0} & \bar{\mathbf{Q}}_2 \end{bmatrix} = \begin{bmatrix} 1 & \mathbf{0} \\ \mathbf{0} & \begin{bmatrix} \mathbf{h}_2^H \\ \mathbf{B}_2^H \end{bmatrix} \end{bmatrix}$$

This matrix has a nested Householder reflection, $\bar{\mathbf{Q}}_2$, that shall let through the $(M-1)\times 1$ vector $\mathbf{r}_1 = \mathbf{B}_1^H \mathbf{R}_0 \mathbf{r}_0$ undistorted. Therefore, if the input-signal vector $\mathbf{x}_0(n)$ is proportional to $\mathbf{R}_0\mathbf{r}_0$, it will go through $\mathbf{Q}_1$ and its lower part $(M-1)\times 1$ will reach $\bar{\mathbf{Q}}_2$ already transformed by $\mathbf{B}_1$. The Householder reflection on $\mathbf{B}_1^H\mathbf{R}_0\mathbf{r}_0$ is obtained with the approach described here if we set the second column of $\mathbf{C}$, say $\mathbf{c}_2$, as $\mathbf{c}_2 = \mathbf{R}_0\mathbf{r}_0$. This assures that the second element of the transformed input-signal vector has the energy relative to the direction of $\mathbf{c}_2$.

The third stage is slightly more involved. The reflection $\bar{\mathbf{Q}}_3$ shall let $\mathbf{r}_2 = \mathbf{B}_2^H\mathbf{B}_1^H\mathbf{R}_0\mathbf{B}_1\mathbf{B}_1^H\mathbf{R}_0\mathbf{r}_0$ through undistorted. But as $\mathbf{B}_1\mathbf{B}_1^H = \mathbf{I} - \mathbf{h}_1\mathbf{h}_1^H$ and the very nature of the second Householder reflection implies that $\mathbf{B}_2^H\mathbf{B}_1^H\mathbf{R}_0\mathbf{r}_0 = \mathbf{0}$, it is easy to verify that $\mathbf{r}_2$ is a vector in the direction of $\mathbf{B}_2^H\mathbf{B}_1^H\mathbf{R}_0^2\mathbf{r}_0$. Therefore, if the input-signal vector $\mathbf{x}_0$ is proportional to $\mathbf{R}_0^2\mathbf{r}_0$, it will go through $\mathbf{Q}_1$ and $\mathbf{Q}_2$ and its lower part $(M-2\times 1)$ will reach $\bar{\mathbf{Q}}_3$ already transformed by $\mathbf{B}_2^H\mathbf{B}_1^H$. The Householder reflection at this stage is obtained with the approach described here if we set the third column of $\mathbf{C}$, say $\mathbf{c}_3$, as $\mathbf{c}_3 = \mathbf{R}_0^2\mathbf{r}_0$. This assures that the third element of the transformed input-signal vector has the energy relative to the direction of $\mathbf{c}_3$.

As the first Householder reflection ensures that $\mathbf{B}_1^H\mathbf{r}_0 = \mathbf{0}$, the second reflection will make $\mathbf{B}_2^H\mathbf{B}_1^H\mathbf{R}_0\mathbf{r}_0 = \mathbf{0}$, the third reflection will make $\mathbf{B}_3^H\mathbf{B}_2^H\mathbf{B}_1^H\mathbf{R}_0^2\mathbf{r}_0 = \mathbf{0}$. As a general rule resulting from the approach we have described above, the ith reflection will make $\mathbf{B}_i^H\cdots\mathbf{B}_1^H\mathbf{R}_0^{i-1}\mathbf{r}_0 = \mathbf{0}$. Furthermore, vector $\mathbf{r}_{i-1}$, and consequently the first row of $\bar{\mathbf{Q}}_i$, is in the direction of $\mathbf{B}_i^H\cdots\mathbf{B}_1^H\mathbf{R}_0^i\mathbf{r}_0$. An input-signal vector proportional to $\mathbf{R}_0^i\mathbf{r}_0$ will go through all reflections from $\mathbf{Q}_1$ up to $\mathbf{Q}_{i-1}$ and its lower part will reach $\bar{\mathbf{Q}}_i$ already transformed by $\mathbf{B}_i^H\cdots\mathbf{B}_1^H$. The Householder reflection at this stage is obtained with the approach described here if we set the ith column of $\mathbf{C}$, say $\mathbf{c}_i$, as $\mathbf{c}_i = \mathbf{R}_0^{i-1}\mathbf{r}_0$. This assures that the ith element in the transformed input-signal vector has the energy relative to the direction of $\mathbf{c}_i$.

