On the Use of Householder Transformation in Adaptive Microphone Array

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ABSTRACT

This paper aims the application and performance evaluation of a class of constrained adaptive filters named Householder-Transform in microphone arrays. A new algorithm, the Householder-Transform Constrained QN, is introduced. Three algorithms – the LMS, the NLMS, and the QN – are then compared. This comparison is carried out in terms of computational complexity, speed of convergence, and directivity pattern, among their constrained versions, their implementations in the so-called GSC structure, and their recently proposed and new Householder transformed implementations.

Keywords: Adaptive microphone array, Householder Transformation, Generalized Sidelobe Canceller (GSC), Quasi-Newton algorithm, Householder Quasi-Newton Algorithm.

1. INTRODUCTION

Topics traditionally studied in antenna theory such as "beamforming" and direction of arrival estimation, have been targets of intensive research in signal processing systems using microphone arrays which present several applications in different fields such as teleconferencing, hands free communications, and hearing aids [1].

One of the techniques used in an adaptive beamformer is the minimization of the variance of the output signal subjected to a specific set of constraints (LCMV). In order to compare the performance of the different adaptive algorithms we have used the coefficient-error norm (speed of convergence), the number of operations per iteration (computational complexity), and an objective measure of similarity between original and received signals for each possible interference direction of arrival (directivity pattern).

The outline of the paper is as follows. In Section 2 we present the basic concepts concerning to beamforming. Section 3 reviews the constrained versions of the LMS and the NLMS algorithms as well as the Generalized Sidelobe Canceller (GSC) scheme. Section 4 presents the Householder transformation applied to constrained adaptive filters in a GSC perspective. Moreover, this section introduces the new algorithm. Simulation results and performance analysis are shown in Section 5. Finally, our conclusions are presented in Section 6. We hereafter assume for all purposes the case of "far field".

2. BASIC CONCEPTS

The term "beamforming" refers to a pencil beam formed by the early spatial filters, which received signals from a group of sensors and provided the reception of a signal from a specific direction and the attenuation of signals from other directions [2].

Fig. 1 depicts an array of M microphones with N tapped-delay lines for each one. The distance between sensors and the angle of arrival of the input signal cause different delays among the microphones; these delays are used here to improve the quality of the signal coming from the desired direction. It is easy to show that the maximum distance between microphones in order to have no ambiguities is $\lambda/2$ [3].

It is worth mentioning that, in this case, the introduction of a tapped-delay line for each sensor produces a space-time filtering well suited for a broadband beamformer. In previous work [4], we have addressed the topic number of sensors versus delays and experimentally obtained the optimum number of delays.

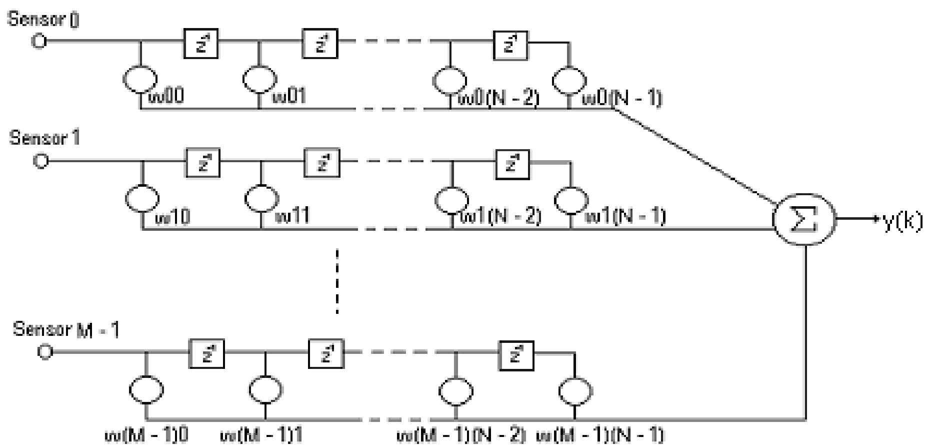

Fig. 1: Array of M microphones with N tapped-delay lines per microphone.

