

# On the performance of new and classical approaches to AOA estimation for near-field acoustic waves

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**Abstract**—Classical AOA estimation methods usually rely on some simplifying assumptions that do not hold in many practical situations, like the far-field condition, for instance. The present work assesses three widely known methods (beamforming, Capon and MUSIC) under the near-field condition. In order to improve such analysis, a simple adaptation to the classical models is proposed and tested for that condition. The analysis comprises also a recently released approach, based on TDOA estimation.

**Index Terms**—AOA, acoustic waves, beamforming, Capon, MUSIC, TDOA-based AOA estimation.

## I. INTRODUCTION

Angle-of arrival (AOA) estimation is a prominent application field of space-time signal processing. Its basic theory applies to harmonic signals in general, comprising physical nature wave phenomena as distinct as electromagnetic fields, acoustic waves, and oceanic waves, among others. Regarding telecommunications applications, for instance, AOA estimation is widely used for location-based services and smart antennas systems [1]-[2]. In the acoustics field, highly directive microphone synthesis and indoor speaker tracking can be picked as applications examples [3]-[4].

There are several AOA estimation methods available in the literature. They can be seen as specializations of the generic spectral estimation problem in which the searched “frequency” is a parameter associated to a spatial information sampling, measured with properly arranged multiple sensors [5]. The simplest algorithms are the Fourier based ones, like the “classical” beamforming [5]. More elaborated approaches are the Capon [6] and the MUSIC (MUltiple Signal Classification) [7] methods. All those procedures assume a complex envelope signal arriving at the sensors. On the other hand, a new class of algorithms referred to as TDOA-based [4], [8] does not require complex signals. Such methods try to exploit the correlation between sensor pairs in order to better estimate the time difference of arrival (TDOA) between the wave fronts and, consequently, the corresponding AOA.

In general, all AOA estimation algorithms rely on a number of simplifying assumptions concerning the source signal (or signals) and the array surroundings. Far-field condition is one of those, as well as phase information availability of the signal’s envelope. Actually, such is the case for many situations, especially outdoor sensors for communication systems. Conversely, for applications like acoustic signals indoor AOA estimation, those basic assumptions are actually not likely to be fulfilled.

Although the above-mentioned situation is not quite unusual, it is hard to find AOA estimation methods specially tailored for that specific scenario or, at least, any references assessing typical estimation algorithms performance when basic hypotheses break down. The present work tried to contribute to “fill that gap” on this matter, proposing simple realistic changes on classical AOA estimation approaches. Simulations of the near-field-real signal problem were carried out, in order to analyze the proposed methods and also to assess its performance in comparison to some classical methods and to a modern TDOA-based approach.

The organization of this paper is as follows. Section II states the main simplifying assumptions typically accepted for AOA estimation. Some widely used estimation methods are briefly reviewed in Section III, which also proposes near-field adapted versions of the classical methods and presents a simple TDOA-based AOA estimation method. The following section comprises a comparative analysis of the previously reviewed approaches under near-field condition, based on simulations. Finally, some concluding remarks are discussed in section V.

## II. TYPICAL SIMPLIFYING ASSUMPTIONS

An almost ubiquitous hypothesis assumed for the AOA estimation problem is the far-field condition, where the wave fronts are plane, and the array is “far enough” from the source. More specifically, such condition may be analytically described as [9]:

$$\begin{cases} r \gg \frac{2D^2}{\lambda} \\ r > \lambda \end{cases} \quad (1)$$

where  $r$  is the separation between the array and the source,  $D$  is the maximum dimension of the array (width or height), and  $\lambda$  is the signal wavelength.

Fig. 1 illustrates a single signal impinging a uniform linear array (ULA) of sensors under the far-field condition.

For outdoor applications, the far-field condition is usually respected with great confidence. On the other hand, in indoor environments the far-field condition may not be respected within a considerable range. As an example, for a 1 kHz tone ( $\lambda = 0.34\text{m}$ ) and a 4-sensors ULA with a  $\lambda/2$  separation between consecutive sensors, the far-field condition corresponds to  $r \gg 1.53\text{ m}$ ; this is barely achievable within a typical room. In this case, a near-field representation would be more suitable, with spherical wave fronts, as illustrated in Fig. 2.