Calculation of the scalar coefficients w_i^* is straightforward. Starting from the first stage, we know that when the input signal vector $\mathbf{x}_0(n)$ is equal to the first column of $\mathbf{C}$, we have

$$f_1 = w_1^* d_1(n)$$

where f_1 is the first element of $\mathbf{f}$. Similarly, for $\mathbf{x}_0(n)$ equal to the second column of $\mathbf{C}$, we have

$$f_2 = w_1^*(d_1(n) - w_2^* d_2(n))$$

from which w_2^* can be obtained. Through a repeated procedure we verify that for $\mathbf{x}_0(n)$ equal to the i-th column of $\mathbf{C}$, we have

$$f_i = w_1^*(d_1(n) - w_2^*(\cdots(d_{i-1}(n) - w_i^* d_i(n))))$$

from which w_i^* can be obtained. The procedure described above can recognized as forward substitution which is trivial and robust to implement.

It is clear that in the case of white input signals, the dimension of the Krylov space is one and the MWF only needs one stage. In the general case of correlated input signals, the number of stages can be reduced to a value $K < M$ with a penalty to the output MSE that is comparable to, and sometimes even smaller than, that of reduced-rank filters that use the cross-spectral metric [3], [10], [11].

Fig.4 depicts the Householder MWF for $M = 4$. Note, from the above discussion, that

$$\begin{aligned} \mathbf{C} &= [\mathbf{r}_0 \quad \mathbf{R}_0\mathbf{r}_0 \quad \mathbf{R}_0^2\mathbf{r}_0 \quad \cdots] \\ \mathbf{f} &= [\mathbf{r}_0\mathbf{R}_0^{-1}\mathbf{r}_0 \quad \|\mathbf{r}_0\|^2 \quad \mathbf{r}_0^H\mathbf{R}_0\mathbf{r}_0 \quad \cdots]^T \end{aligned}$$

In general the generation of the Krylov space through direct multiplication $\mathbf{R}_0^i\mathbf{r}_0$ will result in a computationally complex and potentially unstable implementation. The reason being that the sequence $\{\mathbf{R}_0^i\mathbf{r}_0\}_{i=0}^M$ tends to approximate the same dominant eigenvector of $\mathbf{R}$, and as a consequence $\mathbf{C}$ will be a very ill-conditioned matrix [12]. This was also noted upon in [6] which considered adaptive implementations of the conventional MWF for DS-CDMA interference suppression. In practice a stable an efficient generation of the Krylov space can be obtained using the Conjugate Gradient algorithm [12], [13]. In order to obtain the Householder reflectors necessary for an efficient implementation of the product $\mathbf{Q}\mathbf{x}_0(k)$ [9], we can use the procedure described in Table I.