In order to establish the notation used in this paper, let us define the coefficient vector **W** and the input signal vector $\mathbf{X}_k$.

$$\mathbf{W} = \left[\mathbf{W}_0^T \quad \mathbf{W}_1^T \quad \ldots \quad \mathbf{W}_{M-1}^T\right]^T \quad (1)$$

where $[\mathbf{W}_i]_j = w_{ij}$, $0 \le i \le M-1$ and $0 \le j \le N-1$ as in Fig.1.

$$\mathbf{X}_k = \left[\mathbf{X}_0^T \quad \mathbf{X}_1^T \quad \ldots \quad \mathbf{X}_{M-1}^T\right]^T \quad (2)$$

where $[\mathbf{X}_i]_j = x_i(k-j)$, $0 \le i \le M-1$ and $0 \le j \le N-1$. Note that $(.)^T$ represents the transpose of a vector or matrix and $(.)^H$ represents the Hermitian operator. The output signal is given by

$$y_k = \mathbf{W}^H \mathbf{X}_k \quad (3)$$

The optimal LCMV filter in the sense of the minimum mean output energy (MOE) is the one that minimizes the mean square value of the output signal, subjected to a set of p linear constraints defined by

$$\mathbf{C}^H \mathbf{W} = \mathbf{f} \quad (4)$$

Note that **C** is MN x p, where MN is the size of the coefficient vector, and that **f** is p x 1. For our case, we use only one restriction represented by

$$f = 1 \quad (5)$$

and the steering vector

$$\mathbf{C}^T = \left[\mathbf{S}_0^T \quad \mathbf{S}_1^T \quad \ldots \quad \mathbf{S}_{M-1}^T\right] \quad (6)$$

In Eq. (6), vectors S_i represent the delays between each microphone and each tap or

$$\begin{aligned}
\mathbf{S}_0^T &= \left[1 \;\; e^{-j\varphi} \;\; \ldots \; e^{-j(N-1)\varphi'}\right] \\
\mathbf{S}_1^T &= \left[e^{-j\varphi} \;\; e^{-j(\varphi+\varphi')} \;\; \ldots \;\; e^{-j\left[\varphi+(N-1)\varphi'\right]}\right] \\
&\vdots \\
\mathbf{S}_{M-1}^T &= \left[e^{-j(M-1)\varphi} \;\; \ldots \;\; e^{-j\left[(M-1)\varphi+(N-1)\varphi'\right]}\right]
\end{aligned} \tag{7}$$

where φ’ is the delay between taps given by

$$\varphi' = \frac{2\pi v Ts}{\lambda} \tag{8}$$

where ν is the velocity of sound (340 m/s), Ts is the sampling period, and λ is the wave-length of the incoming signal. In Eq. (7), φ corresponds to the delay between microphones given by [3].

$$\varphi = \frac{2\pi d}{\lambda}\; sin(\theta) \tag{9}$$

with d being the distance between microphones and θ the desired direction of arrival measured from the perpendicular of the linear array plane.

By using Lagrange multipliers to solve the LCMV filter, the result obtained is the optimal coefficient vector given by [2]

$$\mathbf{W}_{opt} = \mathbf{R}^{-1}\mathbf{C}\left(\mathbf{C}^H\mathbf{R}^{-1}\mathbf{C}\right)^{-1}\mathbf{f} \tag{10}$$

Nevertheless, in real time applications, Eq. (10) bears the difficulty of obtaining the inverse of the input signal autocorrelation matrix, $\mathbf{R}^{-1}$. A practical approach is the use of an adaptive filter such that the convergence to the optimal (Wiener) solution may be eventually achieved.

