ISCA Archive
http://www.isca-speech.org/archive

NORSIG '98
IEEE Nordic Signal Processing Symposium
Vigsø, Denmark
June 8-11, 1998

APPLICATION OF CONSTRAINED NORMALIZED ALGORITHMS FOR A MULTIPLE-ANTENNACDMA MOBILE RECEIVER

Stefan Werner[1] *and José Antonio Apolinario Jr.*[2]

[1]Helsinki University of Technology
Laboratory of Telecommunications Technology, Espoo, FINLAND
E-mail: stefan.werner@hut.fi
[2]COPPE/Universidade Federal do Rio de Janeiro
Rio de Janeiro, BRAZIL
E-mail: apolin@aquarius.ime.eb.br

ABSTRACT

Constrained adaptive receiver algorithms are considered in a multiple-antenna CDMA mobile receiver. We study the constrained versions of the Bi-normalized Data-Reusing LMS (BNDR-LMS) and the Normalized LMS (NLMS) and their performance are compared with the classical Frost algorithm. We make use of time-varying step-sequences to further enhance the performance of the receiver. The results show that considerable improvement in the convergence speed can be achieved with the use of the normalized algorithms compared with the conventional Frost algorithm. Using an optimal step-sequence in the update provides a fast convergence and a near optimum steady-state performance.

1. INTRODUCTION

Constrained adaptive filtering finds applications in many fields, e.g., adaptive array processing and interference suppression in CDMA communications. In [1], a blind LMS-type of algorithm was developed for intereference suppression in a CDMA system. The algorithm which is similar to the *gradient projection algorithm* [2] suffers from the same problems as the conventional LMS algorithm, i.e., slow convergence. Furthermore there is no inherent mechanism preventing accumulation of roundoff errors resulting in that the receiver drifts away from the constraint hyper-plane. The problem with error accumulation can be taken care of by using the Frost algorithm [2] which is a simple LMS-type of algorithm ensuring the constraints in every iteration. The Frost algorithm was applied to a CDMA mobile receiver equipped with multiple antennas in [3]. To speed up the convergence speed the use of normalized algorithms, like the NLMS [4] and the BNDR-LMS [5] algorithms, can be used. Normalized algorithms often achieve faster convergence than the conventional LMS at the expense of low increase in the complexity. Another approach is to use least-squares techniques [6] to improve the performance. In a mobile unit it is desirable to keep the signal processing as simple as possible due to complexity constraints. In this paper we develop two normalized LMS-type algorithms suitable for a multiple-antenna mobile CDMA receiver and compare those to the Frost algorithm. The receiver requires knowledge of the spreading waveforms in every antenna and can be seen as a generalization of [1] to include multiple antennas.

The paper is organized as follows. Section 2 briefly describes the signal model. The multiple-antenna receiver is derived in Section 3. In Section 4, the adaptive algorithms used are considered. The performance of the receiver and its adaptive implementations is demonstrated with a simplified example in Section 5, followed by conclusions.

2. SIGNAL MODEL

In downlink transmission (base station to mobile), signals associated with a number of simultaneously active users are transmitted over the same mobile channel. A mobile receiver, equipped with N antennas, will receive the transmitted signal over N different channels.

The system under consideration consists of K users transmitting information with binary antipodal signals with bit duration T_b. We assume an AWGN channel but an extension to more realistic fading multipath channels is straightforward.

The continuous transmitted signal is formed by:

$$x(t) = \sum_{m}\sum_{k=1}^{K}\sqrt{2}A_k b_k(m)s_k(t - mT_b)\cos(\omega_c t + \phi) \tag{1}$$

where for the k^{th} user, $b_k(m)\in\{-1,1\}$ is the m^{th} bit, A_k is the relative amplitude due to power control, $s_k(t)$ is signature sequence (code) with $G=T_b/T_c$ number of chips per bit, ω_c is the carrier frequency and ϕ is the carrier phase. The received signal at the i^{th} antenna after chip-matched filtering can be written as:

$$\mathbf{r}_i(m) = \mathbf{S}_i\mathbf{A}_i\mathbf{b}(m) + \mathbf{n}_i(m) \quad (2)$$

