# Microphone-Array Signal Processing

José A. Apolinário Jr. and Marcello L. R. de Campos

{apolin},{mcampos}@ieee.org

COPPE/Poli/UFRJ

IME – Lab. Processamento Digital de Sinais

# *Outline*

1. Introduction and Fundamentals
2. Sensor Arrays and Spatial Filtering
3. Optimal Beamforming
4. Adaptive Beamforming
5. DoA Estimation with Microphone Arrays

# 4. Adaptive Beamforming

## *4.1 Introduction*

- Scope: instead of assuming knowledge about the statistical properties of the signals, beamformers are designed based on statistics gathered online.

- Scope: instead of assuming knowledge about the statistical properties of the signals, beamformers are designed based on statistics gathered online.
- Different algorithms may be employed for iteratively approximating the desired solution.

- Scope: instead of assuming knowledge about the statistical properties of the signals, beamformers are designed based on statistics gathered online.
- Different algorithms may be employed for iteratively approximating the desired solution.
- We will briefly cover a small subset of algorithms for constrained adaptive filters.

- Linearly constrained adaptive filters (LCAF) have found application in numerous areas, such as spectrum analysis, spatial-temporal processing, antenna arrays, interference suppression, among others.

## *Introduction*

- Linearly constrained adaptive filters (LCAF) have found application in numerous areas, such as spectrum analysis, spatial-temporal processing, antenna arrays, interference suppression, among others.
- LCAF algorithms incorporate into the solution application-specific requirements translated into a set of linear equations to be satisfied by the coefficients.