3. CONSTRAINED ADAPTIVE ALGORITHMS AND THE GSC

The Constrained LMS Algorithm (CLMS)

Frost [5] proposed a method to solve the LCMV problem based on the LMS algorithm. Its updating equation is

$$\mathbf{W}_{k+1} = \mathbf{P}\left[\mathbf{W}_k - \mu\mathbf{X}_k y_k^*\right] + \mathbf{F} \tag{11}$$

where μ is the step-size. The projection matrix **P** and vector **F** are given by

$$\mathbf{P} = \mathbf{I} - \mathbf{C}(\mathbf{C}^H\mathbf{C})^{-1}\mathbf{C}^H \tag{12}$$

$$\mathbf{F} = \mathbf{C}(\mathbf{C}^H\mathbf{C})^{-1}\mathbf{f} \tag{13}$$

The Constrained Normalized LMS Algorithm (CNLMS)

The constrained version of the NLMS algorithm [6] is given by

$$\mathbf{W}_{k+1} = \mathbf{P}\left[\mathbf{W}_k - \mu \frac{\mathbf{X}_k y_k^*}{\mathbf{X}_k{}^H \mathbf{P} \mathbf{X}_k}\right] + \mathbf{F} \qquad (14)$$

where **P** and **F** are as in Eq. (12) and Eq. (13). The step-size in this case is chosen between 0 and 1. For μ = 1, Eq. (14) adapts the coefficient vector such that the *a posteriori* error is zero.

The Constrained Quasi Newton Algorithm (CQN)

The QN algorithm presents a convergence rate similar to the recursive least squares (RLS) algorithm and is known to be stable even under high input signal correlation [7]. Its constrained version [7] is given by

$$\begin{aligned}
e_k &= d_k - \mathbf{W}_{k-1}\mathbf{X}_k \\
\mathbf{t}_k &= \mathbf{R}_{k-1}^{-1}\mathbf{P}\mathbf{X}_k \\
\tau_k &= (\mathbf{P}\mathbf{X}_k)^H \mathbf{t}_k \\
\mu_k &= \frac{1}{2\tau_k} \\
\mathbf{R}_k^{-1} &= \mathbf{R}_{k-1}^{-1} + \frac{\mu_k - 1}{\tau_k}\mathbf{t}_k \mathbf{t}_k^H \\
\mathbf{T}_k &= \mathbf{I} - \mathbf{R}_k^{-1}\mathbf{C}\left[\mathbf{C}^H \mathbf{R}_k^{-1}\mathbf{C}\right]^{-1}\mathbf{C}^H \\
\mathbf{m}_k &= \mathbf{R}_k^{-1}\mathbf{C}\left[\mathbf{C}^H \mathbf{R}_k^{-1}\mathbf{C}\right]^{-1} \\
\mathbf{W}_k &= \mathbf{W}_{k-1} + 2e_k \mathbf{T}_k \mathbf{R}_k^{-1}\mathbf{x}_k + \mathbf{m}_k\left[\mathbf{F} - \mathbf{C}^H \mathbf{W}_{k-1}\right]
\end{aligned} \qquad (15)$$

The GSC Structure

The Generalized Sidelobe Canceller (GSC) structure changes the constrained minimization problem into an unconstrained problem. Given an orthogonal complement of **C** ($\mathbf{C}_a$), i.e. $\mathbf{C}_a{}^T\mathbf{C} = \mathbf{0}$, the following partitioned matrix is defined.

$$\mathbf{U} = [\mathbf{C} : \mathbf{Ca}] \qquad (16)$$

We can then express the coefficient vector in terms of a rotated vector **q** partitioned in a fixed term (**v**) and an adaptive term ($-\mathbf{W}_a$) as follows.

$$\mathbf{W} = \mathbf{U}\mathbf{q} = [\mathbf{C} : \mathbf{Ca}]\begin{bmatrix} \mathbf{v} \\ \Lambda \\ -\mathbf{W_a} \end{bmatrix} = \mathbf{W}_q - \mathbf{C}_a \mathbf{W}_a \qquad (17)$$

where $\mathbf{W}_q = \mathbf{C}\mathbf{v} = \mathbf{C}(\mathbf{C}^H\mathbf{C})^{-1}\mathbf{f}$, and $\mathbf{W}_a$ is unaffected by the constraints such that it can be updated by any unconstrained adaptive algorithm.