where $\mathbf{S}_i$ is the *G×K spreading matrix* containing the spreading sequences for the different users:

$$\mathbf{S}_i = [\mathbf{s}_{1i}\ \mathbf{s}_{2i} \dots \mathbf{s}_{k,i}] \quad (3)$$

where $\mathbf{s}_{k,i}$ is the discrete-time delayed version of the k^{th} user's sampled code sequence at antenna i, $\mathbf{A}_i$ is a diagonal amplitude matrix of the form:

$$\mathbf{A}_i = \mathrm{diag}[a_i A_1\ a_i A_2 \cdots a_i A_k] \quad (4)$$

where $a_i = e^{j(\phi - \omega\tau)}$ is the complex phase factor at the i^{th} antenna. $\mathbf{b}(m)$ is a vector containing the transmitted bits of the users:

$$\mathbf{b}(m) = [b_1(m)\ b_2(m) \dots b_K(m)]^{\mathrm{T}} \quad (5)$$

Finally, $\mathbf{n}_i(m)$ is a vector of the noise components of the antenna elements. These noise components are assumed to be independent, i.e.,

$$E\{\mathbf{n}_i(m)\mathbf{n}_j^{\mathrm{H}}(m)\} = \sigma_i^2 \mathbf{I}_G \delta(i-j) \quad (6)$$

If the antennas are spaced close together, the sampled code sequences will be practically the same in all antennas. In the case of an N-element linear array, the phase factor is simply given by

$$a_i = \exp\left[j\left(\phi - 2\pi(i-1)\frac{\Delta}{\lambda_c}\sin\theta\right)\right] \quad (7)$$

where Δ is the element spacing, λ_c is the wavelength of the carrier and θ is the direction of the incoming signal with respect to the array normal.

3. MULTIPLE-ANTENNA RECEIVER

In this section, a linear single-user multi-antenna (LSUMA) detector is derived. The detector has N antennas and assumes knowledge of the phase factors in every antenna. The structure of the receiver shown in Fig. 1. Each of the N antenna branches contains a linear filter whose coefficients are to be optimized. The filtered signals from each antenna are then added together to form a decision variable. In Fig. 1, $\mathbf{r}_i$ denotes the received signal after chip-matched filtering at antenna i, $\mathbf{h}_i$ contains the complex filter coefficients for the i^{th} antenna, and z is the decision variable formed by adding the filtered outputs from each antenna.

In order to get a compact notation, let us collect the filter coefficients and the received sequences from the antennas in vectors as

$$\mathbf{h} = [\mathbf{h}_1^{\mathrm{T}} \dots \mathbf{h}_N^{\mathrm{T}}]^{\mathrm{T}} \quad (8)$$

$$\mathbf{r} = [\mathbf{r}_1^{\mathrm{T}}\ \mathbf{r}_2^{\mathrm{T}} \dots \mathbf{r}_N^{\mathrm{T}}]^{\mathrm{T}} \quad (9)$$

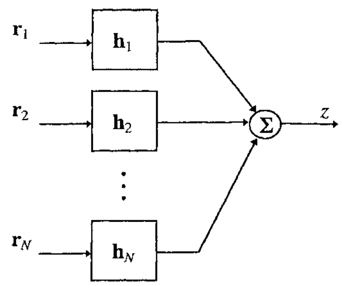

Figure 1: Structure of linear detector

Using the above notation, the output from the receiver can be written as

$$z = \mathbf{h}^{\mathrm{H}}\mathbf{r} \quad (10)$$

The variance of the output, i.e., the output power, is

$$E\{|z|^2\} = E\{\mathbf{h}^{\mathrm{H}}\mathbf{r}\mathbf{r}^{\mathrm{H}}\mathbf{h}\} = \mathbf{h}^{\mathrm{H}}\mathbf{R}\mathbf{h} \quad (11)$$

where $\mathbf{R}$ is the correlation matrix.