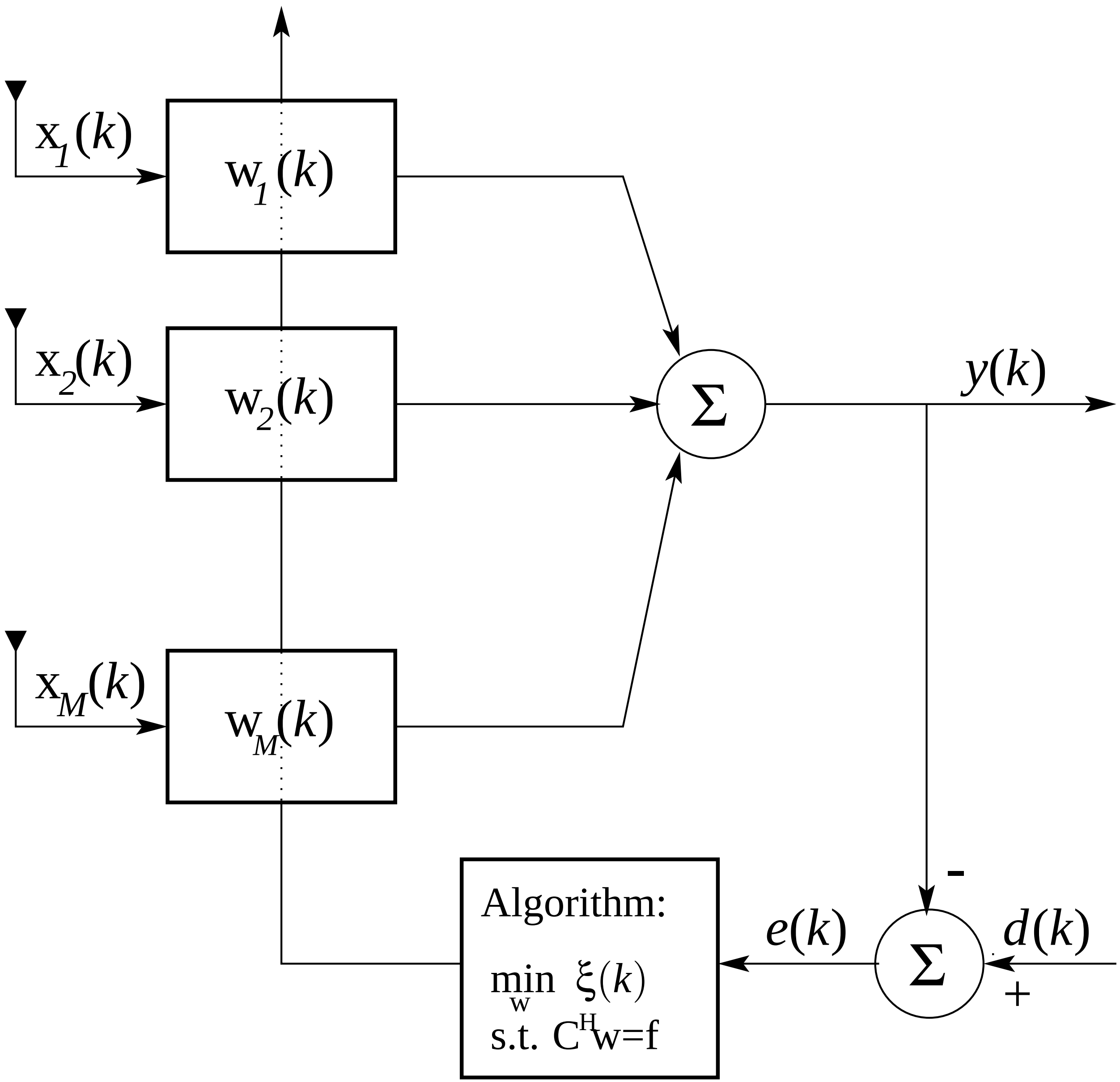
## *Introduction*

- Linearly constrained adaptive filters (LCAF) have found application in numerous areas, such as spectrum analysis, spatial-temporal processing, antenna arrays, interference suppression, among others.
- LCAF algorithms incorporate into the solution application-specific requirements translated into a set of linear equations to be satisfied by the coefficients.
    - For example, if direction of arrival of the signal of interest is known, jammer suppression can take place through spatial filtering without the need of training signal, or

- Linearly constrained adaptive filters (LCAF) have found application in numerous areas, such as spectrum analysis, spatial-temporal processing, antenna arrays, interference suppression, among others.
- LCAF algorithms incorporate into the solution application-specific requirements translated into a set of linear equations to be satisfied by the coefficients.
  - For example, if direction of arrival of the signal of interest is known, jammer suppression can take place through spatial filtering without the need of training signal, or in systems with constant-envelope modulation (e.g., M-PSK), a constant-modulus constraint can mitigate multipath propagation effects.

# 4.2 Constrained FIR Filters

# Broadband Array Beamformer



# *Optimal Constrained MSE Filter*

We look for

$$\min_{\mathbf{w}} \xi(k) \quad \text{s.t. } \mathbf{C}^H\mathbf{w} = \mathbf{f},$$

where

- $\xi(k) = E\left[|e(k)|^2\right]$
- $\mathbf{C}$ is the $MN \times p$ constraint matrix
- $\mathbf{f}$ is the $p \times 1$ gain vector

The optimal beamformer is

$$\mathbf{w}(k) = \mathbf{R}^{-1}\mathbf{p} + \mathbf{R}^{-1}\mathbf{C}\left(\mathbf{C}^H\mathbf{R}^{-1}\mathbf{C}\right)^{-1}\left(\mathbf{f} - \mathbf{C}^H\mathbf{R}^{-1}\mathbf{p}\right)$$

where:

- $\mathbf{R} = E\left[\mathbf{x}(k)\mathbf{x}^H(k)\right]$ and $\mathbf{p} = E\left[d^*(k)\mathbf{x}(k)\right]$
- $\mathbf{w}(k) = \left[\mathbf{w}_1^T(k)\ \mathbf{w}_2^T(k)\ \cdots\ \mathbf{w}_M^T(k)\right]^T$
- $\mathbf{x}(k) = \left[\mathbf{x}_1^T(k)\ \mathbf{x}_2^T(k)\ \cdots\ \mathbf{x}_M^T(k)\right]^T$
- $\mathbf{x}_i^T(k) = \left[x_i(k)\ x_i(k-1)\ \cdots\ x_i(k-N+1)\right]$

## *The Constrained LS Beamformer*

In the absence of statistical information, we may choose

$$\min_{\mathbf{w}} \left[ \xi(k) = \sum_{i=0}^{k} \lambda^{k-i} |d(i) - \mathbf{w}^H \mathbf{x}(i)|^2 \right] \text{ s.t. } \mathbf{C}^H \mathbf{w} = \mathbf{f}$$

with $\lambda \in (0, 1]$,

In the absence of statistical information, we may choose

$$\min_{\mathbf{w}} \left[ \xi(k) = \sum_{i=0}^{k} \lambda^{k-i} |d(i) - \mathbf{w}^H \mathbf{x}(i)|^2 \right] \text{ s.t. } \mathbf{C}^H \mathbf{w} = \mathbf{f}$$

with $\lambda \in (0, 1]$, which gives, as solution, In the absence of statistical information, we may choose

$$\min_{\mathbf{w}} \left[ \xi(k) = \sum_{i=0}^{k} \lambda^{k-i} |d(i) - \mathbf{w}^H \mathbf{x}(i)|^2 \right] \text{ s.t. } \mathbf{C}^H \mathbf{w} = \mathbf{f}$$

with $\lambda \in (0, 1]$, which gives, as solution,

$$\mathbf{w}(k) = \mathbf{R}^{-1}(k)\mathbf{p}(k) + \mathbf{R}^{-1}(k)\mathbf{C}\left(\mathbf{C}^H \mathbf{R}^{-1}(k)\mathbf{C}\right)^{-1}\left[\mathbf{f} - \mathbf{C}^H \mathbf{R}^{-1}(k)\mathbf{p}(k)\right],$$

where

$\mathbf{R}(k) = \sum_{i=0}^{k} \lambda^{k-i} \mathbf{x}(i)\mathbf{x}^H(i)$, and $\mathbf{p}(k) = \sum_{i=0}^{k} \lambda^{k-i} d^*(i)\mathbf{x}(i)$.