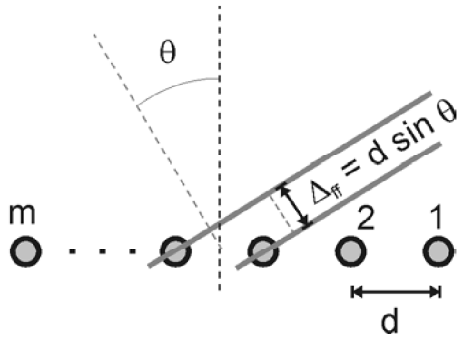


Fig. 1. Uniform linear array under far-field condition.

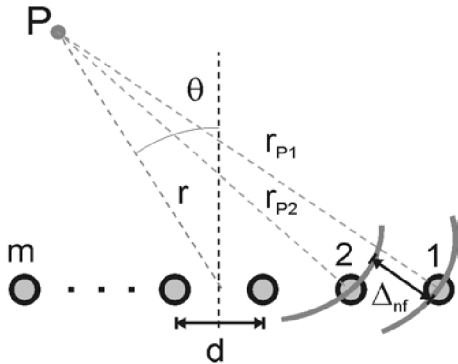


Fig. 2. Uniform linear array under near-field condition.

Another usual assumption when dealing with AOA estimation methods is that both amplitude and phase information of the transmitted signal are available. In other words, a complex envelope signal is assumed impinging the array of sensors. If the problem regards communication systems, this is usually true. But, when acoustic waves are considered, the signal is actually real. Nevertheless, most of the classical estimation methods rely on the complex envelope hypothesis. Thus, when real signals are considered, those methods no longer provide optimum spatial spectrum estimates.

Narrowband signals are usually assumed for most of the classical AOA estimation methods. Though such hypothesis is fairly reasonable for many communication systems problems, for acoustic signal AOA estimation the wideband model occurs more frequently.

### III. ESTIMATION METHODS

#### A. Classical Methods

The classical AOA methods are simply “spatial” versions of some frequently referred spectral estimation methods. Beamforming and Capon are among the simplest and widest used non-parametric algorithms. On the other hand, MUSIC is perhaps the most popular among the parametric methods [5]. The advantage of these two non-parametric methods is that they do not assume anything about the signals statistical properties. On the other hand, in the cases where such information is available, or at least when it is likely that those properties may be partially assumed, parametric methods may present better performances than the non-parametric ones.

In general, classical methods rely on the *array model*, which can be briefly stated as follows [5]. Taking Fig. 1 as reference, if a signal  $s(t)$  impinges a  $m$ -sensors ULA at an AOA  $\theta$ , a vector  $\mathbf{y}(t)$  is formed on the sensors outputs, such that:

$$\mathbf{y}(t) = \mathbf{a}(\theta) \cdot s(t) + \mathbf{n}(t) \quad (2)$$

$$\mathbf{y}(t) = [y_1(t) \cdots y_m(t)]^T \quad (3)$$

$$\mathbf{n}(t) = [n_1(t) \cdots n_m(t)]^T \quad (4)$$

where symbol  $\{\cdot\}^T$  represents the transposition of a vector or matrix,  $y_i(t)$  is the signal at the  $i^{\text{th}}$  sensor, and  $n_i(t)$  is the  $i^{\text{th}}$  sensor noise, usually considered as white Gaussian distributed. Vector  $\mathbf{a}(\theta)$  is frequently known as *steering vector*, and is given by:

$$\mathbf{a}(\theta) = [1 \exp(-j\omega_c \tau_2) \cdots \exp(-j\omega_c \tau_m)]^T \quad (5)$$

$$\tau_k = (k-1) \frac{d \cdot \sin(\theta)}{v} = (k-1) \frac{\Delta_{ff}}{v} \quad (6)$$

where  $\omega_c$  is the signal frequency (a narrowband signal is assumed),  $\tau_k$  is the far-field time delay of arrival (TDOA) between the  $k^{\text{th}}$  sensor and the first sensor, and  $v$  is the phase velocity of the impinging signal.

As it can be noticed in eqs. (5) and (6), the steering vector indeed contains the desired AOA information. How such information is “extracted” depends on the specific formulation of each estimation method. It is also worth mentioning that the array model may also be extended to the multipath case. If the impinging signal arrives at the array from  $n$  different AOAs, then there will be a steering vector for each direction. If the  $n \times m$  steering vectors are put together, a  $m \times n$

matrix is formed, which is commonly referred as *array manifold*. Thus, the basic array model equation for the multipath case may still be represented by eq. (2) if we just replace the steering vector for the array manifold  $\mathbf{A}(\theta)$ .

