SUBBAND ADAPTIVE FILTERING WITH CRITICAL SAMPLING USING THE DATA SELECTIVE AFFINE PROJECTION ALGORITHM

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ABSTRACT

Subband adaptive filtering techniques have been recently developed for a number of applications, such as acoustic echo cancellation and wideband active noise control. Such applications require adaptive filters with hundreds of taps, resulting in high computational complexity and low convergence rate for LMS based algorithms. For fullband systems, a variety of adaptive algorithms, which improve the adaptation convergence rate, have been developed. Most of them (such as the affine projection algorithm), however, present larger complexity than the conventional LMS algorithm. Such computational load can be reduced by making use of subband processing techniques. Considering these matters, we apply the affine projection algorithm (APA) in a recently proposed subband adaptive filter structure.

1. INTRODUCTION

Adaptive filtering techniques, particularly using FIR filters in view of their stability and unimodal performance properties, are used in many applications. However, in some applications such as acoustic echo cancellation and wideband active noise control, the order of the adaptive filter is very high, resulting in a large number of operations for their implementation and hence presenting a slow convergence rate when using LMS-based algorithms.

As an attempt to solve the above problem, subband processing techniques have been proposed for adaptive filters [1]-[2]. The main advantages of subband processing are: (a) the computational complexity is reduced by approximately the number of subbands, because both the number of taps and weight update rate can be decimated in each subband; and (b) the convergence rate is improved because the spectral dynamic range is greatly reduced in each subband.

For the fullband case, a class of algorithms named Affine Projection Algorithm (APA), has been developed [3] in order to improve the convergence rate of applications where the input signal is highly correlated. The main idea of this paper is to implement the affine projection algorithm [5] as well as its set-membership version [4] in the adaptive subband structure proposed in [2].

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This paper is organized as follows. In Section 2, the subband adaptive structure proposed in [2] is described. Section 3 reviews the AP and the Set-Membership AP algorithms. The subband adaptive algorithms based on APA and SM-APA are developed in Section 4. Simulation results are presented in Section 5, and Section 6 contains concluding remarks.

2. THE SUBBAND ADAPTIVE FILTER STRUCTURE

The adaptive subband structure presented in [2] was derived from the filter bank structure with sparse adaptive subfilters of Fig. 1. In a system identification application, such a structure models exactly any FIR system if the sparse adaptive filters $G_k(z)$ satisfy the following equation:

$$\begin{bmatrix} G_0(z) & G_1(z) & \cdots & G_{M-1}(z) \end{bmatrix} = \begin{bmatrix} P_0(z) & P_1(z) & \cdots & P_{M-1}(z) \end{bmatrix} \boldsymbol{F}_p(z) \quad (1)$$

where $P_i(z)$ are the polyphase components of the unknown system transfer function $P(z)$, and $\boldsymbol{F}_p(z)$ is the type-2 polyphase matrix of the synthesis bank which results in perfect reconstruction when associated with the analysis filters $H_k(z)$ of Fig. 1.

By including maximally decimated perfect reconstruction analysis and synthesis banks following each sparse subfilter in Fig. 1, moving the sparse subfilters $G_k(z^M)$ to the right of the decimators, and assuming that non-adjacent filters of the analysis bank have frequency responses which do not overlap, the structure of Fig. 2 is obtained. Observe that, in the resulting structure, the subfilters $G_k(z)$ operate at a rate which is $1/M$-th of the input rate, and that from (1), their lengths should be $K = (N_p + N_f)/M - 1$, where

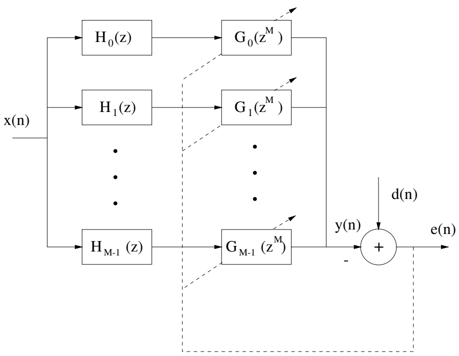

Fig. 1. Adaptive structure with an analysis filter bank and sparse subfilters.

N_p is the length of the system $P(z)$ to be identified and N_f is the length of each synthesis filter $F_k(z)$.

