Microphone-Array Signal Processing

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IME – Lab. Processamento Digital de Sinais

Outline

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1. Introduction and Fundamentals

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2. Sensor Arrays and Spatial Filtering

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4. Adaptive Beamforming
5. DoA Estimation with Microphone Arrays

1. Introduction and Fundamentals

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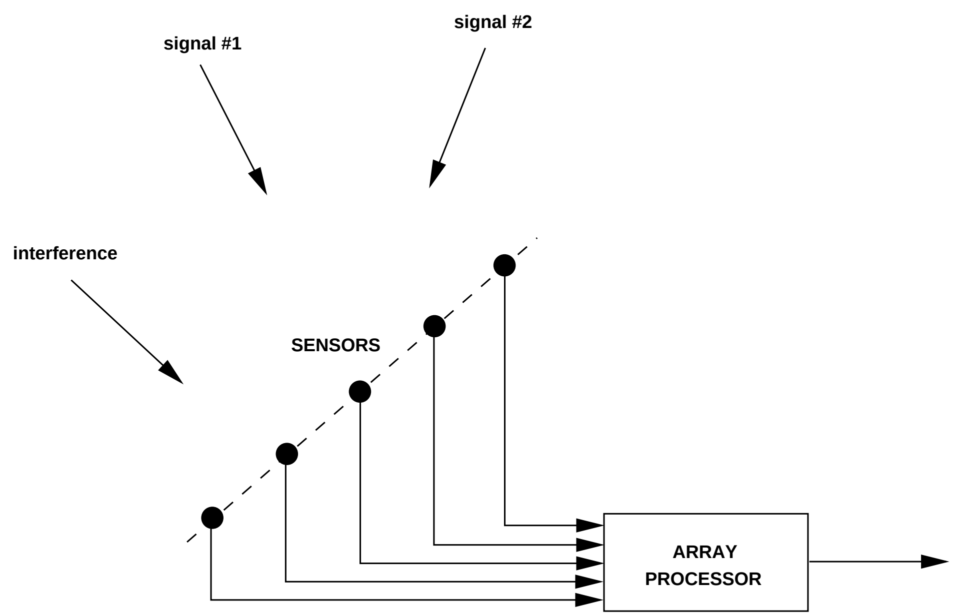

1.2 Signals in Space and Time

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Defining operators

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and

$$\nabla^2_{\mathbf{x}}(\cdot) = \frac{\partial^2(\cdot)}{\partial x^2} + \frac{\partial^2(\cdot)}{\partial y^2} + \frac{\partial^2(\cdot)}{\partial z^2}$$

respectively.

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or, for $s(\mathbf{x}, t)$ a general scalar field,

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where: c is the propagation speed, $\overrightarrow{E}$ is the electric field intensity, and $\mathbf{x} = [x\ y\ z]^T$ is a position vector.

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Note: From this point onwards the terms *wave* and *field* will be used interchangeably.

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where A is a complex constant and k_x, k_y, k_z, and $\omega \geq 0$ are real constants.

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constraints to be satisfied by the parameters of the scalar field

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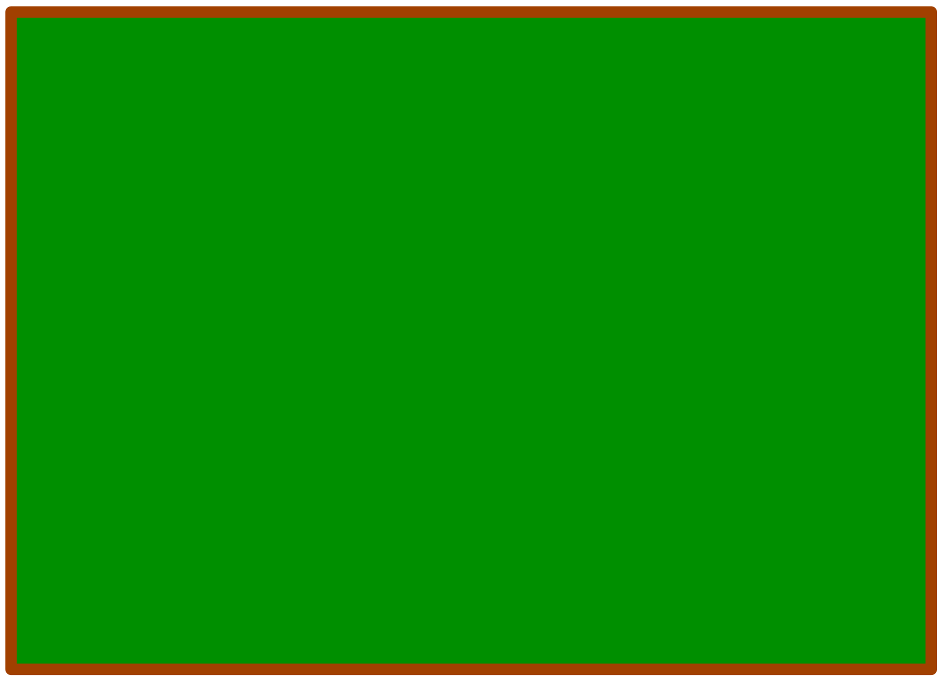

