A SET-MEMBERSHIP APPROACH TO NORMALIZED PROPORTIONATE ADAPTATION ALGORITHMS

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ABSTRACT

Proportionate adaptive filters can improve the convergence speed for the identification of sparse systems as compared to their conventional counterparts. In this paper, the idea of proportionate adaptation is combined with the framework of set-membership filtering (SMF) in an attempt to derive novel computationally efficient algorithms. The resulting algorithms attain an attractive faster converge for both situations of sparse and dispersive channels while decreasing the computational complexity due to the data discerning feature of the SMF approach. Simulations show good results in terms of reduced number of updates, speed of convergence, and final mean-squared error.

1. INTRODUCTION

Frequently used adaptive filtering algorithms like the least mean square (LMS) and the normalized LMS (NLMS) algorithms share the features of low computational complexity and proven robustness. The LMS and the NLMS algorithms have in common that the adaptive filter is updated in the direction of the input vector without favoring any particular direction. In other words, they are well suited for *dispersive*-type systems where the energy is uniformly distributed among the coefficients in the impulse response. On the other hand, if the system to be identified is *sparse*, i.e., the impulse response is characterized by a few dominant coefficients, using different step sizes for each adaptive filter coefficient can improve the initial convergence of the NLMS algorithm. This basic concept is explored in *proportionate adaptive filters* [1, 2, 3], which incorporates the importance of the individual components by assigning weights proportional to the magnitude of the coefficients.

The conventional proportionate NLMS (PNLMS) algorithm [1] experiences fast initial adaptation for the dominant coefficients followed by a slower second transient for the remaining coefficients. Therefore, the slow convergence of the PNLMS algorithm after the initial transient can be circumvented by switching to the NLMS algorithm [4].

Another problem related to the conventional PNLMS algorithm is the poor performance in dispersive or semi-dispersive channels [2]. Refinements of the PNLMS have been proposed [2, 3] to improve performance in a dispersive medium and to combat the slowdown after the initial adaptation. The PNLMS++ algorithm in [2] approaches the problem by alternating the NLMS update with a PNLMS update. The improved PNLMS (IPNLMS) algorithm [3] combines the NLMS and PNLMS algorithms into one single updating expression. The main idea of the IPNLMS algorithm was to establish a rule for automatical switching from one algorithm to the other. Extension of the proportionate adaptation concept to affine projection (AP) type algorithms, proportionate affine projection (PAP) algorithms, can be found in [5, 6].

Using the PNLMS algorithm instead of the NLMS algorithm leads to a 50% increase of the computational complexity. An efficient approach to reduce computations is to employ set-membership filtering (SMF) techniques [7, 8], where the filter is designed such that the output estimation error is upper bounded by a pre-determined threshold.[1] Set-membership adaptive filters (SMAF) feature data-selective (sparse in time) updating, and a time-varying data-dependent step size that provides fast convergence to a low steady-state error. In the following we combine the frameworks of proportionate adaptation and SMF. A set-membership proportionate NLMS (SM-PNLMS) algorithm is proposed as a viable alternative to the SM-NLMS algorithm [7] for operation in sparse scenarios. Following the ideas of the IPNLMS algorithm, an efficient weight-scaling assignment is proposed that utilizes the information provided by the data-dependent step size. Thereafter, we propose a more general algorithm, the set-membership proportionate affine projection algorithm (SM-PAPA) that reuses past input-desired data pairs in the same way as the SM-AP algorithm [10]. Finally, the performances of the proposed algorithms are evaluated through simulations which are followed by conclusions.

2. SMF

Set-membership filtering is a framework applicable to filtering problems that are linear in parameters.[2] A specification on the filter parameters $\mathbf{w} \in \mathbb{C}^N$ is achieved by constraining the magnitude of the output estimation error, $e = d_k - \mathbf{w}^H \mathbf{x}_k$, to be smaller than a deterministic threshold γ, where $\mathbf{x}_k \in \mathbb{C}^N$ and $d_k \in \mathbb{C}$ denote the input vector and the desired output signal, respectively. As a result of the bounded error constraint, there will exist a set of filters rather than a single estimate.