Beamforming is an array model based estimation method that may be seen as a bank of filters, where each sensor is attributed a weight. An optimization criterion is chosen to calculate the filter weights, such that the filter output maximizes only a specified AOA  $\theta$ , equally minimizing all other directions [5]. This principle is very simple and fast to compute, providing a spatial spectral estimate given by:

$$P(\theta) = \mathbf{a}^H(\theta) \cdot \hat{\mathbf{R}} \cdot \mathbf{a}(\theta) \quad (7)$$

where the symbol  $\{\cdot\}^H$  represents the hermitian of a vector or matrix, and  $\hat{\mathbf{R}}$  is an estimate of the signal covariance matrix  $\mathbf{R}$ , usually taken as:

$$\hat{\mathbf{R}} = \frac{1}{N} \sum_{t=1}^N \mathbf{y}(t) \mathbf{y}^H(t) \quad (8)$$

where  $N$  is the number of (time) snapshots of the signal available.

The beamforming method presents resolution limitation as a function of the number of sensors. The least the number of sensors, the worst the capability to distinguish two or more multipath signals arriving at AOAs very close to each other. However, if such situation is unlikely to happen, or if precision is not an issue, this method is adequate enough.

Capon's AOA estimation has the same basic idea of beamforming. The goal of Capon's filter is to maximize a certain direction  $\theta$ , while attenuating any other signals *actually* impinging the array from AOAs  $\neq \theta$  [6]. The beamforming filter, on the other hand, pays uniform attention to all AOAs  $\neq \theta$ , even when there might be no incoming signal at those AOAs, as previously stated. Capon's method is expected to present superior performance compared to beamforming, what is usually confirmed empirically. Capon's spatial spectral estimate is given by:

$$P(\theta) = \left\{ \mathbf{a}^H(\theta) \cdot \hat{\mathbf{R}}^{-1} \cdot \mathbf{a}(\theta) \right\}^{-1} \quad (9)$$

The MUSIC method is a relatively simple and efficient eigenstructure method of AOA estimation [7]. It has many variations and it is perhaps the most studied method in its class. In its standard form, also known as spectral MUSIC, the method estimates the noise subspace from the available samples. This can be done by either eigenvalue or singular value decomposition of the estimated data covariance matrix. Once the noise subspace has been estimated, a search for some directions has to be carried out, looking for steering vectors that are as orthogonal to the noise subspace as possible. More specifically, if  $\mathbf{R}$

is the signal covariance matrix, it can be eigen-decomposed such that:

$$\mathbf{R} = [\mathbf{S} \quad \mathbf{G}] \begin{bmatrix} \lambda_1 & & \\ & \ddots & \\ & & \lambda_m \end{bmatrix} \begin{bmatrix} \mathbf{S}^H \\ \mathbf{G}^H \end{bmatrix} \quad (10)$$

$$\mathbf{S} = [\mathbf{s}_1 \quad \dots \quad \mathbf{s}_n] \\ \mathbf{G} = [\mathbf{g}_1 \quad \dots \quad \mathbf{g}_{m-n}] \quad (11)$$

where  $\lambda_i$  is an eigenvalue of a  $\mathbf{R}$  matrix of rank  $m$ ,  $\mathbf{s}_i$  is a  $m \times 1$  vector representing the actual signal subspace, and  $\mathbf{g}_i$  is a  $m \times 1$  vector representing the noise subspace, with both subspaces orthogonal to each other. Based on such orthogonality, the AOAs may be interpreted as the  $n$  sharpest peaks of the following function:

$$P(\theta) = \left\{ \mathbf{a}^H(\theta) \cdot \hat{\mathbf{G}} \cdot \hat{\mathbf{G}}^H \cdot \mathbf{a}(\theta) \right\}^{-1} \quad (12)$$

where  $\hat{\mathbf{G}}$  is an estimate of the noise subspace matrix  $\mathbf{G}$ .

MUSIC usually presents high accuracy and resolution when the actual signal properties are close to the ones assumed *a priori* in the method. For such reason, it is frequently referred to as a "super-resolution" method. On the other hand, a major drawback of this parametric approach is that it assumes the number of the sources ( $n$ ) as a known parameter, when in fact, knowing that number is an additional estimation problem.