Fig. 2 depicts the GSC structure. It is worth mentioning that the GSC structure reduces the dimension of the adaptive filter.

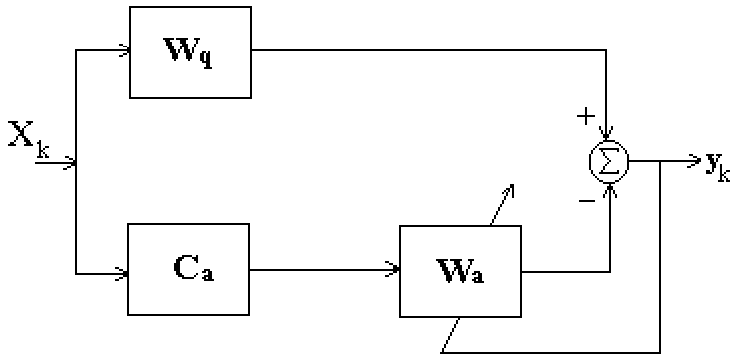

Fig. 2: The Generalized Sidelobe Canceller (GSC)

4. THE HOUSEHOLDER-TRANSFORM CONSTRAINED ALGORITHMS

The Householder Structure

The Householder-Transform Constrained LMS and NLMS algorithms have been recently proposed in [8] and they use an efficient Householder type of transformation in order to reduce the computational complexity with respect to the CLMS (CNLMS) and GSC-LMS (GSC-NLMS) implementations. This efficient technique, which is related to the GSC structure (they are actually equivalent in an infinite precision environment for orthogonal blocking matrices and proper initialization), is here applied to the QN algorithm. The main idea behind this structure is the rotation of the input signal vector by means of an orthogonal rotation matrix **Q**. This matrix **Q** generates a modified coefficient vector $\overline{\mathbf{W}}_k$ that relates to $\mathbf{W}_k$ according to

$$\overline{\mathbf{W}}_k = \mathbf{Q}\mathbf{W}_k \qquad (18)$$

We consider that **Q** is chosen such that $\mathbf{Q}^H\mathbf{Q}=\mathbf{Q}\mathbf{Q}^H=\mathbf{I}$ and

$$\overline{\mathbf{C}}(\overline{\mathbf{C}}^{\mathbf{H}}\overline{\mathbf{C}})^{-1}\overline{\mathbf{C}}^H = \begin{bmatrix} \mathbf{I}_{pxp} & 0 \\ 0 & 0 \end{bmatrix} \qquad (19)$$

then $\overline{\mathbf{C}} = \mathbf{Q}\mathbf{C}$ satisfies $\overline{\mathbf{C}}^{\mathbf{H}}\overline{\mathbf{W}}_{k+1} = \mathbf{f}$. The transformed projection matrix is written as

$$\overline{\mathbf{P}} = \mathbf{I} - \overline{\mathbf{C}}(\overline{\mathbf{C}}^H\overline{\mathbf{C}})^{-1}\overline{\mathbf{C}}^H = \begin{bmatrix} 0_{pxp} & 0 \\ 0 & \mathrm{I} \end{bmatrix} \qquad (20)$$

The rotated coefficient vector ($\overline{\mathbf{W}}_k$) can then be partitioned as follows.

$$\overline{\mathbf{W}}_k = \begin{bmatrix} \hat{\mathbf{W}} \\ \Lambda \\ -\mathbf{W}_H \end{bmatrix} \qquad (21)$$

where $\hat{\mathbf{W}}$ is formed by the first p elements of $\overline{\mathbf{W}}_0 = \mathbf{Q}\mathbf{F}$

Fig. 3 shows that the input vector is first rotated by **Q.** Then its first p elements are filtered by the fixed vector $\hat{\mathbf{W}}$ and its last MN-p elements enter the unconstrained adaptive filter $\mathbf{W}_H$. Matrix **Q** is constructed with successive Householder transformations as described in [8] and summarized in Appendix A. An algorithmic description of the new HCQN algorithm is presented in Table 1.