The filter coefficients $\mathbf{h}$ are found by minimizing the output variance of (11) under the constraints that the desired user's code sequence in every antenna can pass with unity response. To get a compact form we introduce the $GN\times N$ matrix $\mathbf{C}$ and the $N\times 1$ vector $\mathbf{u}$ as

$$\mathbf{C} = \begin{bmatrix} a_1\mathbf{s}_{1,1} & 0 & \cdots & 0 \\ 0 & a_2\mathbf{s}_{1,2} & 0 & \vdots \\ \vdots & 0 & \ddots & 0 \\ 0 & \cdots & 0 & a_N\mathbf{s}_{1,N} \end{bmatrix} \quad (12)$$

$$\mathbf{u} = [|a_1|^2\ |a_2|^2 \dots |a_N|^2]^{\mathrm{T}} \quad (13)$$

The minimization problem can now be formulated as

$$\hat{\mathbf{h}} = \arg\min_{\mathbf{h}} E\{|z|^2\} \quad (14)$$

subject to: $\mathbf{C}^{\mathrm{H}}\mathbf{h} = \mathbf{u}$

The solution to this problem is found by the method of Lagrange multipliers

$$\mathbf{h}_{\mathrm{opt}} = \mathbf{R}^{-1}\mathbf{C}[\mathbf{C}^{\mathrm{H}}\mathbf{R}^{-1}\mathbf{C}]^{-1}\mathbf{u} \quad (15)$$

The minimum output variance is obtained by substituting (15) into (11):

$$E\{|z|^2\} = \mathbf{u}^{\mathrm{H}}[\mathbf{C}^{\mathrm{H}}\mathbf{R}^{-1}\mathbf{C}]^{-1}\mathbf{u} \quad (16)$$

The closed-form solution is not suitable for practical implementation, as we need to estimate the correlation matrix and perform a matrix inversion. In the next section we consider adaptive implementations of the detector.

4. ADAPTIVE ALGORITHMS

In this section, we study three adaptive implementations of Eq. (15). The algorithms are: the Frost algorithm, and the constrained versions of the NLMS and the BNDR-LMS algorithms [7]. The Frost algorithm is simply the constrained version of the conventional LMS algorithm. The BNDR-LMS update performs normalization onto two orthogonal directions obtained from consecutive data pairs within each iteration as compared to the NLMS which only does it for one direction.

All the algorithms make use of a projection matrix $\mathbf{P} = \mathbf{I} - \mathbf{C}(\mathbf{C}^H\mathbf{C})^{-1}\mathbf{C}^H$ and a fixed vector $\mathbf{f} = \mathbf{C}(\mathbf{C}^H\mathbf{C})^{-1}\mathbf{u}$. The projection matrix $\mathbf{P}$ removes all components perpendicular to the plane $\mathbf{C}^H\mathbf{h} = 0$ and the vector $\mathbf{f}$ moves the solution back onto the constraint plane $\mathbf{C}^H\mathbf{h} = \mathbf{u}$. Due to the limited space we only state the algorithms. Detailed derivations of the algorithms can be found in [2,7].

The Frost algorithm is given by

$$\mathbf{h}_F(m+1) = \mathbf{P}\left[\mathbf{h}_F(m) - \mu z^*(m)\mathbf{r}(m))\right] + \mathbf{f} \tag{17}$$

Notice that the expression inside the brackets of Eq. (17) is the unconstrained LMS algorithm (reference signal set to zero).

The constrained NLMS is given by

$$\mathbf{h}_N(m+1) = \mathbf{P}\left[\mathbf{h}_N(m) - \mu \frac{z^*(m)}{\mathbf{r}^H(m)\mathbf{P}\mathbf{r}(m)}\mathbf{r}(m))\right] + \mathbf{f} \tag{18}$$

The expression inside the brackets is similar to the unconstrained NLMS algorithm apart from the introduction of the projection matrix in the denominator of the second term. However, by using the property of $\mathbf{P}^H\mathbf{P} = \mathbf{P}$ we can introduce the rotated vector $\mathbf{r}' = \mathbf{P}\mathbf{r}$ and the expression inside the brackets in Eq. (18) is just the unconstrained NLMS update for the rotated vector $\mathbf{r}'$.