### B. TDOA-based approach

A simple TDOA-based estimation method was addressed in [4], and consists on computing the time delay estimates between sensor pairs of microphones, and then processing those estimates in order to extract the inherent AOA information, which is also function of the site and of the array geometry. This approach is applicable either to wideband or narrowband signals, which is a major advantage compared to the classical methods. However, the TDOA-based methods assume a single path impinging the array. Thus, its application to multipath problems is limited. Computation of time delays between signals from any pair of sensors can be performed by first computing the cross-correlation of the signals at each possible sensors pair. The cross-correlation for the  $i^{\text{th}}$  and  $j^{\text{th}}$  sensors is a  $N \times 1$  vector represented here by  $\phi_{ij}$ , and given by:

$$\phi_{ij} = y_i(t) * y_j(t) \quad (13)$$

where  $N$  is the number of time lags of the resulting convolution. Since a binomial combination gives the number of possible pairs  $np$ , that is:

$$np = \binom{m}{2} = \frac{m!}{2!(m-2)!} \quad (14)$$

a  $N \times np$  cross-correlation matrix  $\phi$  may be computed for the entire array. The cross-correlation may be easily calculated if its corresponding cross-power spectral density (XPSD) is first estimated, since in the spectral domain a single multiplication is needed, rather than a convolution.

The number of time delay lags that maximize  $\phi$  for each pair of sensors corresponds to the TDOA  $\hat{\tau}_{ij}$  between the considered sensors. In other words:

$$\hat{\tau}_{ij} = \arg \max \{ \phi_{ij} \} / f_s \quad (15)$$

where  $f_s$  is the sampling frequency applied to the signal.

Assuming the single-path far-field condition, the AOA estimation is straightforward. Recalling eq. (6),  $np \times 1$  vectors  $\mathbf{d}$  and  $\boldsymbol{\tau}$  may be composed, such that:

$$d_{ij} \sin(\theta) = v \hat{\tau}_{ij} \quad (16)$$

$$\mathbf{d} \sin(\theta) = v \hat{\boldsymbol{\tau}} \quad (17)$$

Finally, the AOA  $\theta$  may be calculated from [4]:

$$\theta = \sin^{-1} \left\{ \left( \mathbf{d}^T \mathbf{d} \right)^{-1} \mathbf{d}^T (v \hat{\boldsymbol{\tau}}) \right\} \quad (18)$$

### C. Adapted Classical Methods

The present work tried to assess some AOA estimation methods for ULA under near-field condition, typical of indoor applications. In order to do that, classical complex-envelope waves algorithms have been adapted to incorporate spherical wave fronts separations, rather than plane wave ones. Regarding the first two sensors of the arrays represented in Figs. 1 and 2, the wave fronts displacements for the near-field and far-field cases are, respectively:

$$\begin{aligned} \Delta_{nf} &= r_{P1} - r_{P2} = \quad (19) \\ &= \sqrt{\left[ \left( \frac{m-1}{2} \right) \cdot d \right]^2 + r^2} + 2 \left( \frac{m-1}{2} \right) \cdot d \cdot r \cdot \sin \theta - \\ &\quad - \sqrt{\left[ \left( \frac{m-1}{2} - 1 \right) \cdot d \right]^2 + r^2} + 2 \left( \frac{m-1}{2} - 1 \right) \cdot d \cdot r \cdot \sin \theta \\ \Delta_{ff} &= d \cdot \sin \theta \quad (20) \end{aligned}$$

Thus, a near-field equivalent of the steering vector given by eq. (5) could be proposed simply replacing  $\Delta_{ff}$  by  $\Delta_{nf}$  in eq. (6). Actually, the near-field model should also comprise an amplitude correction, since the spherical wave front amplitude is proportional to  $1/r$ . However, since phase variation is much more critical than amplitude variation, for small array apertures (small  $m$ ), this spherical amplitude variation

has not been considered for the adapted model proposed.

At last, another adaptation is due to the real domain nature of signals such as audio recorded from each sensor (as in a microphone array): the complex-envelope AOA estimation algorithms do not work properly when real signals are available. In this work, a simple procedure has been adopted in order to transform the original real signal into its complex domain equivalent. Basically, the upper band (mirror from  $\pi$  to  $2\pi$  of the left part 0 to  $\pi$ ) of the signal Fast Fourier Transform (FFT) was nulled, and the result was then inverse transformed (IFFT). The overall result was a good estimate of the desired corresponding complex signal. In other words, with this procedure, a single cosine became a complex exponential.