Adaptive SMF algorithms seek solutions that belong to the *exact membership set* ψ_k constructed from the observed input-signal and desired signal pairs,

$$\psi_k = \bigcap_{i=1}^{k} \mathscr{H}_i \tag{1}$$

where $\mathscr{H}_k$ is referred to as the *constraint set* containing all the vectors $\mathbf{w}$ for which the associated output error at time instant k is upper bounded in magnitude by γ:

$$\mathscr{H}_k = \{\mathbf{w} \in \mathbb{C}^N : |d_k - \mathbf{w}^H \mathbf{x}_k| \leq \gamma\} \tag{2}$$

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[1] For other reduced-complexity solutions, see, e.g., [4] where the concept of partial updating is applied.

[2] This includes nonlinear problems like Volterra modelling, see, e.g., [9].

Adaptive approaches leading to algorithms with low peak complexity compute a point estimate through projections using information provided by past constraint sets [7, 8, 10].

3. SM-PNLMS

In this section the idea of proportionate adaptation is applied to SMF in order to derive a data-selective algorithm, the SM-PNLMS, suitable for sparse environments.

3.1 Algorithm derivation

The SM-PNLMS algorithm uses the information provided by the constraint set $\mathcal{H}_k$ and the coefficient updating to solve the optimization problem employing the criterion

$$\mathbf{w}_{k+1} = \arg\min_{\mathbf{w}} \|\mathbf{w} - \mathbf{w}_k\|^2_{\mathbf{G}_k^{-1}} \text{ subjected to:}$$
$$\mathbf{w} \in \mathcal{H}_k \tag{3}$$

where $\mathbf{G}_k$ is here chosen as a diagonal weighting matrix of the form:

$$\mathbf{G}_k = \text{diag}\{g_{1,k}, \cdots, g_{N,k}\} \tag{4}$$

The elements values of $\mathbf{G}_k$ will be discussed Subsection 3.2. The optimization criterion in Eq. (3) states that if the previous estimate already belongs to the constraint set, $\mathbf{w}_k \in \mathcal{H}_k$, it is a feasible solution and no update is needed. However, if $\mathbf{w}_k \notin \mathcal{H}_k$, an update is required. Following the principle of *minimal disturbance*, a feasible update made such that it ends up on the nearest boundary of $\mathcal{H}_k$ is given by

$$\mathbf{w}_{k+1} = \mathbf{w}_k + \alpha_k \frac{e_k^* \mathbf{G}_k \mathbf{x}_k}{\mathbf{x}_k^H \mathbf{G}_k \mathbf{x}_k} \tag{5}$$

where

$$\alpha_k = \begin{cases} 1 - \frac{\gamma}{|e_k|}, & \text{if } |e_k| > \gamma \\ 0, & \text{otherwise} \end{cases} \tag{6}$$

is a time-varying data-dependent step-size, and $e(k)$ is the *a priori* error given by

$$e_k = d_k - \mathbf{w}_k^H \mathbf{x}_k. \tag{7}$$

For the proportionate algorithms considered in this paper, matrix $\mathbf{G}_k$ will be diagonal. However, with other choices of $\mathbf{G}_k$, we can identify from Eq. (5) different types of SMAFs available in literature. For example, choosing $\mathbf{G}_k = \mathbf{I}$ gives the SM-NLMS algorithm [7], setting $\mathbf{G}_k$ equal to a weighted covariance matrix will result in the BEACON recursions [11], and choosing $\mathbf{G}_k$ such that it extracts the $P \leq N$ elements in $\mathbf{x}_k$ of largest magnitude gives a partial-updating SMF [12]. Next we consider the weighting matrix used with the SM-PNLMS algorithm.

3.2 Choice of weighting matrix $\mathbf{G}_k$

This section proposes a weighting matrix $\mathbf{G}_k$ suitable for operation in sparse environments.