TABLE I
CONSTRUCTION OF THE HOUSEHOLDER VECTORS

```
Available at start:
  C is the M × p constraint matrix to be triangularized
Initialize:
  V = 0_{M×p};
for i = 1 : p
{  x = C(i : M, i);
   e1 = [1 0_{1×(M−i)}]^T
   v = −||x|| e1 + x;
   v = v/||v||;
   C(i : M, i : p) = C(i : M, i : p) − 2v (v^T C(i : M, i : p)) ;
   V(i : M, i) = v; }
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IV. SIMULATIONS

In this section, we apply the MWF using Householder reflections to the case of single-user detection in DS-CDMA mobile communications systems. The goal of this example

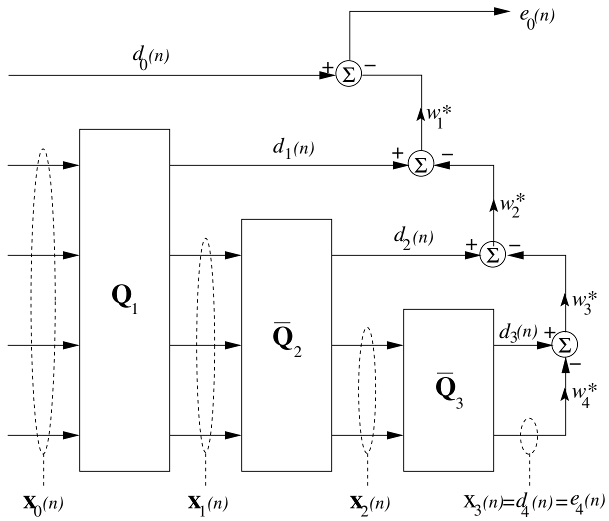

Fig. 4. An example of the Householder MWF with M=4.

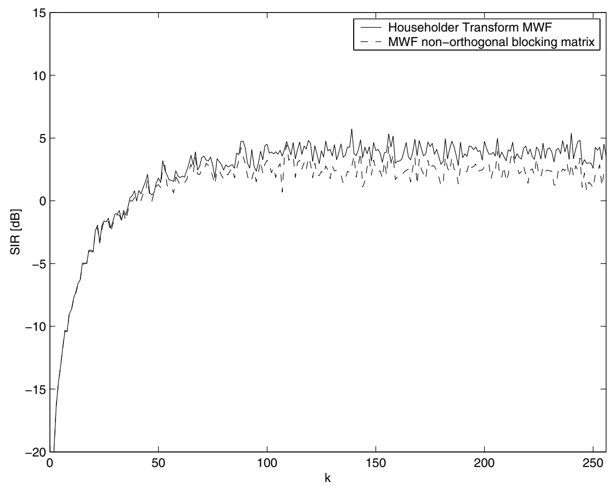

Fig. 5. SIR versus iterations for $M = 2$ stages, 32-bit floating-point arithmetic, $2\mathcal{E}_i/N_0 = 10$ dB.

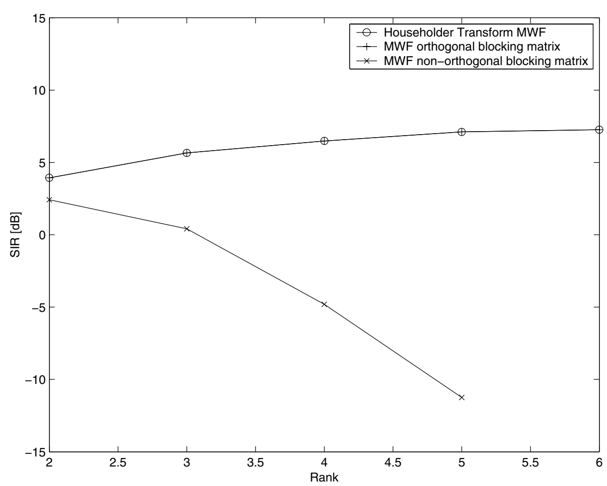

Fig. 6. SIR versus the rank M, 32-bit floating-point arithmetic, $2\mathcal{E}_i/N_0 = 10$ dB.

is to demonstrate the robust implementation of the proposed scheme when implemented in finite precision using 32-bit floating-point arithmetic. The results are compared with those of the conventional MWF [3]. In order to have a similar computational complexity of the two structures, the conventional MWF used the low-complexity non-orthogonal blocking matrix proposed in [3] in each stage. Using an orthogonal blocking matrix in the MWF would render an implementation of much higher complexity than the proposed scheme.

The received signal for a system with K simultaneous users can be written as

$$\mathbf{x}(k) = \sum_{i=1}^{K} \sqrt{\mathcal{E}_i} b_i(k)\mathbf{s}_i + \mathbf{n}(k)$$

where for the ith user, $\mathcal{E}_i$ is the energy per bit, $\mathbf{s}_i \in \mathbb{R}^L$ is the spreading code with $\|\mathbf{s}_i\|^2$=1, $b_i(k) \in \{\pm 1\}$ is the transmitted user information, and $\mathbf{n}(k)$ is the noise vector. In the case of single-user detection, we are only interested in detecting one user (here assumed to be $i = 1$). The system considered contained $K = 16$ users. The spreading codes of length $L = 32$ were taken as random binary codes, where the user codes were changed for each of the 100 realizations. The signal-to-noise ratio (SNR) was such that $2\mathcal{E}_i/N_0 = 10$dB, and user amplitudes were assumed to be equal.

The input-signal auto-correlation matrix $\mathbf{R}$ was obtained recursively using the unbiased estimation rule $\hat{\mathbf{R}}(k) = (1-\alpha)\mathbf{R}(k-1) + \alpha\mathbf{x}(k)\mathbf{x}^T(k)$ [8], with $\alpha = 0.02$.

The signal-to-interference ratio (SIR) versus the number iterations for a for a 32 bits finite precision implementation with $M = 2$ stages is shown in Figure 5. The proposed scheme is able to rapidly converge to a value slightly below 5 dB whereas the conventional MWF using non-orthogonal blocking matrices already for $M = 2$ suffers from finite precision effects and does not reach the same level.

The signal-to-interference ratio (SIR) versus the number of stages (rank) for a 32 bits finite precision implementation is shown in Figure 6. As can be seen from the figure the proposed scheme is robust to finite precision effects and already after 5 stages a close-to-optimal performance is achieved. The MWF using orthogonal blocking matrices resulted in identical result as the proposed scheme, but having a much higher complexity as was noted before. On the other hand, the MWF using nonorthogonal blocking matrices is very sensitive to finite precision effects and special care must be taken to obtain a robust implementation.

V. CONCLUSIONS

This paper describes an efficient implementation of Multi-stage Wiener Filter via nested Householder transformations. The approach utilized is based on a modified version of the linearly-constrained Householder transformed Wiener filter recently proposed. The method produces in a straightforward way the analysis and synthesis parts of the MWF through a suitable choice of the constraint matrix and gain vector. The constraint matrix is constructed with the bases of the Krylov space ($\mathbf{R}_0\mathbf{r}_0$). It was shown in an DS-CDMA interference suppression example that for finite precision implementation the proposed Householder Transform MWF can outperform the conventional MWF.

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