## IV. NEAR-FIELD COMPARATIVE PERFORMANCE ANALYSIS

The near-field performance analysis comprised the classical beamforming, Capon and MUSIC methods, as well as the TDOA-based approach described in the previous section. Both the original far-field versions and the proposed near-field adaptations have been assessed. Simulations have been carried out for an additive white noise corrupted narrowband signal (1 kHz tone), arriving at a 4-sensors ULA from several directions. The 4 sensors choice concerns future tests with a practical electrets array that is being assembled.

Still concerning future measurements, the simulations tried to reproduce the near-field main conditions of a small acoustic chamber available in our signal processing lab: source to array separation  $r$  around 0.5 to 2 m, signal-to-noise ratio (SNR) varying from 0 to 40 dB, and distance between consecutive sensors  $d = 10$  cm (less than the  $\lambda/2$  Nyquist limit [5]). A sample rate  $f_s = 180$  kHz were assumed instead of a more realistic 44 kHz frequency (“wav” files acquisition rate), in order to focus the assessment on the estimation methods. Lower rates result in higher quantization errors, what could mask some minor differences between the estimation approaches. The total sampling time for each simulation was 0.1 s.

The simulation results indicated an unexpected robustness of the original far-field classical algorithms to the near-field condition, as observed in Figs. 3 and 4. Actually, only for distances smaller than 0.5 m the far-field methods errors were noticeable, as exemplified in Figs. 5 and 6. Regarding SNR influence, beamforming and Capon were more susceptible than MUSIC, as Figs. 7 and 8 suggest. The lower the SNR, the broader the peak “skirt” width, though the AOA estimation error remained as small as for the higher SNR case. In general, the MUSIC algorithm presented the best results among the classical methods, while beamforming presented the worst (though also with relatively small errors).

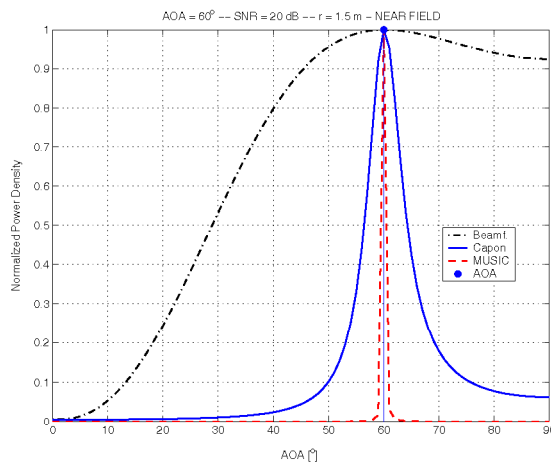


Fig. 3. Simulation results using near-field methods for AOA =  $60^\circ$ ,  $r=1.5$  m and SNR = 20 dB.

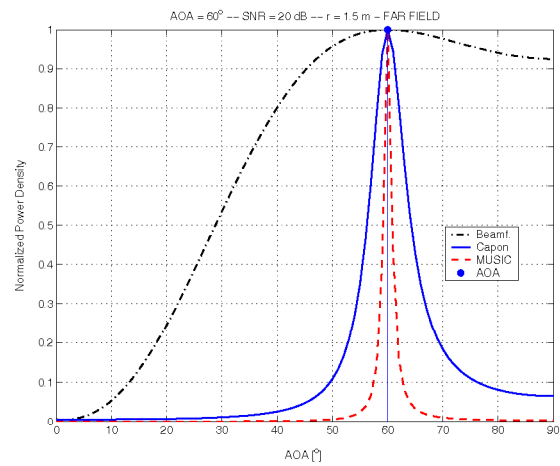


Fig. 4. Simulation results using far-field methods for AOA =  $60^\circ$ ,  $r=1.5$  m and SNR = 20 dB.