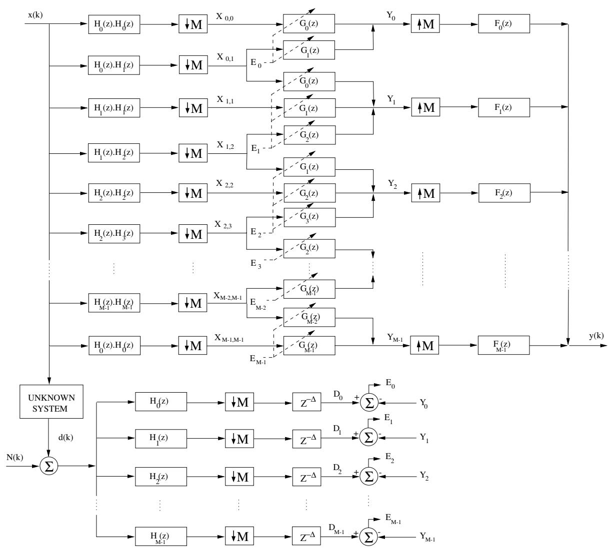

Fig. 2. Adaptive subband structure with critical sampling applied to system identification problems.

3. THE AFFINE PROJECTION ALGORITHM

In this section, we review the derivation of the Affine Projection Algorithm for the usual single channel case and establish the notation used hereafter. For a full band adaptive filter, $x(k)$ denotes the input signal, $d(k)$ is the reference signal, and $\mathbf{w}(k)$ is the coefficient vector. Moreover, $e(k) = d(k) - y(k)$ where $y(k) = \mathbf{w}^T(k)\mathbf{x}(k)$ is the output signal with $\mathbf{x}(k) = [x(k)\ x(k-1)\ \ldots\ x(k-N)]^T$ being the input signal vector.

For the APA, we define the *a priori* error vector as

$$\mathbf{e}(k) = \mathbf{d}(k) - \mathbf{X}^T(k)\mathbf{w}(k-1) \tag{2}$$

where

$$\mathbf{d}(k) = \begin{bmatrix} d(k) \\ d(k-1) \\ \vdots \\ d(k-L+1) \end{bmatrix} \tag{3}$$

and

$$\mathbf{X}(k) = [\mathbf{x}(k)\ \mathbf{x}(k-1)\ \ldots\ \mathbf{x}(k-L+1)] \tag{4}$$

with L being the number of projections, that is, the number of data pairs (reference signal samples and input vectors) used in the L-order APA.

In the Affine Projection algorithm, we minimize the expression $\|\ \mathbf{w}(k) - \mathbf{w}(k-1)\ \|^2$ subject to

$$\mathbf{d}(k) = \mathbf{X}^T(k)\mathbf{w}(k) \tag{5}$$

that is, the *a posteriori* error vector is made zero. Now using (5) in (2), we have

$$\begin{aligned} \mathbf{e}(k) &= \mathbf{X}^T(k)[\mathbf{w}(k) - \mathbf{w}(k-1)] \\ &= \mathbf{X}^T(k)\Delta\mathbf{w}(k) \end{aligned} \tag{6}$$

which leads to the solution

$$\Delta\mathbf{w}(k) = \mathbf{X}(k)[\mathbf{X}^T(k)\mathbf{X}(k)]^{-1}\mathbf{e}(k) \tag{7}$$

With the introduction of a step-size, the final updating equation for the APA is given by

$$\mathbf{w}(k) = \mathbf{w}(k-1) + \mu\mathbf{X}(k)[\mathbf{X}^T(k)\mathbf{X}(k)]^{-1}\mathbf{e}(k) \tag{8}$$

In the Set-Membership Affine Projection algorithm [4], an upper bound for the estimation error is set, resulting in the following update equation:

$$\mathbf{w}(k) = \begin{cases} \mathbf{w}(k-1) + \\ \mu\mathbf{X}(k)[\mathbf{X}^T(k)\mathbf{X}(k)]^{-1}\mathbf{e}(k), |\mathbf{e}(k)| > \gamma \\ \mathbf{w}(k-1),\ \text{otherwise} \end{cases} \tag{9}$$

which corresponds to the conventional APA whenever the coefficient vector is within the so-called constraint set[1].