The constrained BNDR-LMS algorithm is given by

$$\mathbf{h}_B(m+1) = \mathbf{P}\left[\mathbf{h}_B(m) + \mu_{1k}\mathbf{r}(m) + \mu_{2k}\mathbf{r}(m-1)\right] + \mathbf{f} \tag{19}$$

$$\mu_{1k} = \frac{e_1\mathbf{r}^H(m-1)\mathbf{P}\mathbf{r}(m-1) - e_2\mathbf{r}^H(m-1)\mathbf{P}\mathbf{r}(m)}{den}$$

$$\mu_{2k} = \frac{e_1\mathbf{r}^H(m)\mathbf{P}\mathbf{r}(m) - e_2\mathbf{r}^H(m-1)\mathbf{P}\mathbf{r}(m)}{den}$$

$$e_1 = -\mathbf{r}^H(m)\mathbf{h}_B(m)$$

$$e_2 = -\mathbf{r}^H(m-1)\mathbf{h}_B(m)$$

$$den = \mathbf{r}^H(m)\mathbf{P}\mathbf{r}(m)\mathbf{r}^H(m-1)\mathbf{P}\mathbf{r}(m-1) - \left(\mathbf{r}^H(m-1)\mathbf{P}\mathbf{r}(m)\right)^2$$

Also here the expression inside the brackets of Eq. (19) is the unconstrained BNDR-LMS update for the rotated vector $\mathbf{r}'$.

All the algorithms are initialized with $\mathbf{h}(0) = \mathbf{f}$. It is easy to check that the initial value satisfies the constraints in Eq. (14).

5. NUMERICAL RESULTS

In this section, the receiver algorithms are simulated and their performance is compared. The antennas are structured as a uniform linear array (ULA) with spacing of half the wavelength and the direction of arrival is set to 15°. The system used in the example consists of 5 users with spreading sequences taken as Gold codes of length 7. The signal-to-noise ratio for the desired user was set to 8 dB (in absence of MAI). The simulation is carried out for one and two antennas.

In order to achieve fast convergence and small misadjustment we use time-varying step size. The step sizes correspond to the optimal sequences presented in [8].

As the performance measure we use the signal-to-interference ratio (SIR) at the output of the receiver which are presented in Figs. 2-3 for the Frost, NLMS, and BNDR-LMS as a function of the number of iterations. All the results are averaged over 500 runs. In order to be able to compare the algorithms, the Frost algorithm is plotted for two different step sizes, one that results in a steady-state value comparable to the normalized algorithms and another that gives faster convergence.

In the plots the horizontal dashed line shows the optimum SIR value and the solid line corresponds to the conventional matched filter solution.

Figs. 2 show the results for one antenna. The step sizes used in the Frost algorithm are: $\mu=2\cdot10^{-3}$ and $\mu=10\cdot10^{-3}$.

From Fig. 2 we can see that the performance of the NLMS and the BNDR-LMS algorithms is about the same but has considerable faster than that of the Frost algorithm. It is of course possible to speed up the Frost algorithm even more by choosing a larger step size. However, in order to have a convergence speed close to the normalized algorithms will result in a very large misadjustment.

In Fig. 3, the results for two antennas are plotted. The step sizes used in the Frost algorithm are: $\mu=1\cdot10^{-3}$ and $\mu=5\cdot10^{-3}$.

As can be seen from Fig. 3, the normalized algorithms can still provide a convergence speed superior to that of the Frost algorithm. We can also see here that the misadjustment becomes higher as the step size in the Frost algorithm increases. In fact, the misadjustment is so high when using $\mu=5\cdot10^{-3}$ that the resulting steady-state is slightly lower than for the NLMS and BNDR-

LMS using only one antenna. Furthermore, the convergence speed is about the same. As a consequence, the same performance in terms of convergence speed and steady-state value is achieved by only using one antenna and the normalized algorithms instead of using two antennas and the Frost algorithm.