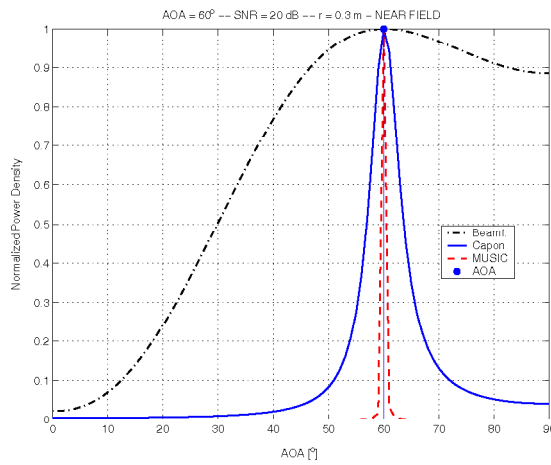


Fig. 5. Simulation results using near-field methods for AOA =  $60^\circ$ ,  $r=0.3$  m and SNR = 20 dB.

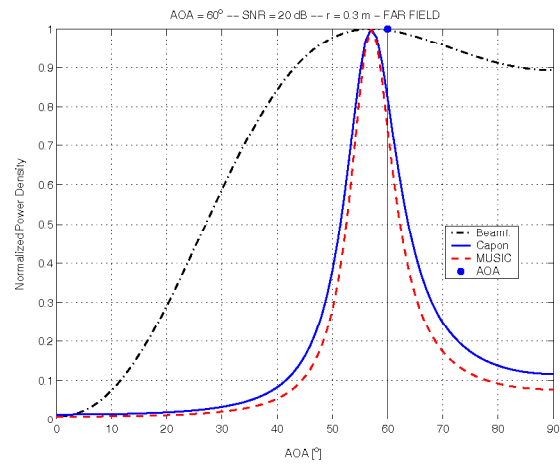


Fig. 6. Simulation results using far-field methods for AOA =  $60^\circ$ ,  $r=0.3$  m and SNR = 20 dB.

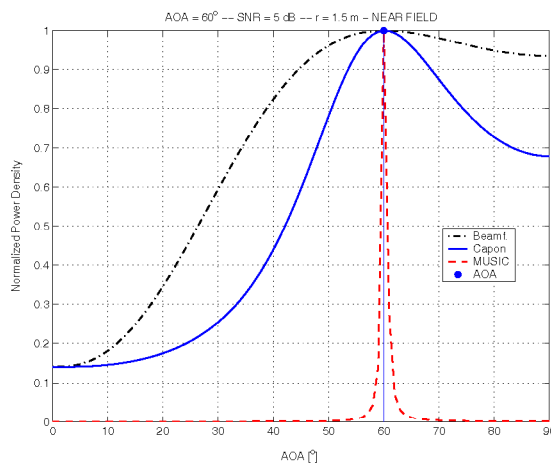


Fig. 7. Simulation results using near-field methods for AOA =  $60^\circ$ ,  $r=1.5$  m and SNR = 5 dB.

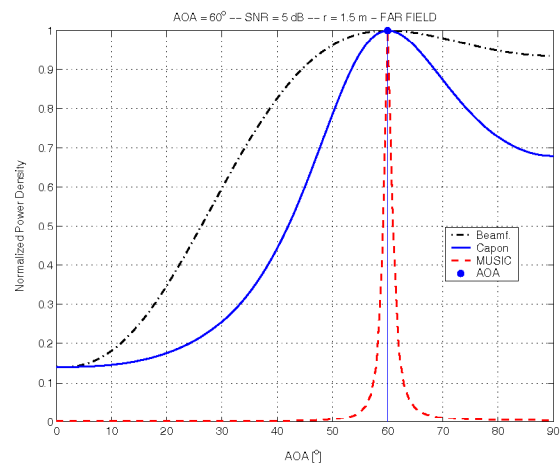


Fig. 8. Simulation results using far-field methods for AOA =  $60^\circ$ ,  $r=1.5$  m and SNR = 5 dB.

Besides the previously mentioned classical methods, the simple version of the new TDOA-based approaches described in Section III [4] has also been assessed for the same indoor scenario. Unlike those

methods, rather than spatial spectra, the TDOA-based approach provided directly an AOA estimate. Thus, in order to compare the performances of all the simulated algorithms, a maximum peak search was applied to the classical methods to extract their respective AOA estimates. Tables I and II present absolute error comparisons for the situations partially depicted in Figs. 5-6 and 7-8 (for SNR = 20 dB,  $r = 0.3$  m and for SNR = 5 dB,  $r = 1.5$  m, respectively), comprising 10 distinct angles of arrival (AOAs) from  $0^\circ$  to  $90^\circ$ .