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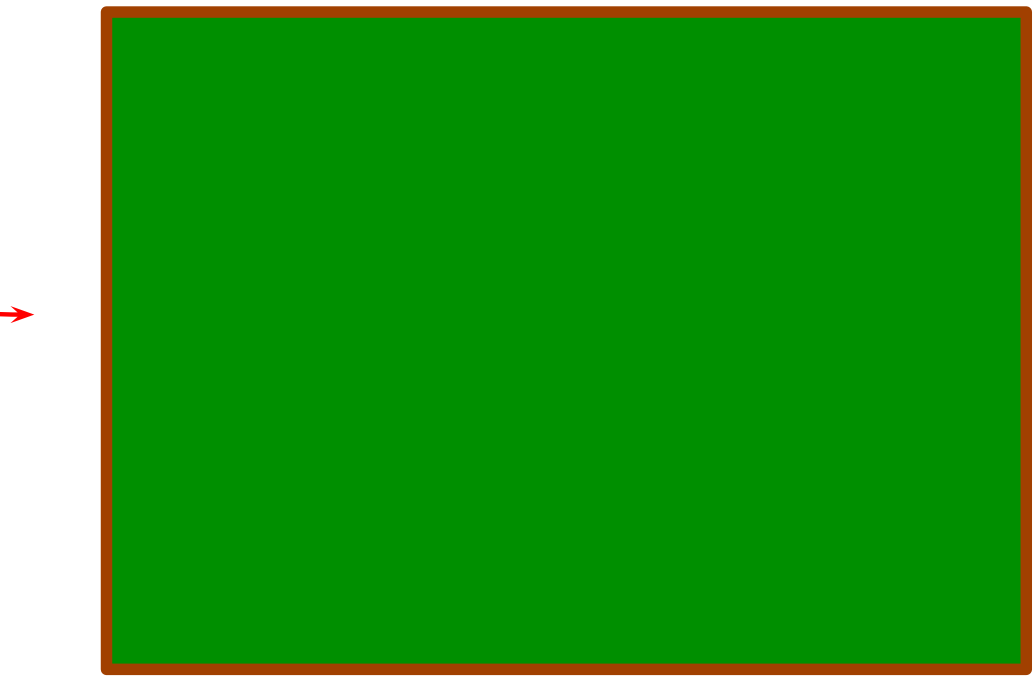

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$$k_x x + k_y y + k_z z = C$$

where C is a constant.

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The planes where $s(\mathbf{x}, t)$ is constant are perpendicular to the wavenumber vector $\mathbf{k}$

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$$\Longrightarrow \omega\delta t - \mathbf{k}^T\delta\mathbf{x} = 0$$

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Remember the constraints?
$\|\mathbf{k}\|^2 = \omega^2/c^2$

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The wavenumber vector, $\mathbf{k}$, may be considered a *spatial frequency* variable, just as ω is a *temporal frequency* variable.

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We may rewrite the wave equation as

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As $c = \omega/\|\mathbf{k}\|$, vector $\boldsymbol{\alpha}$ has a magnitude which is the reciprocal of c.

Periodic propagating periodic waves

Any arbitrary periodic waveform $s(\mathbf{x}, t) = s(t - \boldsymbol{\alpha}^T \mathbf{x})$ with fundamental period ω_0 can be represented as a sum:

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The coefficients are given by

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We will come <u>back</u> to this later...