Following the same line of thought as in the IPNLMS algorithm, the diagonal elements of $\mathbf{G}_k$ are computed to provide a good balance between the SM-NLMS algorithm and a solution for sparse systems. The goal is to reduce the length of the initial transient for estimating the dominant peaks in the impulse response and, thereafter, to emphasize the input-signal direction to avoid a slow second transient. Furthermore, the solution should not be sensitive to the assumption of a sparse system. From the expression for α_k in Eq. (6) we see that if the solution is far from the constraint set we have $\alpha_k \to 1$, while close to the steady-state $\alpha_k \to 0$. Therefore, a suitable weight assignment rule emphasizes dominant peaks when $\alpha_k \to 1$ and the input-signal direction (SM-PNLMS update) when $\alpha_k \to 0$. As α_k is a good indicator of how close a steady-state solution is, we propose to use

$$g_{i,k} = \frac{1 - \kappa\alpha_k}{N} + \frac{\kappa\alpha_k |w_{i,k}|}{\|\mathbf{w}_k\|_1} \tag{8}$$

where $\kappa \in [0, 1]$ and $\|\mathbf{w}_k\|_1 = \sum_i |w_{i,k}|$ denotes the l_1 norm with $\sum_i |w_{i,k}| = 1$ as in [1, 3].. The constant κ is included to increase the robustness for estimation errors in $\mathbf{w}_k$, and from the simulations provided in Section 5, $\kappa = 0.5$ shows good performance for both sparse and dispersive systems. For $\kappa = 1$, the algorithm will converge faster but will be more sensitive to the sparse assumption. The IPNLMS algorithm uses a similar strategy, see [3] for details. The updating expressions in Eqs. (8) and (5) resembles those of the IPNLMS algorithm except for the time-varying step size α_k. From Eq. (8) we can observe: 1) during initial adaptation (during the transient) the solution is far from the steady-state solution, and consequently α_k is large, and more weight will be put on the strongest components of the adaptive filter's impulse response; 2) as the error decreases, α_k gets smaller, all the coefficients become equally important, and the algorithm behaves the SM-NLMS algorithm.

4. SM-PAPA

In this section we extend the results from the previous section to derive an algorithm that utilizes the L most recent constraint sets $\{\mathcal{H}_i\}_{i=k-L+1}^k$. The algorithm, SM-PAPA, includes the SM-AP algorithm [10] as a special case and is suitable whenever the input signal is highly correlated. As with the SM-PNLMS algorithm, the main idea is to allocate different weights to the filter coefficients using a weighting matrix $\mathbf{G}_k$. The SM-PAPA is derived so that its coefficient vector after updating belongs to the set ψ_k^L corresponding to the intersection of L past constraint sets, i.e.,

$$\psi_k^L = \bigcap_{i=k-L+1}^k \mathcal{H}_i \tag{9}$$

If the previous estimate belongs to the L past constraint sets, i.e., $\mathbf{w}_k \in \psi_k^L$ no coefficient update is required. Otherwise, the SM-PAPA performs an update according to the following optimization criterion

$$\mathbf{w}_{k+1} = \arg\min_{\mathbf{w}} \|\mathbf{w} - \mathbf{w}_k\|^2_{\mathbf{G}_k^{-1}} \text{ subjected to:}$$
$$\mathbf{d}_k - \mathbf{X}_k^T \mathbf{w}^* = \mathbf{p}_k \tag{10}$$

where vector $\mathbf{d}_k \in \mathbb{C}^L$ contains the desired outputs from the L last time instants, vector $\mathbf{p}_k \in \mathbb{C}^L$ has components that obey $|p_{i,k}| < \gamma$ and so specify the point in ψ_k^L, and matrix $\mathbf{X}_k \in \mathbb{C}^{N \times L}$ contains the corresponding input vectors, i.e.,

$$\mathbf{p}_k = [p_{1,k}\, p_{2,k} \cdots p_{L,k}]^T$$
$$\mathbf{d}_k = [d_k\, d_{k-1} \ldots d_{k-L+1}]^T \tag{11}$$
$$\mathbf{X}_k = [\mathbf{x}_k\, \mathbf{x}_{k-1} \cdots \mathbf{x}_{k-L+1}]$$

Applying the method of Lagrange multipliers for solving the minimization problem of Eq. (10), the update equation of the most general SM-PAPA version is obtained as

$$\mathbf{w}_{k+1} = \begin{cases} \mathbf{w}_k + \mathbf{G}_k \mathbf{X}_k [\mathbf{X}_k^H \mathbf{G}_k \mathbf{X}_k]^{-1} [\mathbf{e}_k^* - \mathbf{g}_k^*], & \text{if } |e_k| > \gamma \\ \mathbf{w}_k & \text{otherwise} \end{cases} \tag{12}$$