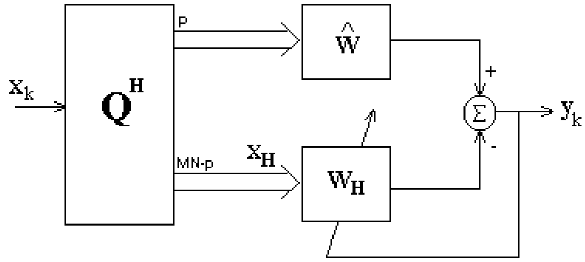

Fig. 3: The HouseHolder Structure

Table 1: The HCQN Algorithm

Initialization: α, $\mathbf{R}_{H(0)}^{-1}$ and $\overline{\mathbf{W}}_0 = \mathbf{QC}\left(\mathbf{C}^H\mathbf{C}\right)^{-1}\mathbf{f}$

For each k

$\overline{\mathbf{X}}_k = \mathbf{QX}_k$; according to an efficient procedure

$\hat{\mathbf{X}}_k$ = first p elements of $\overline{\mathbf{X}}_k$;

$\mathbf{X}_{H(k)}$ = last MN-p elements of $\overline{\mathbf{X}}_k$;

$$\overline{\mathbf{W}}_{k-1} = \left[\frac{\hat{\mathbf{W}}_0}{-\mathbf{W}_{H(k-1)}}\right];$$

$$e_k = \hat{\mathbf{W}}_0^H\hat{\mathbf{X}}_k - \mathbf{W}_{H(k)}^H\mathbf{X}_{H(k)};$$

% Update of the coefficient error according

% to the unconstrained algorithm:

$$\mathbf{t}_k = \mathbf{R}_{H(k-1)}^{-1}\mathbf{X}_{H(k)};$$

$$\tau_k = \mathbf{X}_{H(k)}^H\mathbf{t}_K;$$

$$\mu_K = \frac{1}{2\tau_k};$$

$$\mathbf{R}_{H(k)}^{-1} = \mathbf{R}_{H(k-1)}^{-1} + \frac{\mu_k - 1}{\tau_K}\mathbf{t}_k\mathbf{t}_k^H;$$

$$\mathbf{W}_{H(k)} = \mathbf{W}_{H(k-1)} + \alpha\frac{e_k}{\tau_k}\mathbf{t}_k;$$

End

5. SIMULATION RESULTS AND PERFORMANCE COMPARISON

In order to test and compare the performance of the proposed algorithm, the following computer experiment was carried out. We simulated the beamformer with 2 microphones separated by a distance of $\lambda/2$. In [4] we have found 4 as the optimum number of delays per microphone in order to have the best compromise between performance and computational load. In this experiment we used this number of delays and the output was the sum of each tapped delay-line. The simulation was such that we had two speech signals arriving in the microphone array, both with the same power and initially coming from directions 0 rad and $\pi/3$ rad, respectively. The desired output of the array was set to 0 rad.

We then ran the experiments for each algorithm (LMS, NLMS, and QN) and for each implementation (constrained, GSC, and Householder), and found that the three implementations present identical results in an infinite precision environment with an equivalent initialization.

In order to obtain the directivity pattern, we simulated an interference signal with several different angles of arrival and measured the distortion caused by this signal on the signal that had the desired direction of arrival. This distortion measurement was obtained by measuring the spectral distance between the original and received signals. Many spectral distance measures could be used [9] but the log spectral distance was particularly used due to its perceptual relevance and computational efficiency. Consider S(w) and S'(w) the spectra of the signals, the difference between the two spectra measured in logarithmic magnitude versus frequency is defined by

$$V(w) = \log S(w) - \log S'(w) \qquad (22)$$

One natural choice for a distance measure between S(w) and S'(w) is the set of L_q norms defined by

$$d(S,S') = \int_{-\pi}^{\pi} |V(w)|^q \frac{dw}{2\pi} \qquad (23)$$

When q=1, Eq. (21) defines the mean absolute log spectral distance. For q=2 we have the rms log spectral distance that has found application in many speech processing systems. When q approaches infinity, this equation is reduced to the peak log spectral distance. We can therefore state that a larger spectral distance implies a stronger difference between the signals.