4. THE APA IN SUBBANDS

For the subband structure of Fig. 2, we define the i-th subband error vector as

$$\mathbf{E}_i(k) = \mathbf{D}_i(k) - \mathbf{Y}_i(k) \tag{10}$$

[1]Set of all vectors $\mathbf{w}$ with estimation errors upper bounded in magnitude by γ.

where

$$\mathbf{E}_i(k) = \begin{bmatrix} E_i(k) \\ E_i(k-1) \\ \vdots \\ E_i(k-L+1) \end{bmatrix} \tag{11}$$

$$\mathbf{D}_i(k) = \begin{bmatrix} D_i(k) \\ D_i(k-1) \\ \vdots \\ D_i(k-L+1) \end{bmatrix} \tag{12}$$

and

$$\mathbf{Y}_i(k) = \begin{cases} \boldsymbol{\mathcal{X}}_{0,0}^T(k)\mathbf{G}_0(k-1) \\ \quad + \boldsymbol{\mathcal{X}}_{0,1}^T(k)\mathbf{G}_1(k-1), \; i=0 \\ \boldsymbol{\mathcal{X}}_{i-1,i}^T(k)\mathbf{G}_{i-1}(k-1) + \boldsymbol{\mathcal{X}}_{i,i}^T(k)\mathbf{G}_i(k-1) \\ \quad + \boldsymbol{\mathcal{X}}_{i,i+1}^T(k)\mathbf{G}_{i+1}(k-1), \; 0 < i < M-1 \\ \boldsymbol{\mathcal{X}}_{\text{M-2,M-1}}^T(k)\mathbf{G}_{\text{M-2}}(k-1) \\ \quad + \boldsymbol{\mathcal{X}}_{\text{M-1,M-1}}^T(k)\mathbf{G}_{\text{M-1}}(k-1), \; i = M-1 \end{cases} \tag{13}$$

with

$$\boldsymbol{\mathcal{X}}_{i,j} = [\mathbf{X}_{i,j}(k)\,\mathbf{X}_{i,j}(k-1)\,\ldots\,\mathbf{X}_{i,j}(k-L+1)] \tag{14}$$

and $\mathbf{X}_{i,j} = [X_{i,j}(k)\; X_{i,j}(k-1)\; \ldots\; X_{i,j}(k-N)]^T$.

The coefficients of the subband adaptive filters $\mathbf{G}_i(k)$ are updated such that the *a posteriori* error vector is null for every subband, or equivalently,

$$\mathbf{D}_i(k) = \begin{cases} \boldsymbol{\mathcal{X}}_{0,0}^T(k)\mathbf{G}_0(k) \\ \quad + \boldsymbol{\mathcal{X}}_{0,1}^T(k)\mathbf{G}_1(k), \; i=0 \\ \boldsymbol{\mathcal{X}}_{i-1,i}^T(k)\mathbf{G}_{i-1}(k) + \boldsymbol{\mathcal{X}}_{i,i}^T(k)\mathbf{G}_i(k) \\ \quad + \boldsymbol{\mathcal{X}}_{i,i+1}^T(k)\mathbf{G}_{i+1}(k), \; 0 < i < M-1 \\ \boldsymbol{\mathcal{X}}_{\text{M-2,M-1}}^T(k)\mathbf{G}_{\text{M-2}}(k) \\ \quad + \boldsymbol{\mathcal{X}}_{\text{M-1,M-1}}^T(k)\mathbf{G}_{\text{M-1}}(k), \; i = M-1 \end{cases} \tag{15}$$

Substituting (13) and (15) in (10), and defining $\Delta\mathbf{G}_i(k) = \mathbf{G}_i(k) - \mathbf{G}_i(k-1)$, we obtain the following system of ML equations:

$$\begin{aligned} &\boldsymbol{\mathcal{X}}_{0,0}^T(k)\Delta\mathbf{G}_0(k) + \boldsymbol{\mathcal{X}}_{0,1}^T(k)\Delta\mathbf{G}_1(k) = \mathbf{E}_0(k) \\ &\boldsymbol{\mathcal{X}}_{i-1,i}^T(k)\Delta\mathbf{G}_{i-1}(k) + \boldsymbol{\mathcal{X}}_{i,i}^T(k)\Delta\mathbf{G}_i(k) \\ &\quad + \boldsymbol{\mathcal{X}}_{i,i+1}^T(k)\Delta\mathbf{G}_{i+1}(k) = \mathbf{E}_i(k), 0 < i < M-1 \\ &\boldsymbol{\mathcal{X}}_{\text{M-2,M-1}}^T(k)\Delta\mathbf{G}_{\text{M-2}}(k) + \boldsymbol{\mathcal{X}}_{\text{M-1,M-1}}^T(k)\Delta\mathbf{G}_{\text{M-1}}(k) = \mathbf{E}_{\text{M-1}}(k) \end{aligned} \tag{16}$$