The choice of $p_i(k)$ can vary for different problems. Following the ideas of [10], a particulary simple SM-PAPA version is obtained if $p_{i,k}$ for $i \neq 1$ corresponds to the *a posteriori* error $d_{k-i+1} - \mathbf{w}_k^H \mathbf{x}_{k-i+1}$ and $p_{1,k} = e_k/|e_k|$. The simplified SM-PAPA version has the recursions given by

$$\mathbf{w}_{k+1} = \mathbf{w}_k + \mathbf{G}_k \mathbf{X}_k [\mathbf{X}_k^H \mathbf{G}_k \mathbf{X}_k]^{-1} \alpha_k e_k^* \mathbf{u}_1 \tag{13}$$

where $\mathbf{u}_1 = [1\, 0 \cdots 0]^T$ and α_k is given by Eq. (6).

Table 1: The Set-Membership Proportionate Affine-Projection Algorithm.

SM-PAPA
for each k
{
$e_k = d_k - \mathbf{x}_k^{\mathrm{H}} \mathbf{w}_k$
if $\lvert e_k \rvert > \gamma$
{
$\alpha_k = 1 - \gamma / \lvert e(k) \rvert$
$g_{i,k} = \frac{1 - \kappa \alpha_k}{N} + \frac{\kappa \alpha_k \lvert w_{i,k} \rvert}{\sum_{i=1}^{N} \lvert w_{i,k} \rvert}, \ i = 1, \dots, N$
$\mathbf{G}_k = \mathtt{diag}[g_{1,k} \cdots g_{N,k}]$
$\mathbf{X}_k = [\mathbf{x}_k \ \mathbf{U}_k]$
$\phi_k = \mathbf{x}_k - \mathbf{U}_k (\mathbf{U}_k^{\mathrm{H}} \mathbf{G}_k \mathbf{U}_k)^{-1} \mathbf{U}_k^{\mathrm{H}} \mathbf{G}_k \mathbf{x}_k$
$\mathbf{w}_{k+1} = \mathbf{w}_k + \alpha_k e_k^* \frac{1}{\phi_k^{\mathrm{H}} \mathbf{G}_k \phi_k} \mathbf{G}_k \phi_k$
}
else
{
$\mathbf{w}_{k+1} = \mathbf{w}_k$
}
}

Figure 1: The amplitude of two impulse responses used in the simulations: a sparse microwave channel[4] (top), and a dispersive (bottom).

Due to the special solution involving $\mathbf{u}_1$ in Eq. (13), a computationally efficient expression for the coefficient update is obtained by partitioning the input signal matrix as[3]

$$\mathbf{X}_k = [\mathbf{x}_k \ \mathbf{U}_k] \qquad (14)$$

where $\mathbf{U}_k = [\mathbf{x}_{k-1} \cdots \mathbf{x}_{k-L+1}]$. Substituting the partitioned input matrix in Eq. (13) and carrying out the multiplications, we get after some algebraic manipulations (see Appendix)

$$\mathbf{w}_{k+1} = \mathbf{w}_k + \frac{\alpha_k e_k^*}{\phi_k^{\mathrm{H}} \mathbf{G}_k \phi_k} \mathbf{G}_k \phi_k \qquad (15)$$

where vector ϕ_k is defined as

$$\phi_k = \mathbf{x}_k - \mathbf{U}_k (\mathbf{U}_k^{\mathrm{H}} \mathbf{G}_k \mathbf{U}_k)^{-1} \mathbf{U}_k^{\mathrm{H}} \mathbf{G}_k \mathbf{x}_k \qquad (16)$$

This representation of the SM-PAPA is computationally attractive as the dimension of the matrix to be inverted is reduced from $L \times L$ to $(L-1) \times (L-1)$. As with the SM-PNLMS algorithms, $\mathbf{G}_k$ is a diagonal matrix whose elements are computed according to (8). Table 1 shows the recursions for the SM-PAPA.