A (cheaper) alternative cost function is

$$\min_{\mathbf{w}} \left[\xi(k) = \|\mathbf{w}(k) - \mathbf{w}(k-1)\|^2 + \mu|e(k)|^2\right] \text{ s.t. } \mathbf{C}^H\mathbf{w}(k) = \mathbf{f},$$

A (cheaper) alternative cost function is

$$\min_{\mathbf{w}} \left[\xi(k) = \|\mathbf{w}(k) - \mathbf{w}(k-1)\|^2 + \mu|e(k)|^2\right] \text{ s.t. } \mathbf{C}^H\mathbf{w}(k) = \mathbf{f},$$

which gives, as solution,

# *The Constrained LMS Algorithm*

A (cheaper) alternative cost function is

$$\min_{\mathbf{w}} \left[\xi(k) = \|\mathbf{w}(k) - \mathbf{w}(k-1)\|^2 + \mu|e(k)|^2\right] \text{ s.t. } \mathbf{C}^H\mathbf{w}(k) = \mathbf{f},$$

which gives, as solution,

$$\mathbf{w}(k) = \mathbf{w}(k-1) + \mu e^*(k)\left[\mathbf{I} - \mathbf{C}\left(\mathbf{C}^H\mathbf{C}\right)^{-1}\mathbf{C}^H\right]\mathbf{x}(k),$$

where $e(k) = d(k) - \mathbf{w}^H(k-1)\mathbf{x}(k)$, $\mu$ is a positive small constant called step size.

# *The Constrained LMS Algorithm*

A (cheaper) alternative cost function is

$$\min_{\mathbf{w}} \left[\xi(k) = \|\mathbf{w}(k) - \mathbf{w}(k-1)\|^2 + \mu|e(k)|^2\right] \text{ s.t. } \mathbf{C}^H\mathbf{w}(k) = \mathbf{f},$$

which gives, as solution,

$$\mathbf{w}(k) = \mathbf{P}\left[\mathbf{w}(k-1) + \mu e^*(k)\mathbf{x}(k)\right] + \mathbf{F},$$

where $e(k) = d(k) - \mathbf{w}^H(k-1)\mathbf{x}(k)$, $\mu$ is a positive small constant called step size, $\mathbf{P} = \mathbf{C}\left(\mathbf{C}^H\mathbf{C}\right)^{-1}\mathbf{C}^H$, and $\mathbf{F} = \mathbf{C}\left(\mathbf{C}^H\mathbf{C}\right)^{-1}\mathbf{f}$.

# *The Constrained AP Algorithm*

We may wish to trade complexity for speed of convergence:

$$\min_{\mathbf{w}} \left[\xi(k) = \|\mathbf{w}(k) - \mathbf{w}(k-1)\|^2\right] \text{ s.t. } \begin{cases} \mathbf{X}^T(k)\mathbf{w}^*(k) = \mathbf{d}(k) \\ \mathbf{C}^H\mathbf{w}(k) = \mathbf{f}, \end{cases}$$

where

- $\mathbf{d}(k) = [d(k)\ d(k-1)\ \cdots\ d(k-L+1)]^T$
- $\mathbf{X}(k) = [\mathbf{x}(k)\ \mathbf{x}(k-1)\ \cdots\ \mathbf{x}(k-L+1)]^T$

We may wish to trade complexity for speed of convergence:

$$\min_{\mathbf{w}} \left[\xi(k) = \|\mathbf{w}(k) - \mathbf{w}(k-1)\|^2\right] \text{ s.t. } \begin{cases} \mathbf{X}^T(k)\mathbf{w}^*(k) = \mathbf{d}(k) \\ \mathbf{C}^H\mathbf{w}(k) = \mathbf{f}, \end{cases}$$

which gives, as solution,

# The Constrained AP Algorithm

We may wish to trade complexity for speed of convergence:

$$\min_{\mathbf{w}} \left[\xi(k) = \|\mathbf{w}(k) - \mathbf{w}(k-1)\|^2\right] \text{ s.t. } \begin{cases} \mathbf{X}^T(k)\mathbf{w}^*(k) = \mathbf{d}(k) \\ \mathbf{C}^H\mathbf{w}(k) = \mathbf{f}, \end{cases}$$

which gives, as solution,

$$\mathbf{w}(k) = \mathbf{P}\left[\mathbf{w}(k-1) + \mu\mathbf{X}(k)\mathbf{t}(k)\right] + \mathbf{F}$$

where

- $\mathbf{e}(k) = \mathbf{d}(k) - \mathbf{X}^T(k)\mathbf{w}^*(k-1)$
- $\mathbf{t}(k) = \left[\mathbf{X}^H(k)\mathbf{P}\mathbf{X}(k)\right]^{-1}\mathbf{e}^*(k)$