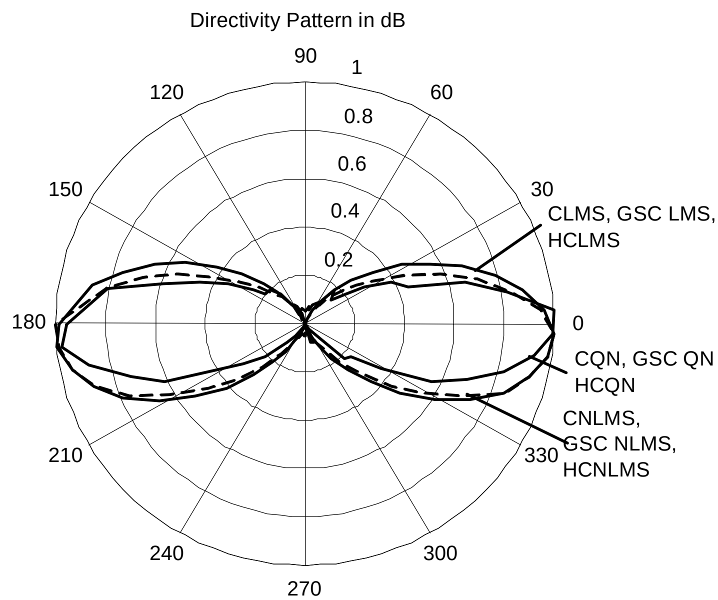

Fig. 4: Directivity Pattern of the Quasi Newton into Householder Transformed and GSC structures

Fig. 4 depicts the directivity pattern (q=2) for angles between $-\pi$ rad and π rad where the desired response is clearly 0 rad as expected. It can be noted that due to the geometry of the array used, the directivity pattern is symmetric to the plane of the microphone array.

For the comparison of the convergence speed, we used the norm of the coefficient error vector. The results are presented in Fig. 5. Note that there is a faster convergence for the HCQN algorithm comparing to the convergence speed of the HCLMS and HCNLMS.

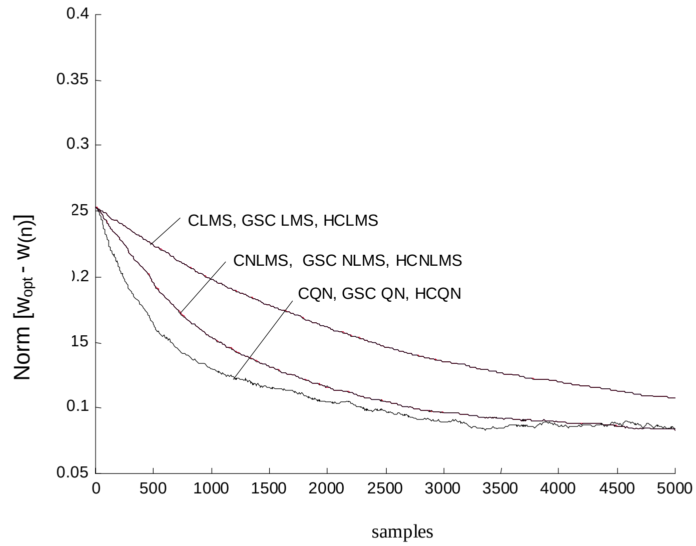

Fig. 5: Norm of coefficient-error vectors

Computational Complexity

Table 1 shows the computational complexity of the before mentioned algorithms in terms of MN (the size of the coefficient vector) with p (the number of restrictions) set to 1. Considering the most efficient implementation, the complexity of the LMS, NLMS, and QN algorithms are presented in their three forms: Constrained, GSC, and Householder-Transform structures.