which can be expressed as

$$\boldsymbol{\mathcal{X}}^T(k)\Delta\mathbf{G}(k) = \mathbf{E}(k) \tag{17}$$

where $\boldsymbol{\mathcal{X}}^T(k)$, $\Delta\mathbf{G}(k)$, and $\mathbf{E}(k)$, defined below [2], have dimensions $ML \times MK$, $MK \times 1$, and $ML \times 1$, respectively:

$$\boldsymbol{\mathcal{X}}^T(k) = \begin{bmatrix} \boldsymbol{\mathcal{X}}_{0,0}^T & \boldsymbol{\mathcal{X}}_{0,1}^T & \mathbf{0} & \cdots & \mathbf{0} \\ \boldsymbol{\mathcal{X}}_{0,1}^T & \boldsymbol{\mathcal{X}}_{1,1}^T & \boldsymbol{\mathcal{X}}_{1,2}^T & \ddots & \vdots \\ \mathbf{0} & \boldsymbol{\mathcal{X}}_{1,2}^T & \boldsymbol{\mathcal{X}}_{2,2}^T & \ddots & \mathbf{0} \\ \vdots & \ddots & \ddots & \ddots & \boldsymbol{\mathcal{X}}_{\text{M-2,M-1}}^T \\ \mathbf{0} & \cdots & \mathbf{0} & \boldsymbol{\mathcal{X}}_{\text{M-2,M-1}}^T & \boldsymbol{\mathcal{X}}_{\text{M-1,M-1}}^T \end{bmatrix} \tag{18}$$

$$\Delta\mathbf{G}(k) = \mathbf{G}(k) - \mathbf{G}(k-1) \tag{19}$$

$$\mathbf{E}(k) = [\mathbf{E}_0^T(k)\; \mathbf{E}_1^T(k)\; \cdots\; \mathbf{E}_{\text{M-1}}^T(k)]^T \tag{20}$$

with

$$\mathbf{G}(k) = [\mathbf{G}_0^T(k)\; \mathbf{G}_1^T(k)\; \cdots\; \mathbf{G}_{\text{M-1}}^T(k)]^T \tag{21}$$

Finally, we express the weight updating equation for the subband APA as

$$\mathbf{G}(k) = \mathbf{G}(k-1) + \mu\boldsymbol{\mathcal{X}}(k)[\boldsymbol{\mathcal{X}}^T(k)\boldsymbol{\mathcal{X}}(k)]^{-1}\mathbf{E}(k) \tag{22}$$

In an attempt to achieve both fast convergence and low misadjustment using this new algorithm, we introduce the concept of set-membership filtering also as a mean to accomplish an average reduction of the computational complexity. Using the trivial choice for vector $\mathbf{g}(k)$, from (9), and from (22), we write the updating equation of the coefficient vector as

$$\mathbf{G}(k) = \begin{cases} \mathbf{G}(k-1) + \\ \boldsymbol{\mathcal{X}}(k)[\boldsymbol{\mathcal{X}}^T(k)\boldsymbol{\mathcal{X}}(k)]^{-1}\mathbf{E}(k), |\mathbf{E}(k)| > \gamma \\ \mathbf{G}(k-1), \text{ otherwise} \end{cases} \tag{23}$$

The parameter γ is application dependent but we can suggest $1/M$ times the value used in the full band case [4], yielding $\gamma = \sqrt{5\sigma_n^2}/M$, where σ_n^2 corresponds to the variance of the observation error.

5. SIMULATION RESULTS

The identification of a length $N = 256$ FIR system is considered. Experiments were performed with the subband structure of Fig. 2 with $M = 2, 4, 8$ and 16 subbands and perfect-reconstruction cossine modulated filter banks with prototype filters of lengths $N_h = 32, 64, 128$ and 256, respectively. The input signal was a colored noise sequence generated by passing a gaussian white noise sequence through the IIR filter: $H_c(z) = 1/(1+0.999z^{-1}+0.99z^{-2}+.995z^{-3}+0.99z^{-4})$.