5. SIMULATION RESULTS

In this section, the performances of the SM-PNLMS algorithm and SM-PAPA are evaluated in a system identification experiment. The performance of the NLMS, the IPNLMS, the SM-NLMS, and the SM-AP algorithms are also compared.

The first experiment was carried out with an unknown plant with sparse impulse response that consisted of an $N = 50$ truncated FIR model from a digital microwave radio channel.[4] Thereafter, the algorithms were tested for a dispersive channel, where the plant was a complex FIR filter whose coefficients were generated randomly. Figure 1 depicts the absolute values of the channel impulse responses used in the simulations. For the simulation experiments, we have used the following parameters: $\mu = 0.4$ for the NLMS and the PAP algorithms, $\gamma = \sqrt{2\sigma_n^2}$ for all SMAFs, $\kappa = 0.5$ for all

proportionate algorithms. Note that for the IPNLMS and the PAP algorithms $g_{i,k} = \frac{1-\kappa}{N} + \frac{\kappa \lvert w_{i,k} \rvert}{\|\mathbf{w}_k\|}$ correspond to the same updating as in [3] when $\kappa \in [0, 1]$. The parameters were set in order to have fair comparison in terms of the final steady-state error. The input signal $x(k)$ was a complex-valued noise sequence, colored by filtering a zero-mean white complex-valued Gaussian noise sequence $n_x(k)$ through the fourth-order IIR filter $x(k) = n_x(k) + 0.6617x(k-1) + 0.3402x(k-2) + 0.5235x(k-3) - 0.8703x(k-4)$, and the *SNR* was set to 40dB.

The learning curves shown in Figures 2–3 are the result of 500 independent runs and smoothed by a low pass filter. From the learning curves in Figure 2 for the sparse system, it can be seen that the SMF algorithms converge slightly faster than their conventional counterparts to the same level of MSE. In addition to the faster convergence, the SMF algorithms will have a reduced number of updates. In 20000 iterations, the number of times an update took place for the SM-PNLMS, the SM-PAPA, and the SM-AP algorithms were 7730 (39%), 6000 (30%), and 6330 (32%), respectively. This should be compared with 20000 updates required by the IPNLMS and PAP algorithms. From Figure 2, we also see that the proportionate SMF algorithms converges faster than those without proportionate adaptation.

Figure 3 shows the learning curves for the dispersive channel identification, where it can be observed that the performance of the SM-PNLMS algorithm and SM-PAPA is very close to the SM-AP and SM-NLMS algorithms, respectively. In other words, the SM-PNLMS algorithm and the SM-PAPA are not sensitive to the assumption of having a sparse impulse response. In 20000 iterations, the SM-PAPA and the SM-PNLMS algorithm updated 32% and 50%, respectively, while the SM-AP and SM-NLMS algorithms updated 32% and 49%, respectively.

6. CONCLUSIONS

This paper presented novel set-membership filtering (SMF) algorithms suitable for applications in sparse environments. The set-membership proportionate NLMS (SM-PNLMS) algorithm and the set-membership proportionate affine projection algorithm (SM-PAPA) are proposed as viable alternatives to the SM-NLMS and SM-AP algorithms. The algorithms benefit from the reduced average computational complexity from the SMF strategy and fast convergence for sparse scenarios resulting from proportionate updating. Simulations were presented for both sparse and dispersive

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[3]The same approach can be used to reduce the complexity of the Ozeki Umeda's AP algorithm for the case of unit step size [13].