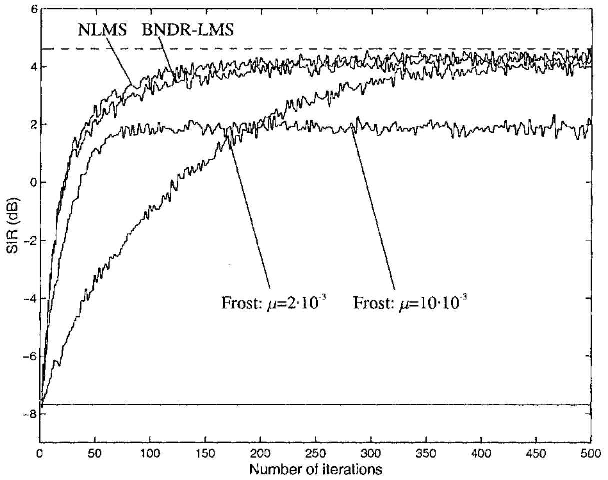

*Figure 2: SIR as a function of the number of iterations for the algorithms in a system with one antenna and **five users**. The interfering users transmit at 10 dB higher power than the desired user.*

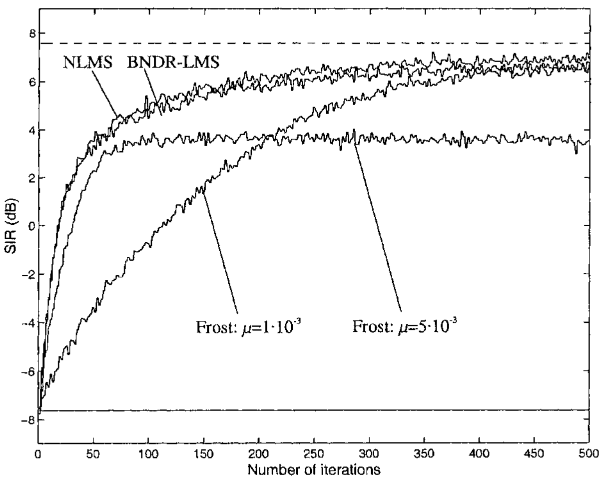

*Figure 3: SIR as a function of the number of iterations for the algorithms in a system with two antennas and **five users**. The interfering users transmit at 10 dB higher power than the desired user.*

6. CONCLUSIONS

In this paper, we have presented and compared three adaptive algorithms suitable for a DS-CDMA multi-antenna receiver. The results showed that constrained versions of the normalized algorithms outperforms the conventional Frost algorithm in terms of both convergence speed and misadjustment.

ACKNOWLEDGMENT

This work is part of a research project of the Institute of Radio Communication (IRC) funded by Technology Development Center (TEKES), NOKIA Research Center, Telecom Finland and the Helsinki Telephone Company.

REFERENCES

[1] M. Honig, U. Madhow, S. Verdú, "Blind adaptive multiuser detection", *IEEE Transactions on Information Theory,* vol. 41, pp. 944-960, Jul 1995.

[2] O. L. Frost, III, "An algorithm for linearly constrained adaptive array processing", *Proceedings of IEEE*, vol. 60, pp. 926-935, Aug 1972.

[3] S. Werner, T. Laakso, J. Lilleberg, "Multiple-antenna receiver for CDMA mobile reception," *International Conference on Communications*, June 7-11 1998, Atlanta, Georgia, USA.

[4] P. S. R. Diniz, *Adaptive Filtering: Algorithms and Practical Implementations,* Kluwer Academic Publishers, Boston, MA, USA, 1997.

[5] J. A. Apolinário Jr., M. L. R. de Campos, P. S. R. Diniz, "The binormalized data-reusing LMS algorithm," *Proceedings of the XV Simpósio Brasileiro de Telecomunicações,* Recife, Brazil, 1997.

[6] L. S. Resende, J. M. T. Romano, and M. G. Bellanger, "A fast least-squares algorithm for linearly constrained adaptive filtering," *IEEE Transactions on Signal Processing*, vol. 44, pp. 1168-1174, May 1996.

[7] J.A. Apolinario Jr., S. Werner, T. Laakso, P.S.R Diniz, "Constrained normalized adaptive filtering for CDMA mobile communications," Submitted to *European Signal Processing Conference*, Sept 8-11, 1998, Island of Rhodes, Greece.

[8] J.A. Apolinario Jr., P.S.R Diniz, T. Laakso, M. L. R. de Campos, "Step-size optimization of the BNDR-LMS algorithm," Submitted to *European Signal Processing Conference*, Sept 8-11, 1998, Island of Rhodes, Greece.