Figure 3 presents the MSE evolution for the subband algorithm with $M = 8$ subbands and different numbers of projections L. As expected, as L increases, the convergence

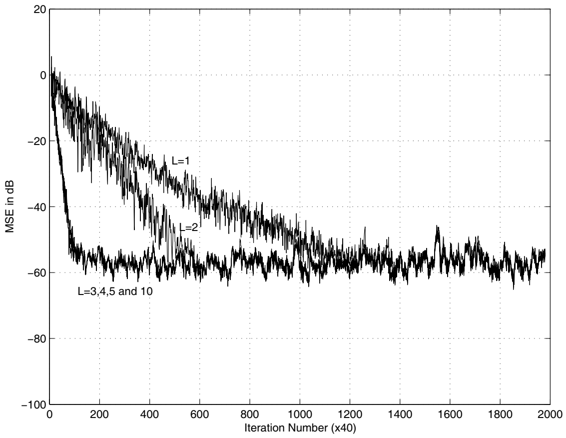

Fig. 3. Subband APA with $M = 8$ and different number of projections L.

rate becomes faster. However, for L larger than 3, there is no noticeable alteration in the convergence rate.

Figure 4 presents the MSE evolution of the subband and fullband APA for $L = 2$ projections, varying the number of subbands M. From this plot, we observe that, for small L, the subband algorithm has a better convergence rate than the fullband algorithm, and that the convergence rate increases with the number of subbands. For $M > 2$, the subband structure converges to an MSE of the order of the stopband attenuation of the analysis filter (which is around -58 dB for the prototype filters used in the simulations).

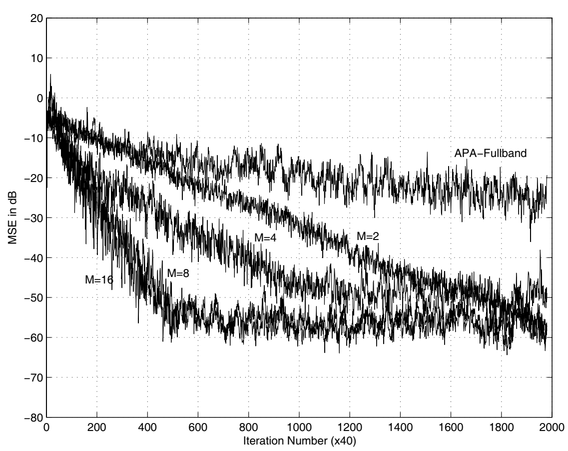

Fig. 4. Subband APA with $L = 2$ and different number of subbands M.

Figure 5 presents the MSE evoltuion of the subband and fullband APA for $L = 3$. In this case, the convergence rate of the subband APA is still better than that of the fullband APA, and it does not change as the number of subbands increases.

[2]The index k of the matrices $\boldsymbol{\mathcal{X}}_{k,l}(k)$ was ommited in the defi nition of $\boldsymbol{\mathcal{X}}^T(k)$ in order to simplify the notation.

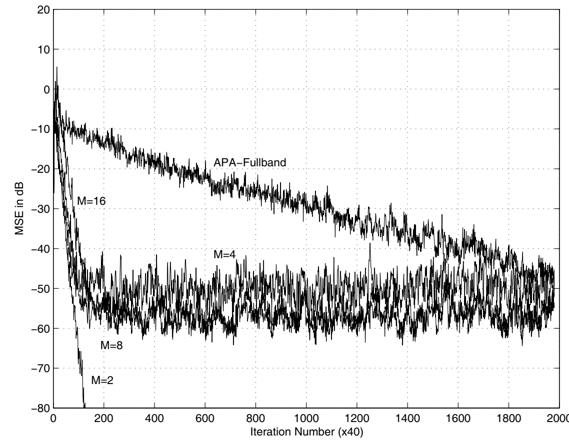

Fig. 5. Subband APA with $L = 3$ and different number of subbands M.

6. CONCLUSIONS

We have derived a new APA and set-membership APA for a subband structure with critical sampling. Experimental results have shown that the convergence rate is improved when compared to the fullband affine projection algorithm with small number of projections when the input signal is highly correlated.

7. REFERENCES

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