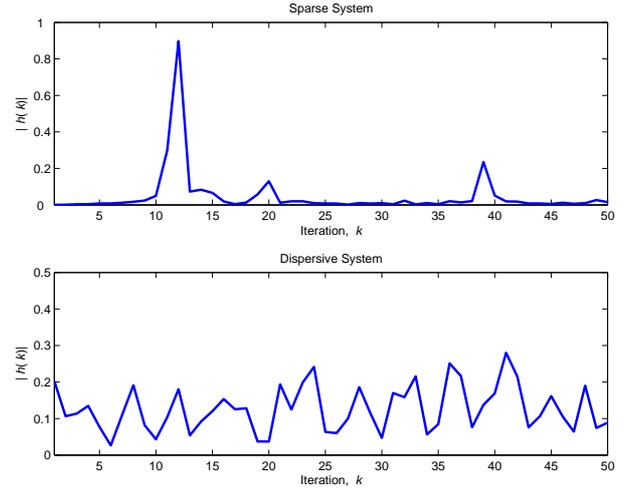[4]The coefficients of this complex-valued baseband channel model can be downloaded from http://spib.rice.edu/spib/microwave.html
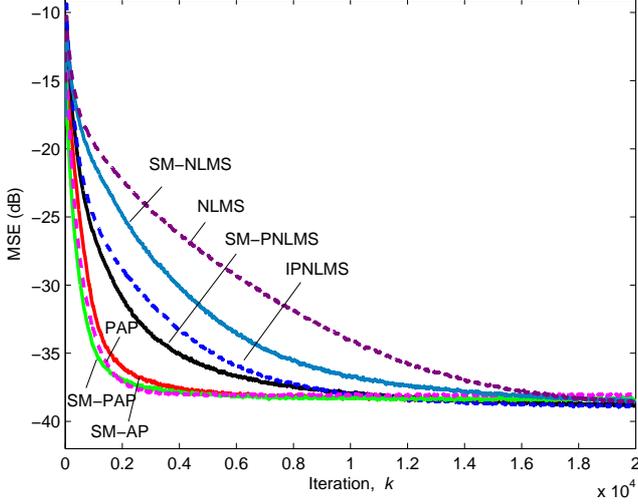
Figure 2: Learning curves in a **sparse system** for the SM-PNLMS, the SM-PAPA ($L = 2$), the SM-NLMS, the NLMS, the IPNLMS, the PAP ($L = 2$) algorithms. $SNR = 40$ dB, $\gamma = \sqrt{2}\sigma_n$, and $\mu = 0.4$.
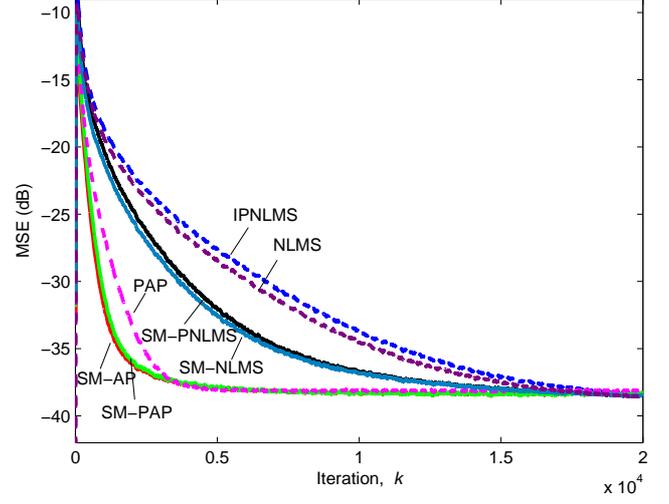

Figure 3: Learning curves in a **dispersive system** for the SM-PNLMS, the SM-PAPA ($L = 2$), the SM-NLMS, the NLMS, the IPNLMS, the PAP ($L = 2$) algorithms. $SNR = 40$ dB, $\gamma = \sqrt{2}\sigma_n$, and $\mu = 0.4$.

impulse responses. It was verified that not only the proposed SMF algorithms can further reduce the computational complexity when compared with their conventional counterparts, the IPNLMS and PAP algorithms, but also present a faster convergence to the same level of MSE when compared with the SM-NLMS and the SM-AP algorithms. The weight assignment of the proposed algorithms utilize the information provided by a time-varying step size typical for SMF algorithms and is robust to the assumption of sparse impulse response.