In both Tables I and II, the previously referred behavior of the classical methods is more explicit. It is also noticeable that the TDOA-based approach from [4] presented the worst performance of all the simulated algorithms.

TABLE I  
ABSOLUTE ERRORS [ $\theta - \hat{\theta}$  IN  $^\circ$ ], SNR = 20 dB,  $R = 0.3$  M

$\theta$ [in $^\circ$ ]	FF Beam	FF Cap	FF MUS	NF Beam	NF Cap	NF MUS	TDOA
0	-1.00	-1.00	-1.00	-1.00	-1.00	-1.00	-30.66
10	1.00	1.00	1.00	0.00	0.00	0.00	-9.19
20	1.00	1.00	1.00	0.00	0.00	0.00	1.84
30	2.00	2.00	2.00	0.00	0.00	0.00	2.43
40	3.00	3.00	3.00	0.00	0.00	0.00	3.15
50	3.00	3.00	3.00	0.00	0.00	0.00	3.19
60	3.00	3.00	3.00	0.00	0.00	0.00	3.00
70	2.00	2.00	2.00	0.00	0.00	0.00	2.67
80	1.00	1.00	1.00	0.00	0.00	0.00	0.82
90	1.00	1.00	1.00	1.00	1.00	1.00	0.00

TABLE 2  
ABSOLUTE ERRORS [ $\theta - \hat{\theta}$  IN  $^\circ$ ], SNR = 5 dB,  $R = 1.5$  M

$\theta$ [in $^\circ$ ]	FF Beam	FF Cap	FF MUS	NF Beam	NF Cap	NF MUS	TDOA
0	-1.00	-1.00	-1.00	-1.00	-1.00	-1.00	-30.66
10	0.00	0.00	0.00	0.00	0.00	0.00	-0.12
20	0.00	0.00	0.00	0.00	0.00	0.00	0.01
30	0.00	0.00	0.00	0.00	0.00	0.00	0.21
40	0.00	0.00	0.00	0.00	0.00	0.00	0.04
50	0.00	0.00	0.00	0.00	0.00	0.00	0.09
60	0.00	0.00	0.00	0.00	0.00	0.00	0.53
70	0.00	0.00	0.00	0.00	0.00	0.00	0.46
80	0.00	0.00	0.00	0.00	0.00	0.00	-0.08
90	4.00	4.00	4.00	4.00	4.00	4.00	6.01

V. FINAL COMMENTS

The present work analyzed some typical AOA estimation methods, and also a new TDOA-based approach, under near-field conditions, especially for acoustic signals propagation. Besides the geometric implications of that condition, the impact of the real nature of acoustic signals to the frequently referred AOA estimation approaches has also been discussed.

A simple near-field adaptation has been proposed to be used in AOA estimation techniques based on classical methods such as beamforming, Capon, and MUSIC. The new proposal has been assessed with the help of simulations of a narrowband single arriving path signal impinging a small aperture array of sensors, under near-field conditions. Although the proposed method represented more accurately the near-field scenario, typical of indoor environments, the far-field original approaches presented an

unexpected robustness. The small error increase observed only for situations where the source is extremely close to the array was an indication that even for near-field conditions, the far-field approaches provide accurate AOA estimates.

A simple version of a new TDOA-based approach has also been simulated, in order to compare its performance with the classical methods. Though such methods are claimed to present superior performance [8], specially for wideband signals, the narrowband single-path simulations led to a slightly different conclusion. Anyway, except for the poor estimation behavior at the spatial spectrum edges ( $0^\circ$  and  $90^\circ$ ), the TDOA-based approach performed quite as well as the far-field classical methods.

A multipath analysis has also been carried out with the same algorithms described in this work. However, since the real-to-complex domain signal transformation described in Section III does not apply to a sum of cosines, the expected performance was poor, what has been confirmed for 2 or 3 paths simultaneously arriving at the array. Different approaches still have to be tested in order to incorporate the multipath effect to the AOA estimation methods analysis for the case of real signals. The other major issue still to be considered for the next stage of this work is wideband signal analysis.

Despite the limitations above, the main simulated results presented in this paper have been recently corroborated experimentally. Preliminary tests with a 4-microphones ULA in a small acoustic chamber provided fairly accurate results, though obviously not as accurate as those from the theoretical simulations.

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