Table 2: Computational Complexity

ALG.	ADD.	MULT.	DIV.
CLMS	$4MN - 2$	$4MN + 1$	0
GSC LMS	$(MN)^2 + MN - 2$	$(MN)^2 + 2MN - 1$	0
HCLMS	$4MN - 3$	$4MN$	0
NCLMS	$6MN - 3$	$6MN + 1$	1
GSC NLMS	$(MN)^2 + 2MN - 4$	$(MN)^2 + 3MN - 2$	1
NHCLMS	$5MN - 5$	$5MN - 1$	1
CQN	$(MN)^3 + 7(MN)^2 + 9MN - 8$	$(MN)^3 + 8(MN)^2 + 12MN + 1$	$MN + 4$
GSC QN	$3(MN)^2 - 4$	$3(MN)^2 - MN$	3
HCQN	$2(MN)^2 - MN - 4$	$2(MN)^2$	3

6. CONCLUSIONS

This paper introduces a new constrained adaptive algorithm, the Householder-Transform Constrained QN algorithm, and applies it to a microphone array beamformer. This work also compares the performance of the QN algorithm with the LMS and the NLMS algorithms in three different implementations named the constrained, the GSC, and the Householder-Transform.

It was seen that the three different structures are equivalent in infinite precision if the initializations are equivalent. Moreover, the HCQN algorithm has faster convergence speed, as expected, in comparison to the LMS and NLMS algorithms. The computational burden of the Householder structure is lower than the constrained or GSC implementations for each algorithm.

We can also conclude that the Householder structure can actually be used with any unconstrained algorithm and in this type of application (beamforming), this structure is expected to have a better directivity pattern than the GSC structure using a non orthogonal blocking matrix.

APPENDIX A

Matrix **Q** is constructed with successive Householder transformations applied to each of the p columns of matrix **CL**, where **L** is the square-root factor of $(\mathbf{C}^H\mathbf{C})^{-1}$ or $\mathbf{L}\mathbf{L}^H = (\mathbf{C}^H\mathbf{C})^{-1}$. **Q** can be written as

$$\mathbf{Q} = \mathbf{Q}_p \ldots \mathbf{Q}_2 \mathbf{Q}_1,$$

where

$$\mathbf{Q}_i = \begin{bmatrix} \mathbf{I} & 0^T \\ 0 & \overline{\mathbf{Q}}_i \end{bmatrix}$$

and $\mathbf{Q}_i$ is a (MN-i+1)x(MN-i+1) Householder transformation matrix given by $\overline{\mathbf{Q}}_\mathbf{i} = \mathbf{I} - 2\mathbf{V}_i\mathbf{V}^T{}_i$, with $\mathbf{V}_i$ being the Householder reflectors.

If vectors $\mathbf{V}_i$ are in the following form, they can be easily obtained with the Matlab® command V = housevec(C*sqrtm(inv(C’*C))) where housevec(.) is a simple function listed in Table 3.

$$\mathbf{V} = \begin{bmatrix} 0 & & 0_{p-1} \\ \mathbf{V_1} & \ldots & \\ & \mathbf{V_2} & \mathbf{V_p} \end{bmatrix}$$

Table 4 shows an efficient procedure for the multiplication $\mathbf{QX}_k$ using these Householder reflectors.

Table 3. Computation of Matrix V

```
Function [V] = housevec (A)
% function B = housevec (A)
% IN: A matrix to be triangularized
% OUT: Matrix with Householder Vectors
[MN,p]=size(A);
V=[ ];
for i=1:p
   vtemp=zeros(MN,1);
   x=A(i:MN,i);
   e1=eye(size(x));
   v=sign(x(1))*norm(x)*e1+x;
   v=v/norm(v);
   vtemp=i:MN)=v;
   A(i:MN,i:p)=A(i:MN,i:p)-2*v*(v’*A(i:MN,i:p));
   V=[V  vtemp];
end;
```

Table 4. Computation of $\overline{\mathbf{X}}_k = \mathbf{Q}\mathbf{X}_k$

$\overline{\mathbf{X}}_k = \mathbf{X}_k;$

for i = 1 : p

{

$\overline{X}_k(i:MN) = \overline{X}_k(i:MN) -$

$2V(i:MN,i)\left[V^H(i:MN,i)\overline{X}_K(i:MN)\right]$

}

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