7. APPENDIX

The inverse in Eq. (13) can be partitioned as

$$\left(\mathbf{X}_k^H \mathbf{G}_k \mathbf{X}_k \right)^{-1} = \left(\left[\mathbf{x}_k \ \mathbf{U}_k \right]^H \mathbf{G}_k \left[\mathbf{x}_k \ \mathbf{U}_k \right] \right)^{-1} = \begin{bmatrix} a & \mathbf{b}^H \\ \mathbf{b} & \mathbf{C} \end{bmatrix} \quad (17)$$

where

$$a = \frac{1}{\phi_k^H \mathbf{G}_k \mathbf{x}_k} = \frac{1}{\phi_k^H \mathbf{G}_k \phi_k}$$

$$\mathbf{b} = -a \left(\mathbf{U}_k^H \mathbf{G}_k \mathbf{U}_k \right)^{-1} \mathbf{U}_k^H \mathbf{G}_k \mathbf{x}_k \quad (18)$$

with ϕ_k defined as in Eq. (16). Therefore,

$$\begin{aligned}
\mathbf{G}_k \mathbf{X}_k \left(\mathbf{X}_k^H \mathbf{G}_k \mathbf{X}_k \right)^{-1} \mathbf{u}_1 &= \mathbf{G}_k \mathbf{X}_k \begin{bmatrix} a \\ \mathbf{b} \end{bmatrix} \\
&= \frac{\mathbf{G}_k \mathbf{x}_k - \mathbf{G}_k \left(\mathbf{U}_k^H \mathbf{G}_k \mathbf{U}_k \right)^{-1} \mathbf{U}_k^H \mathbf{G}_k \mathbf{x}_k}{\phi_k^H \mathbf{G}_k \phi_k} \\
&= \frac{\mathbf{G}_k \phi_k}{\phi_k^H \mathbf{G}_k \phi_k}.
\end{aligned} \quad (19)$$

REFERENCES

[1] D. L. Duttweiler, "Proportionate normalized least mean square adaptation in echo cancelers," *IEEE Trans. Speech Audio Processing*, vol. 8, pp. 508–518, Sept. 2000.

[2] S. Gay, "An efficient, fast converging adaptive filter for network echo cancellation," *Proc. Asilomar Conference on Signals, Systems, and Computers*, USA, 1998.

[3] J. Benesty, S. Gay, "An improved PNLMS algorithm," *ICASSP'2002*, Orlando, USA, 2002.

[4] K. Doğançay and O. Tanrıkulu, "Selective-partial-update normalized least-mean-square algorithm for network echo cancellation," *ICASSP'2002*, Orlando, USA, 2002.

[5] J. Benesty, T. Gänsler, D.R. Morgan, M.M. Sondhi, S. Gay, *Advances in Network and Acoustic Echo Cancellation*, New York: Springer Verlag, 2001.

[6] O. Hoshuyama, R. A. Goubran, A. Sugiyama, "A generalized proportionate variable step-size algorithm for fast changing acoustic enviroments," *ICASSP'2004*, Montreal, Canada, 2004.

[7] S. Gollamudi, S. Nagaraj, S. Kapoor, and Y.-F. Huang, "Set-membership filtering and a set-membership normalized LMS algorithm with an adaptive step size," *IEEE Signal Processing Lett.*, vol. 5, pp. 111-114, May 1998.

[8] P. S. R. Diniz and S. Werner, "Set-membership binormalized LMS data-reusing algorithms," *IEEE Trans. Signal Processing*, vol. 51, Jan. 2003.

[9] A. V. Malipatil, Y. F. Huang, S. Andra, and K. Bennett, "A kernalized set-membership approach to nonlinear adaptive filtering," *ICASSP'2005*, Philadelphia, PA, USA, 2005.

[10] S. Werner and P. S. R. Diniz, "Set-membership affine projection algorithm," *IEEE Signal Processing Lett.*, vol. 8, pp. 231–235, Aug. 2001.

[11] S. Nagaraj, S. Gollamudi, S. Kapoor, and Y. F. Huang, "BEACON: An adaptive set-membership filtering technique with sparse updates," *IEEE Trans. Signal Processing*, vol. 47, no. 11, pp. 2928–2941, November 1999.

[12] S. Werner, M. L. R. de Campos, and P. S. R. Diniz, "Partial-update NLMS algorithms with data-selective updating," *IEEE Trans. Signal Processing*, vol. 52, pp. 938–949, Apr. 2004.

[13] M. Rupp, "A family of adaptive filtering algorithms with decorrelating properties," *IEEE Trans. Signal Processing*, vol. 46, no. 3, pp. 771–775, March 1998.